Are rogue waves beyond conventional predictions ?

M. Olagnon and M. Prevosto

Ifremer, Centre de Brest, France

Abstract. There is no doubt that waves of unexpected severity happen at sea. Many authors have developed models that would explain the supposed abnormally high numbers of such occurrences, most of them based on a Benjamin-Feir instability related to the characteristics of the sea states where they were observed. Those models are as realistic as conventional ones, since they provide waves very similar to the few measurements of extreme waves that could be made. However, the question remains of how to verify that some of the extreme waves observed in nature do derive from those models and from the hypotheses that they are based on. First, it is necessary to verify that a number of high waves would not be explained by the common models, and then that the assumptions used in the non-conventional models are met on numbers of cases of the same magnitude as those that fail to be explained. In the present study, we investigate on a North Sea database the distribution tail of extreme wave heights, in order to quantify the possible differences between observed occurrence frequencies and those predicted using common second-order estimations. On the base of the instability models and of the conditions required for them to apply, we examine whether it is likely that those models should be called to explain the possible differences between observed and commonly predicted numbers of extremes. We take special account of the aleatory variability of characteristics within a stationary sea state and of the time-scales implied by the phenomena. This paper does not in any way deny that in some complex specific metocean situations, very unusual waves or series thereof may occur, but we conclude that such circumstances are not frequent enough to influence the long-term statistics of extreme waves.

1. Introduction

The occurrence of waves of abnormal height or severity raises two main issues: are the design values used for ships and offshore structures in accordance with the safety expectations, and, independently, can methods be devised to predict increases in the probabilities of occurrence and to send warnings in such cases. Both these questions lead to study whether common models for long term-statistics of ocean waves correctly estimate the probablities of occurrence of high waves. Especially, if in such a preliminary step outliers from the conventional distributions, “rogue” or “freak” waves, can be identified, then it would be easier to test the assumptions of the various generation models against them, and to relate them to some parameters at the time scale of the sea state or of the storm that could lead to warning possibilities.

Surprisingly, many authors study the occurrence of rogue waves in the perspective of short- or medium-term statistics, and very few of them examine their long-term statistics. *Robin and Olagnon* [1991] from the point of view of observations, and *Nerzic and Prevosto* [1998] from the point of view of statistical models suggest that extreme wave occurrences would not depart from the proportions predicted by conventional methods. We investigate and confirm this assumption here through the analysis of a larger database from the same Frigg field in the North Sea and consideration of whole storms rather than only short-duration records.

Adherence of occurrences to the predicted long-term distribution does not imply that extreme waves offer no surprise with respect to the individual properties of the neighboring ones, nor on the opposite, that they are unrelated to the characteristics of the sea state or of the storm when they happen. However, it should be noted that many characteristics of a sea state are affected when an extreme wave is included in it from that mere presence, and that the change may thus be useless for risk forecasting if it is only when the wave occurs that the characteristics vary. We study changes in the charac-

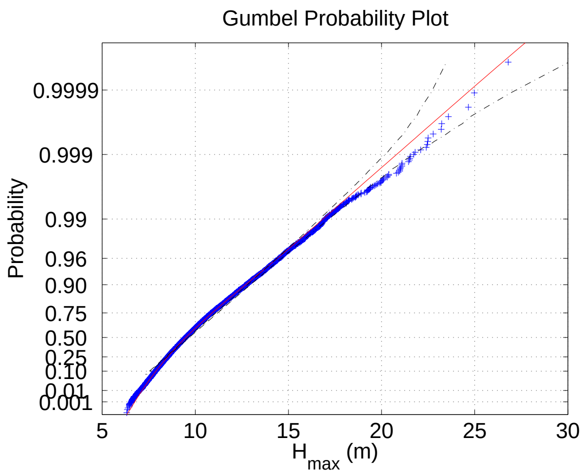

Figure 1. Empirical distribution of maximum wave heights H_{max} of sea states with $H_{1/3} > 5$ compared to the predicted one using the model of *Nerzic and Prevosto* [1998] with 95% confidence interval as a dotted line.

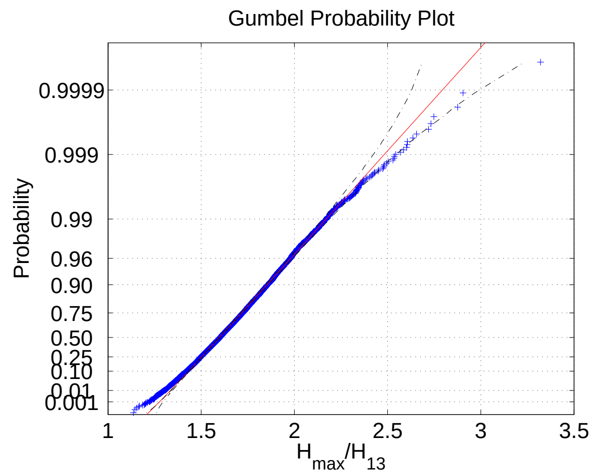

Figure 2. Empirical distribution of normalized maximum wave heights $H_{max}/H_{1/3}$ of sea states with $H_{1/3} > 5$ compared to the predicted one using the model of *Nerzic and Prevosto* [1998] with 95% confidence interval as a dotted line.

teristics of the sea states and storms when extreme waves can be found in the database, and we try to distinguish between potential causes and consequences of the extreme wave occurrence.

2. Database

The database used in *Robin and Olagnon* [1991] and complemented as reported in *Olagnon and Magnusson* [2004] was further supplemented with statistical parameters when data had not been recorded and during the period 1991 to 1999. It can thus be split into two levels of detail.

At the first level of detail about 18800 17-minute records of time-histories of the water surface elevation are available with a 2 Hz sampling frequency, as measured with a radar distance meter from the QP platform on the Frigg field and, when the corresponding sensors are operating, wind speed and direction and current speed and direction time-histories are also available with the same sampling frequency.

For the second level of detail, only some statistical parameters are available. Most of the time, significant wave height ($H_{1/3}$), maximum wave height (H_{max}) and zero-crossing peak period (T_Z) at least are available. Unfortunately, some parameters of interest such as maximum crest height, or spectral width, were neither computed nor archived. Depending on the time period, the computed parameters may be available every 3 hours, every hour, or every 20 minutes. On some occasions, no data are available.

The measurement system was changed during the period 1989-1990, and the new downlooking radar was decommissioned at the end of 1999.

A global database of H_{max}, $H_{1/3}$, and T_Z was built in the following manner:

- When time-series of the water surface elevation were available (7% of the final base), the parameters were computed from those time-series.
- Otherwise, when at least both H_{max} and $H_{1/3}$ were available in the second level database, they were used.
- Incomplete data were discarded.

In addition, for records belonging to storms identified as explained in section 5 (3.2% of the overall database), it was ensured that periods would be available for all records in the following manner:

- If the zero-crossing period T_Z was missing from the database, but significant period $T_S = T_{H_{1/3}}$ was available (9.4% of the storms time), T_Z was computed from the latter using the regression formula derived from the analysis of the time-histories:

$$T_Z = 0.71 T_S + 0.85$$

- If both T_Z and T_S were missing (1.7% of the storms time), a value was drawn from a simple empirical model of the joint ($H_{1/3}$, T_Z) distribution derived from the analysis of the time-histories:

$$T_Z = \sqrt{8.3 H_{1/3} + 20.5} + 0.56 \mathcal{R}(0,1)$$

where $\mathcal{R}(0,1)$ is a normalized Rayleigh distributed random value.

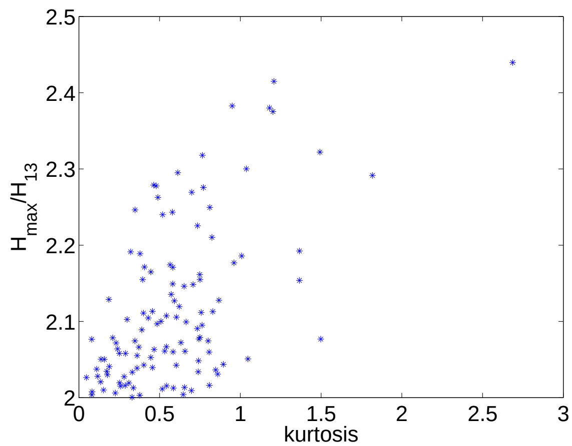

Figure 3. Normalized maximum wave height ($\frac{H_{max}}{H_{1/3}}$) as a function of sea state kurtosis.

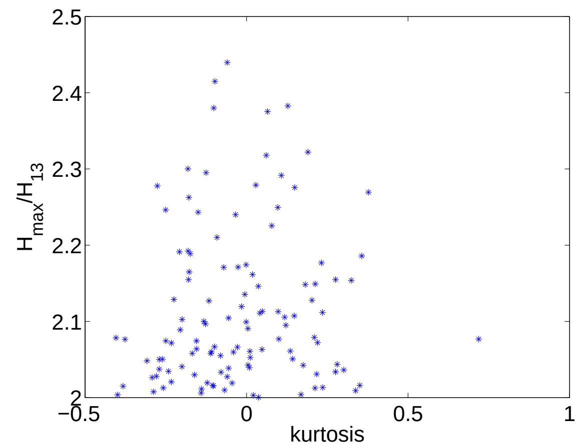

Figure 4. Normalized maximum wave height ($\frac{H_{max}}{H_{1/3}}$) as a function of sea state kurtosis computed with omission of the maximum wave

Through that procedure, it may happen that the true history of a few time-windows is poorly reproduced, but long-term statistics over the whole set of data should not depart significantly from reality. The final data consist thus of 265147 values of ($H_{1/3}$, H_{max}, T_Z, T_S) spanning the period from 1979/01/01 to 1999/12/31, with intervals ranging from 3 hours to 20 minutes and some gaps. For 18825 of those, a 17-minute record of the time-history of the water surface elevation is also available.

Using the average number of waves per record of *Robin and Olagnon* [1991], one may estimate that the total number of waves that were analyzed to derive the parameters is close to 50 million.

3. Distribution of maximum wave heights

Nerzic and Prevosto [1998] proposed a model for the distribution of the short-term maximum wave-height conditional to $H_{1/3}$ and T_Z.

The model provides the mode and scale parameters of a parametrized Gumbel law

$$G(h) = Prob(H \leq h) = exp\left(-exp\left(-\frac{h - a_{xN}}{b_{xN}}\right)\right)$$

as

$$\begin{aligned} a_{xN} &= a_N\left(1 + \frac{3}{8}(k_z a_N)^2\right) \\ b_{xN} &= b_N\left(1 + \frac{9}{8}(k_z a_N)^2\right) \end{aligned}$$

and

$$\begin{aligned} a_N &= \theta_w H_S Log\,(N)^{\frac{1}{\beta}} \\ b_N &= \frac{a_N}{\beta Log(N)} \end{aligned}$$

with N the number of waves in the sea state and $\theta_w = 0.77$ and $\beta = 2.38$ scale and shape parameters coming from the fit of $G(h)$ to the empirical maximum wave-height distribution observed on Frigg data.

A small subset only of the database of the present paper was used in constructing the statistical model, so it seemed valuable to examine whether the additional sea states, more than tenfold as numerous as the initial ones, fit with the proposed long-term distribution.

After computation of the maximum wave-heights distribution, it appears clearly from examination of Figures 1 and 2 that no extreme waves can be observed in excess of the model predictions.

A slight deviation appears for normalized maxima, that may likely be attributed to sample variability and underestimation of $H_{1/3}$ as much as to higher H_{max} than normal.

4. Sea state properties

The question arises of a parameter that could be characteristic of a sea state, and that would exhibit some changes when the probability of occurrence of rogue waves increases. The work of *Janssen* [2002] and *Mori and Janssen* [2005] suggests that given the correlation between kurtosis and the Benjamin-Feir instability index, one such parameter could be the Benjamin-Feir instability index. However, it must be noted that in order to be useful either for the prediction or for the interpretation of the freak wave phenomenon, as studied in *Olagnon and Magnusson* [2004], the change needs to be sufficiently clear to be distinguished from aleatory random variations. In addition, the change needs to be more than a mere consequence of the occurrence of a measured freak wave.

Given the relationship between kurtosis and Benjamin-Feir instability, and the expectations with respect to the latter as a causal factor in occurrence of extreme waves, we investigated the water surface elevation kurtosis of the sea states with high waves ($H_{max}/H_{1/3} > 2$). Figure 3 confirms a correlation of kurtosis with the occurrence of extreme waves for the time histories of the previously described database. However, if the maximum wave is removed from the kurtosis computation, and kurtosis is estimated from the remaining of the record as in Figure 4, no further correlation can be seen and the maximum normalized wave height is even reached for a zero value of the kurtosis.

We can thus infer from the above results that the correlation between occurrence of extreme waves and kurtosis, and thus likely Benjamin-Feir instability, is a very local one and is not reflected on statistics at the time scale of a sea state.

5. Storm properties

If forecast of extreme waves is considered, and since very local properties cannot be forecast accurately if they are not related to longer duration properties, the time-scale of interest may very well be extended to the whole storm duration. As a matter of fact, sailors are concerned by the wave of maximum height or severity that they may encounter in a given storm, with respect to the maximum forecast significant wave height. Whether the most severe wave is simultaneous to the maximum significant wave height or occurs with some time difference is usually a secondary matter: given the uncertainties on the exact time of occurrence of the maximum $H_{1/3}$, the decisions with respect to avoiding or accepting the storm will take a safety margin to allow for such time-shifts.

Thus, whereas many studies consider the ratio $\frac{H_{max}}{H_{1/3}(H_{max})}$ or $\left(\frac{H}{H_{1/3}}\right)_{max}$, we define a storm normalized maximum as the ratio $R_{max} = \frac{H_{max}}{\left(H_{1/3}\right)_{max}}$. Maxima are relative to the storm duration.

Storms are defined as durations of at least 12 consecutive hours where significant wave height remains above 5 meters; 187 such storms could be identified in the Frigg database.

Examples of those storms are given in Figure 5 for a case in the first period of measurement, when time-histories are availble and allow us to compute crest height parameters. It can be seen that significant wave height is in rather good agreement, but that some differences may appear on the detection of extreme waves, that we suspect could be attributed to the use of zero-up-crossing wave definition in the parameters database instead of the recommended down-crossing one. Figure 6 shows a storm of the second measurement period, when only statistical parameters are available. One may raise some doubts about the quality of the measurements for low $H_{1/3}$, given the very frequent overpassing of

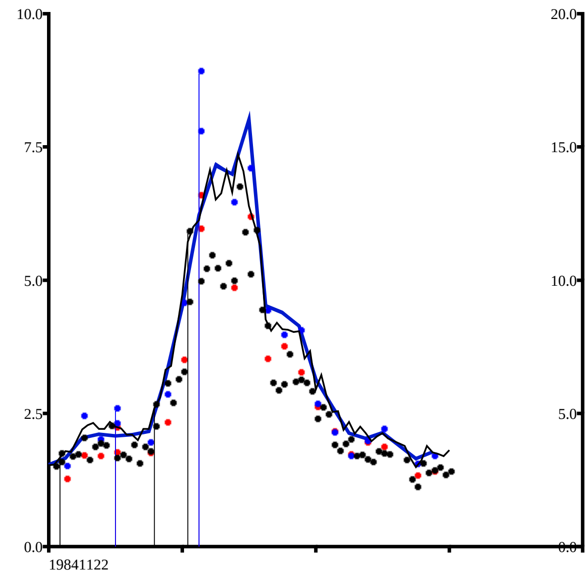

Figure 5. Parameters in an identified storm when time-histories are available - dots: max., right scale - line: significant, left scale - black: heights from parameters database - blue: heights from time histories - red: max. crests from time histories - vertical lines are drawn when maximum overpasses “rogue” threshold

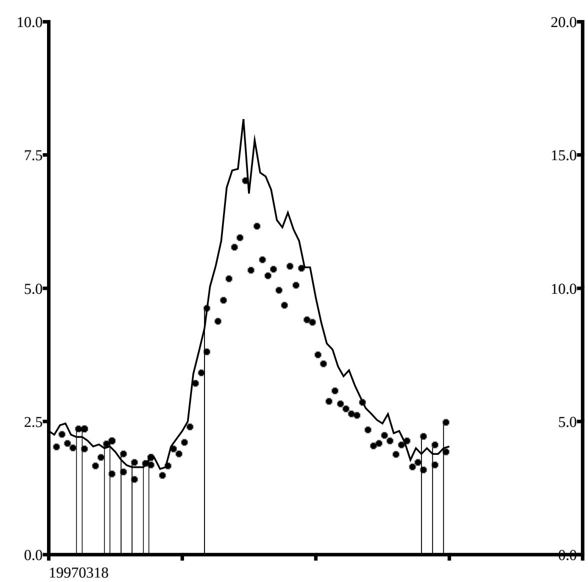

Figure 6. Parameters when no time history is available, second recording period

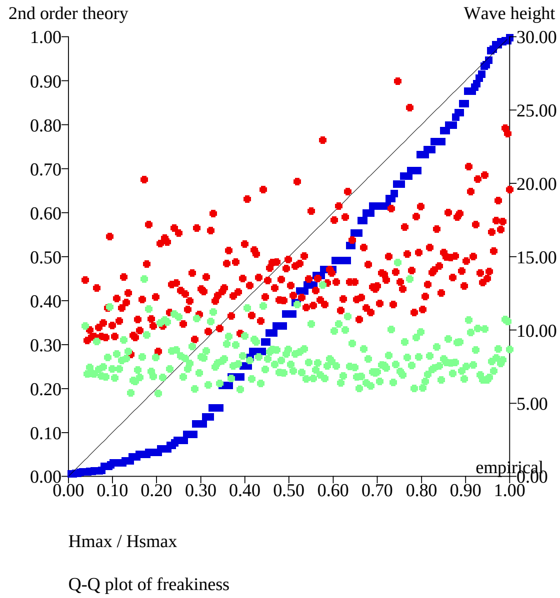

Figure 7. Q-Q plot of the observed storm "freakiness" distribution relative to second-order prediction (blue squares), and of corresponding maximum $H_{1/3}$(green dots) and H_{max} (red dots) on right axis scale

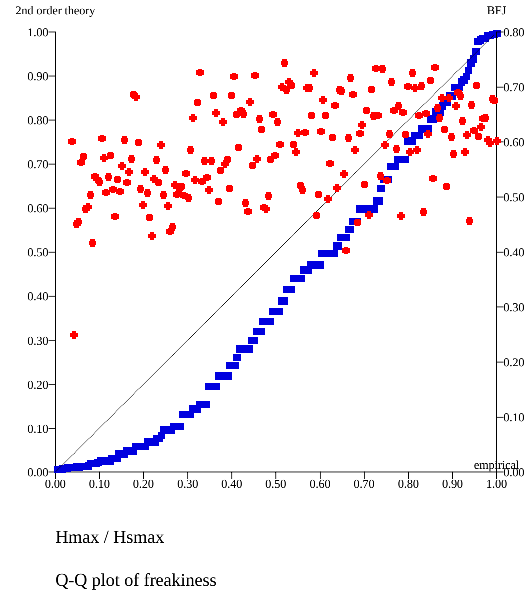

Figure 8. Q-Q plot of the observed storm "freakiness" distribution relative to second-order prediction (blue squares), and of corresponding mean Benjamin-Feir indices over the storms (red dots)

the "rogueness" threshold set to twice the $H_{1/3}$ value. However, the rate of overpassing appears reasonable when $H_{1/3}$ is above 5 meters, and thus storm data above that threshold seem reliable.

5.1. Construction of a conventional reference

In order to have a reference describing the expected extremes according to conventional methods, 1000 random simulations of time-histories were performed for each storm and analysed to extract their statistical parameters.

Since spectra were not available for many of the storms, it was decided to use a constant spectral shape, *i.e.* a Jonswap spectrum with $\gamma = 3$ whatever the sea state. That assumption is realistic according to the findings that storm spectra in the North Sea are single peaked [*Olagnon and van Iseghem*, 2000] and that they derive from very similar shapes at all times [*Olagnon*, 2001]. It should also lead to less variability than in nature, and thus provide a "conservative" reference.

The synthetic storms were then analysed. A second-order correction was applied to the maximum wave height of each simulated record according to the formula:

$$H_{corr} = H\left(1 + \frac{3}{32}(kH)^2\right)$$

5.2. Comparison of extremes with reference

Given the above simulations, a measure of the "freakiness" of the maximum wave in a storm can be obtained from the empirical quantile of the observed storm normalized maximum R_{max} in the distribution of the simulated R_{max} values for the 187 $\times$ 1000 storms. This approach of the definition of "freakiness" has the advantage that it should be uniformly distributed over the [0, 1] interval if the storms followed the simulated model.

Figure 7 compares the empirical freakiness distribution with the second-order prediction. For each storm represented as a blue square, abscissa corresponds to the proportion of observed storms that are not as freaky, and the ordinate to the proportion of simulated storms. It appears that only the ten most freaky storms or so are up to the model expectations.

It is also clear from Figure 8 that the trend of the Benjamin-Feir instability index, averaged over the storm, to increase with freakiness is still far from sufficient for any practical forecast purpose.

One may wonder whether such a relative mildness is related to the definition of freakiness. Figure 9 shows the corresponding distribution when freakiness is measured with the more conventional $\frac{H_{max}}{H_{1/3}(H_{max})}$ ratio. This distribution fits much better to the second order simulated one, probably because of the effect of the intra-storm variability that is underestimated in the simulations through the use of a constant shape and width spectrum.

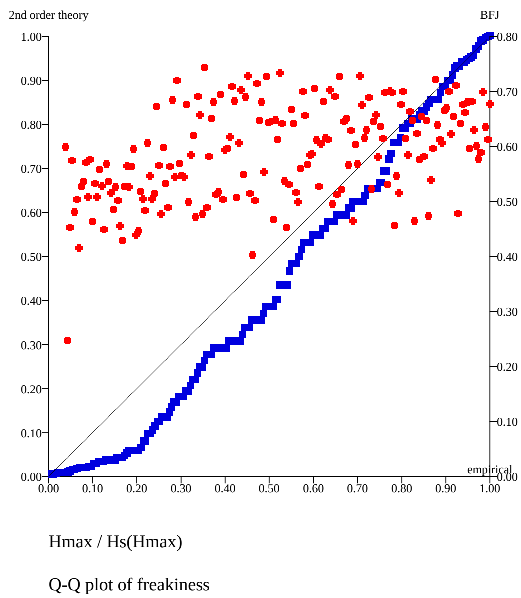

Figure 9. Q-Q plot of the observed storm "freakiness" based on $\frac{H_{max}}{H_{1/3}(H_{max})}$ relative to second-order prediction (blue squares), and of corresponding mean Benjamin-Feir indices over the storms (red dots)

In any case, a simple forecast using the history of $H_{1/3}$ and T_Z in the storm, a Jonswap spectrum with γ=3, and second order approximation provides the same distribution of extreme maximum height to significant height ratios as could be observed in 187 storms over nearly 20 years on the Frigg field in the North Sea. For less extreme maximum height ratios, the prediction is even more severe than the observation. That point might be a factor of risk by favouring underestimated extrapolations if only empirical data are used.

Forecast histories of $H_{1/3}$ and T_Z used in conjunction with a simple second order model should thus be prefered to naive extrapolation from observations. It is also likely that rogue wave occurrences are only related to very local properties of the sea states where they occur, and the prediction of the risk of such occurrences is all the more a difficult task.

6. Conclusions

A 20-year database of wave measurements in the North Sea has been used to investigate the distribution of high waves with respect to conventional predictions, and the relationship between high wave occurrences and characteristics of the prevailing sea conditions.

It appears that the change observed in signal kurtosis at the time of a high wave occurrence can be satisfactorily explained by the high wave alone, and that no further relation can be found at larger time-scales.

No extreme height event was found in excess of the statistics given by the *Nerzic and Prevosto* [1998] model.

Attempts to compare high wave occurrences with second-order predictions at the time-scale of whole storms, carried out over 187 storms with significant wave height above 5 meters for more than 12 hours, concluded to almost identical probabilities for the largest normalized maximum wave heights, and lower probabilities in reality than in the simple model for the other ones.

Extreme waves are thus not found more frequently than expected from conventional analyses. Yet, they are not found less frequently either. It is likely that the generation mechanisms affect only a short duration with respect to the time-scales of the non-directional parameters that can be forecast by meteorological models, and especially of the spectral ones.

We believe that more imaginative research efforts are thus still needed before methods can be found to issue advance warnings. Special attention should be given to directional information, since nondirectional parameters do not seem to allow discrimination between risky and more benign conditions.

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M. Olagnon and M. Prevosto, Ifremer, Centre de Brest, B.P. 70, F-29280 Plouzané, France