Understanding rogue waves: Are new physics really necessary?

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Abstract. The standard model of ocean waves describes them as the superposition of many wavelets with different frequencies and directions of travel. Nonlinearities are assumed to be small enough that they can be handled by a perturbation expansion. This model has served us well, leading to accurate predictions of directional wave spectra, wave statistics, and wave kinematics. Yet there remain some observations which are difficult to explain with the standard model. Strongly nonlinear physical processes have been invoked to explain the existence of very large, or rogue waves. Are these new models really necessary to explain the observations? There are several reasons why we may not need to invoke new physics. First, measuring rare events in extreme conditions in the ocean is very difficult. There is a significant possibility of substantial instrument error. There are well- documented cases where carefully calibrated wave recorders on the same platform gave very different readings. The most striking examples of rogue waves in the recent literature are unusually asymmetrical with high crests compared to the depth of their troughs. Second order perturbation theory produces crests in steep waves that are at least 10% higher than those given by the Rayleigh distribution, and higher order approximations show further increases. If only the short record in which a large wave occurs is considered, then it will stand out as an outlier. When more data is considered, these very large waves seem less aberrant. Sometimes waves are measured over an area, as by remote sensing, or estimated over an area, as by damage observed on the deck of a platform. In such cases, statistical theory shows that we should expect higher maximum wave crests than those measured at a point measurement since more waves are effectively sampled.

Introduction

A rogue wave can be defined as one that is surprisingly large compared to the usual run of waves in a seaway. It stands outside the rest of the population. If we use this definition, most people who have spent time at sea have observed rogue waves.

Because waves are a random process, there is a wide variation in height from one wave to another. The standard model of ocean waves describes them as the superposition of many wavelets with different frequencies and directions of travel. In the simplest version of that model, the wave phases are uniformly distributed. As a result, wave heights have a Rayleigh distribution. That distribution predicts that one wave out of about 3000 will even exceed twice the significant wave height. Is this amount of variability enough to explain observations of rogue waves, or are there phenomena which are not described by the standard model?

One very famous observation is the New Years wave measured at the Draupner platform in 1995 (*Haver*, this volume). The probability of a wave with this crest height arising in a sea described by a second order model is 6×10^{-7}. While this is not impossible, such a low probability naturally raises suspicions that some physical process other than a low order perturbation expansion is responsible for the wave.

One possibility is that nonlinear self focusing similar to the Benjamin-Feir instability is responsible for some rogue waves. *Osborne* (this volume) conducted experiments in a long towing tank in Trondheim with a narrow spectrum input at the paddle. Instabilities produced a strong modulation in the wave envelope. The highest waves were much higher than those predicted by the Rayleigh distribution.

Similar instability processes are not necessarily important in the ocean where the waves are directionally spread. *Dysthe et al*. (2005) have simulated the development of waves over an area using a high order nonlinear Schrödinger equation. The observed distribution of crest heights was very similar to that expected from second order theory.

The purpose of this paper is to take a critical look at whether high order nonlinearities are actually needed to explain observations of rogue waves in the ocean. First we examine the ability of measurements to accurately observe extreme waves. Then we investigate the statistical evidence for rogue waves. Are the observed high waves really less probable than predicted by standard statistics? Finally, we study the statistics of waves over an area.

Measuring high waves is difficult

The sea surface in a storm is violent and complicated. Accurate measurements in these conditions is very difficult. The problem comes not only from the difficulty of devising an instrument to measure the position of the sea surface but also from distortion of the waves by the structures supporting the instruments. Analysis of waves measured at the Tern platform in two storms in the North Sea illustrates this problem.

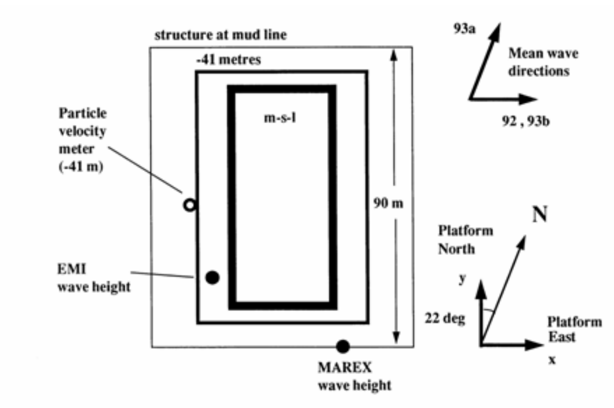

Figure 1. Outline plan of the Tern platform with locations of the wave height sensors and the mean wave directions for the storms.

Figure 1 shows the locations of the two wave sensors. A Marex wave radar was mounted under the southeast corner of the platform deck and an EMI laser wave sensor was mounted under the deck on the southwest corner. Both of these sensors measure the distance from the instrument to the instantaneous water surface.

Figures 2 and 3 show the probability distributions of the wave crest heights for two storms at Tern. The crest heights are normalized by the significant wave height during the hour the crest was measured. For both storms, the significant wave height remained over 10 m for over 8 hours. The solid lines in the figures show the Rayleigh distribution that the crest height would follow if the surface elevation had a Gaussian distribution. As expected, the sample distributions from the measurements have higher crests because of the nonlinearity of steep waves.

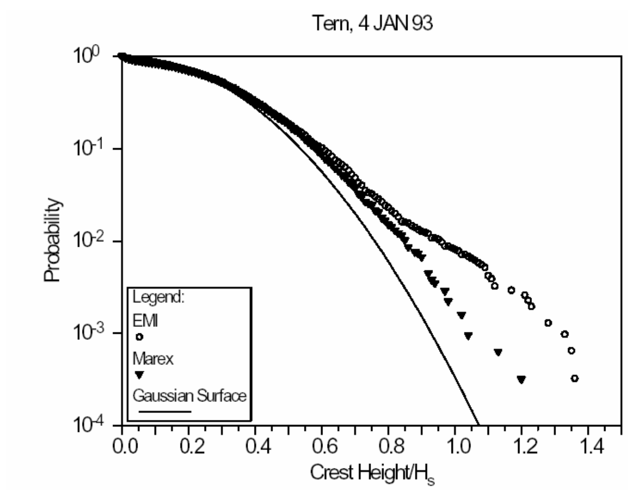

Figure 2. Probability distribution of normalized crest heights measured at Tern during the storm on 4 Jan 1993. The crest heights are normalized by the significant wave height during each hour of measurements.

The sample distributions also show significant disagreement between the results from the two wave sensors. In Figure 2, the crests measured by the EMI laser are 10-20% higher than those measured by the Marex radar. In Figure 3, however, the situation is reversed. It seems quite likely that the difference between the storms can be explained by the difference in wave directions shown in Figure 2.

Figure 2 shows wave in storm 93a when the waves were propagating to the north, so that their crests passed the Marex radar before encountering any structural elements on the platform. On the other hand, the leg on the southwest corner of the platform is up-wave from the EMI sensor, and it is quite likely that spray caused by a wave crest hitting that leg would sometimes pass under the EMI laser. Figure 3 shows waves from storm 92 that were propagating to the east, so that the EMI laser was on the windward side of the platform while the Marex radar was in the lee of structural members. Apparently, both of the sensors measured crests higher than the ambient waves due to spray from structural members, although there is no obvious evidence of spray in the measured time series. The location of a wave sensor with respect to the platform it is mounted is at least as important as the response characteristics of the sensor itself.

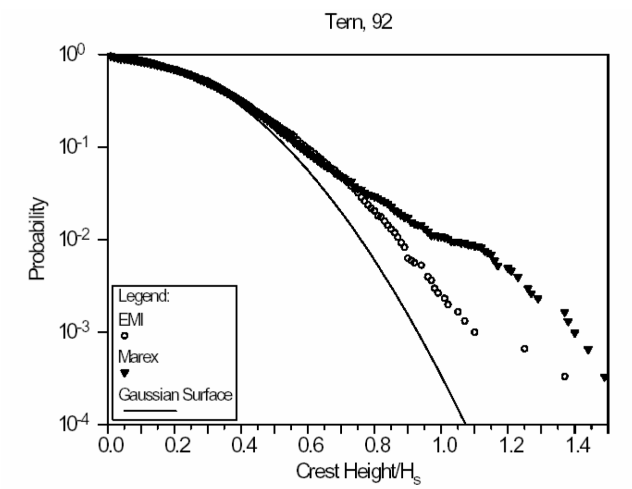

Figure 3. Probability distribution of normalized crest heights measured at Tern during the storm on 1 Jan 1992.

Different sensors do give different results, particularly for wave crests. There is a definite need to understand which sensors are most reliable. In response to this need, the Wave Crest Sensor Intercomparison Study WACSIS Joint Industry Project (Forristall et al., 2004) was begun in 1997. The key to the experiment was to place all of the popular sensors on the same platform located where they were likely to experience large waves in one season.

The intrinsic difficulty in establishing which wave sensors are most accurate is succinctly given in a quotation from Kinsman (1965):

> "I have never seen an account of a calibration procedure for a wave-gauge system that persuaded me of its validity; nor have I myself been able to devise one that satisfied me."

There is no standard wave for calibration. When we test an instrument, we simply do not know the correct answer. The best we can do, and the intention of WACSIS, is to achieve a consensus.

Figure 4 shows the problems involved in comparing wave sensors. The sensors shown by the lavender and blue lines were 15 or 20 m away from the others, but the other three measured within 2 m of each other. Yet there are substantial differences in the detailed shape and height of the wave they measured. Two video cameras were mounted on the platform in hopes of understanding the effect of spray on the instruments. Despite extensive study of the images, they proved to be of little use in understanding the differences in the measurements.

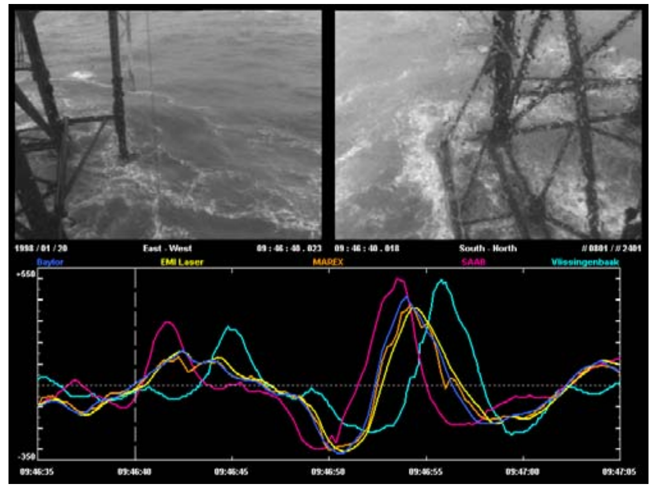

Figure 4. Sample of video records and wave measurements from WACSIS. The highest wave in the record is approximately 8 m high.

The statistics of extreme waves

Many reports of rogue waves, and in particular the New Years wave, are distinguished by having extremely high crests. WACSIS provided good information on the statistics of crest heights in steep waves because of the near agreement of several of its instruments.

Figure 5 show crest height statistics from approximately 100 hours of the highest waves measured during the WACSIS project. The abscissa gives the number of crests which exceed the height given on the ordinate. The Saab, Vlissing, and Baylor instruments give similar results. The Marex radar reported many crests which far exceeded the crest heights from the other instruments. Marex radars have given apparently good measurements in other locations; it may be that the one used in WACSIS was improperly adjusted. Nevertheless, the results pointedly demonstrate that measurements from one instrument cannot conclusively show the presence of a rogue wave.

The lower orange line in Figure 5 shows the crest heights expected from Gaussian statistics. The observed crests are higher, but they are matched well by the results of second order simulations.

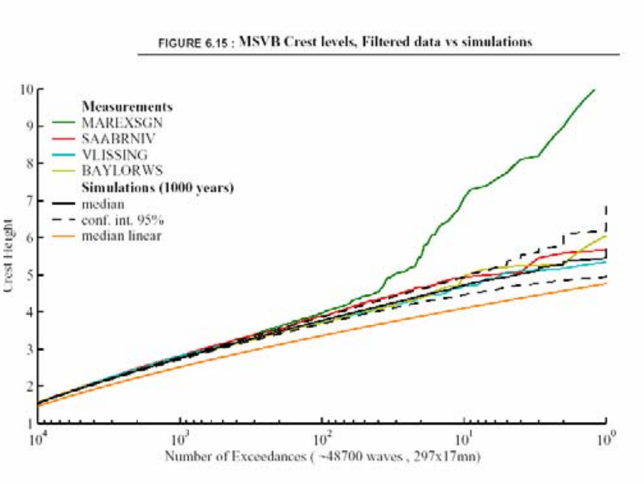

Figure 5. Number of exceedances of crest heights in about 100 hours of the highest waves in the WACSIS project.

Predicting crest heights from second order theory has now become standard practice in engineering design. Though nonlinear, it is a part of the standard perturbation theory of random waves. But second order theory is a poor explanation of the Draupner New Years wave. The probability of the measured wave crest is only is 6 x 10^{-7}. Prevosto and Beauffandeau (2002) showed that the probability of the New Years wave crest was 4.4 x 10^{-6} if the Improved Linear Representation (Henyey, 2005) was used to transform the wave profiles. Taylor and Williams (2002) modified a NewWave profile up to fifth order. The probability of the New Year wave crest was then about 1 in 10^{-5}.

The highest digitized point in the New Years wave profile was over 2.5 m higher than the other points measured in that wave. If that one digitized point is removed from the record, the probability of the wave crest is 2 x 10^{-5}. There is no objective evidence of errors in the Draupner measurements. The point is that an error in one point in the time series would reduce the crest height to a plausible level.

Large waves are not necessarily rogue waves, and a wave which stands out as unusual in a short record may be expected if we look long enough. Measurements made in Hurricane Ivan in the Gulf of Mexico in September 2004 illustrate these points.

Figure 6 shows the significant wave height, maximum wave height, and maximum crest height during each hour as the storm passed the Marlin platform. The significant wave height reached 15.4 m between 1600 and 1630. During that half hour, the maximum wave height was 26.3 m. That is an extremely large wave, but it does not really deserve to be called a rogue wave. It was only 1.71 times the significant wave height at the time.

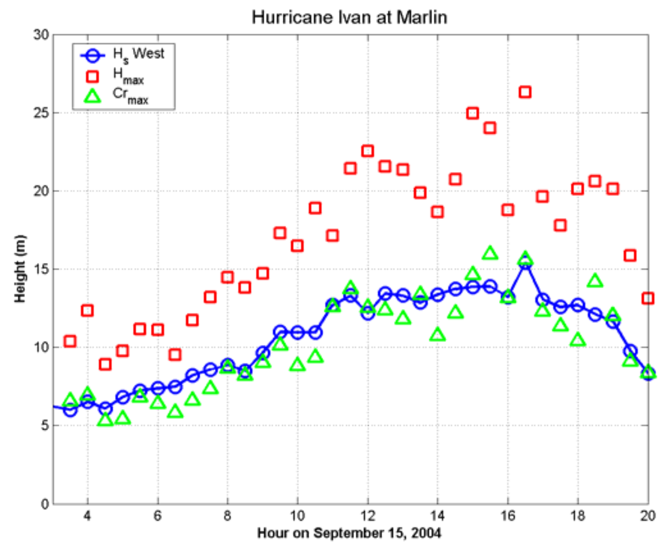

Figure 6. Wave and crest heights measured during Hurricane Ivan at the Marlin platform in the Gulf of Mexico.

The highest ratio of individual wave height to significant wave height during the measurements at Marlin was at 0400. The distribution of individual wave heights during that record is shown in Figure 7. The wave heights were normalized by the significant wave height. The sample distribution generally lies below the Rayleigh distribution and in fact fits the empirical distribution of Forristall (1978) rather well. But one wave is an outlier with a ration of 1.89. It might well be called a rogue wave.

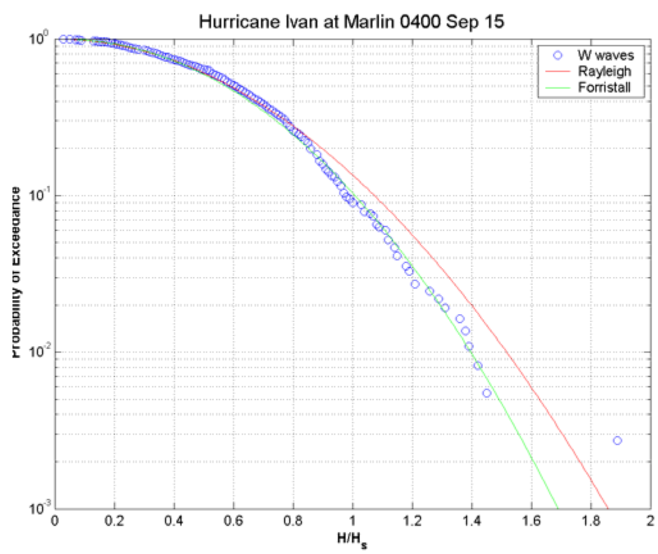

Figure 7. Probability distribution of wave heights during one hour starting at 0400 during Hurricane Ivan at the Marlin platform in the Gulf of Mexico. The wave heights are normalized by the significant wave height.

But when taken in the context of a longer record, the large wave at 0400 fits ordinary statistics. Figure 8 shows the distribution of all of the individual waves during Hurricane Ivan at Marlin. They are normalized by the significant wave height in each half hour record. The wave that appeared so unusual in a single short record falls right on the empirical distribution line. If we wait a long time, Gaussian statistics can produce a very large wave.

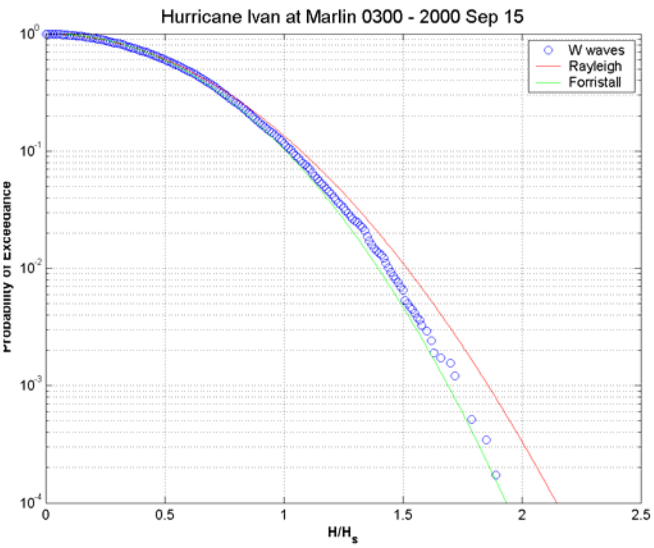

Figure 8. Probability distribution of wave heights throughout Hurricane Ivan at the Marlin platform in the Gulf of Mexico. The wave heights in each half hour were normalized by the significant wave height in that half hour.

Maximum wave heights over an area

The maximum wave crest somewhere in a large area is expected to be larger than the maximum crest at a single point. Piterbarg (1996) found asymptotic distributions for Gaussian processes over large multi-dimensional spaces. Krogstad et al. (2004) applied Petierbarg's theorems to the estimation of maximum wave crests. The most probable extreme in a Gaussian field containing a large number of waves is

$$E[\max(x)] = h_n + 0.5772 / h_n$$

where (1)

$$h_n = \sqrt{2\ln N + (n-1)\ln(2\ln N)}$$

and N is the equivalent number of waves in the field and n is its dimension. For fields involving space and time, the space-time correlation must be considered in estimating N. We see that the number of dimensions of the field as well as the number of waves influence the expected maximum.

Krogstad et al. (2004) give an example of a snapshot of waves over a 100 x 100 km area. They calculated that the expected maximum linear crest is $1.32H_s$. In comparison, the expected Gaussian maximum for a 1000 wave time series is only $0.93H_s$. The theory for waves over an area makes the observations of large crests made by orbiting radars seem much more reasonable.

High crests are also expected to occur over smaller areas. Figure 9 shows the results of simulations of linear waves over square areas with the side lengths given on the abscissa of the graph. The simulations were based on a Jonswap spectrum with a peak period of 10 sec. The mean period was thus 8.35 sec and the associated wave length was 109 m. The waves were directionally spread using the spreading functions for fetch limited waves given by Ewans (1998). Each simulation lasted 1024 sec and at least 100 simulations were made for each area.

The values shown in Figure 9 are the averages over the simulations of the maximum crest height anywhere in the area. The crest heights are normalized by the standard deviation of the wave elevation. They are substantially higher than the highest crest expected at a point even for areas of 50-100 m on a side.

According to Piterbarg (1996), the equivalent number of waves is given by

$$N = 2\pi \frac{L_1 L_2 T}{V} \tag{2}$$

where L_1 and L_2 are the lengths of the sides of the area, T is the duration of the simulation, and V is given by

$$V = \alpha \lambda_1 \lambda_2 T_z \tag{3}$$

where λ_1 and λ_2 are the mean wavelength and mean crest length, T_z is the mean zero-crossing period, and the factor α accounts for the space-time correlation. For the directional spreading used in our simulations, $\lambda_2 \approx 3\lambda_1$. Fitting equations (1) – (3) to the simulations for the larger areas gave an empirical estimate of $\alpha = 1.25$. The maximum crest heights from equation (1) using this value of α are shown as the red line in Figure 9.

Equation 1 fits the simulations well when the side of the simulated area is greater than about 150 m. Piterbarg's theorem is asymptotic in the sense that it applies for large areas. It is not surprising that it fails for side lengths smaller than one wavelength. For such small areas, it is likely that a local maximum of the surface will not appear in the area at any given time.

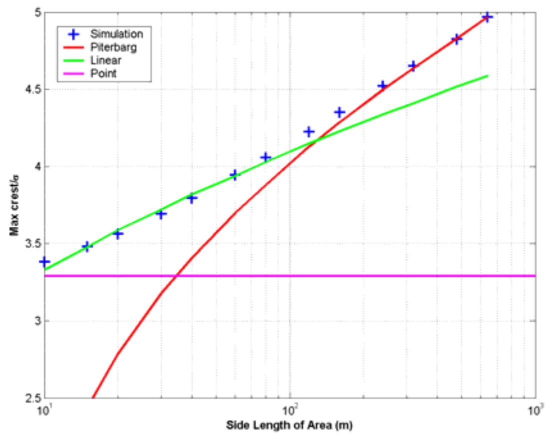

Figure 9. Expected maximum crest heights over an area with the side length given on the abscissa.

Empirically we found that for small areas a good fit to the simulations can be found by taking

$$N = 2\frac{L}{\lambda_1}\frac{T}{T_z} \tag{4}$$

and applying equation (1). The results from using equations (4) and (1) are shown as the green line in Figure 9. For small areas, the equivalent number of waves is thus proportional to the length of the side rather than to the area. We may speculate that this is because the maximum value of the wave surface is likely to be on the edge of the area when the edge is shorter than one wavelength.

The expectation of high crests over relatively small areas has important implications for the air gap under the decks of fixed structures. Consider a structure with a deck 50 m by 50 m. For the wave spectrum used in the simulations, the maximum crest somewhere under that deck is expected to be 1.18 times higher than the maximum crest at a single fixed point such as a wave staff. If, for example, the significant wave height is 13 m, then the expected maximum in 1024 seconds will be 2.4 m higher than the conventional estimate from point statistics.

Localized damage has sometimes been observed on the lower decks of platforms after storms. This damage has often been hard to reconcile with hindcast significant wave heights and standard crest statistics. The fact that the expected maximum over the area of a deck is substantially greater than the expected maximum at a point may go a long way to explaining this anomaly.

Localized damage during a severe storm does not necessarily mean that a platform was poorly designed. A small amount of water in the deck does not affect the structural integrity of the platform. Occasional damage might be repaired at less cost than building and installing a platform with a higher deck. But designers should be aware that the potential for green water at some location in the deck is much greater than would be estimated from statistics of crests at one point.

Conclusions

Evidence for rare events is very difficult to obtain. The behavior of wave sensors in extreme conditions is basically unknown. If a very large wave appears in a record, it is difficult to distinguish from noise. The noise may be electronic or the result of interference from the structure that supports the wave sensor.

An unusually large wave will always stand out as a rogue wave in a short record. Yet it may fit standard statistics perfectly well if the statistics from many hours of storms are combined.

Perturbation theories yield wave crests that are significantly higher than given by linear theory. Many observations of high crests can be explained by such theories.

The highest crest expected over an area is naturally higher than that expected at one point. It is somewhat surprising though that the crests expected over an area are so much higher than those at a point. These predictions may well explain observations of very high crests in radar images as well as the localized damage that has been seen on the decks of fixed platforms.

Strongly nonlinear focusing of waves may well exist in the ocean. But existing observations do not demand such a mechanism. More investigation of sensor performance, careful measurements, and statistical calculations are needed to establish whether the standard model remains adequate.

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