

# Rogue waves and wave breaking – How are these phenomena related?

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**Abstract.** According to prevailing ideas, the formation of rogue waves and the onset of wave breaking share a common physical framework: energy focusing within nonlinear wave groups. For 2D (one horizontal spatial direction) wave packet evolution, this correspondence appears to be very close. However, the situation is less clear for 3D propagation, and in the open ocean, observed breaking of dominant wind waves seems to occur more frequently than rogue waves. This contribution seeks to highlight the role of the modulation process in these two “extreme” wave phenomena from observational and computational perspectives, with the aim of stimulating discussion on what differentiates the more commonly observed phenomenon of wave breaking from the more elusive rogue waves.

## Introduction

The occurrence and consequences of rogue waves and large breaking waves have been a long-term concern for maritime operations, both in deep and coastal waters. Such waves generate the highest impact forces on ships and coastal structures, and represent amongst the most hazardous conditions for seafarers, as is evident in Figure 1. In order to be able to provide reliable forecasts of these extreme events, even in a statistical sense, it is essential to first understand the underlying physical mechanism(s) involved. Recent research on rogue waves (e.g., Janssen, 2003; Onorato *et al.*, 2005) and on wave breaking (e.g., Banner *et al.*, 2000; Song and Banner, 2002) suggests that both of these extreme wave phenomena might arise from similar nonlinear wave group behaviour. This contribution aims to examine this tentative viewpoint by reviewing recent developments in rogue wave formation and wave breaking onset. In any event, the issue remains as to whether a rogue wave, once formed, will break.

## Rogue wave formation

It is known that rogue waves in the open ocean can develop in a variety of ways. These have been identified in the literature and include geometric focusing of dispersive wave components, through waves encountering opposing currents or waves propagating into regions of strong lateral convergence. For these cases, the amplification of waves can be addressed through conservation of wave action, but it is evident that once the wave

steepness becomes finite, the influence of nonlinearity needs to be included. Note that wind forcing establishes the mean steepness, and hence the nonlinearity, of the waves. Local aerodynamically induced steepening is usually not the primary cause of the formation of rogue wave events.



**Figure 1.** A very large breaking wave inundating a tanker during storm conditions at sea.

Recent attention has turned to rogue wave formation by nonlinear hydrodynamical wave energy focusing in wave groups, both in 2D and 3D situations. The relative importance of this mechanism in the open ocean context is not yet known, and resolving this issue is a very challenging task, given the low statistical occurrence and/or detection rates for rogue waves.

There were several presentations at this workshop (by Janssen, Dysthe, Osborne, amongst others) featuring computations with different variants of the 2D

Nonlinear Schrödinger (NLS), or similar, equation. Depending on initial conditions, spectral bandwidth and degree of nonlinearity, these calculations all show transient formation of isolated large wave events. The occurrence statistics of such events were investigated systematically by some of these authors, but it is not clear whether the computed statistics of these rogue wave events were in any sense ‘exceptional’, a difficulty shared with actual statistics of observed rogue wave events.

Evolving wave fields can also be generated using an efficient computational approach based on the exact equations of motion. The large-scale, phase-resolved direct simulations presented at this workshop by *Wu, Liu and Yue* (this volume) are of particular interest. They used higher order spectral (HOS) techniques to investigate the evolution of a relatively wide bandwidth sea state, specified initially by a JONSWAP spectrum with broad initial directional spreading and random phase. They showed examples of steep wave formation in which they detected, and then tracked, the evolution of individual waves which attained wave heights exceeding twice the significant wave height  $H_s$ . They carried out a detailed analysis of the growth rates of these steepest waves using the method proposed by *Song and Banner* (2002, hereafter SB02). For the cases they examined, they found that the steepest waves were often associated with intersecting wave packets, yet the rapid steepening was remarkably uni-directional in terms of energy fluxes driving the growth process. The magnitude of the growth rates appeared to exceed the unidirectional threshold found by SB02 appreciably, yet the 3D model waves were below a geometrical breaking threshold steepness.

A central issue that arises in such calculations is how accurately such approximate HOS calculations represent rapidly steepening waves that would otherwise evolve to break. The same issue arises with nonlinear analytical models (e.g., the NLS, Dysthe or Zakharov equations). Since breaking is excluded in these models, how reliably do they predict extreme wave heights close to breaking?

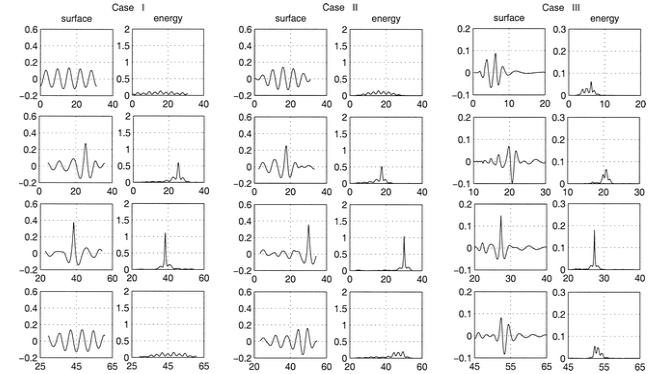
## Wave breaking onset and wave groupiness

The association of breaking waves at sea with wave group structure was reported over thirty years ago by *Donelan, Longuet-Higgins, and Turner* (1972). In a subsequent observational study, *Holthuijsen and Herbers* (1986) observed a large number ( $\sim 1500$ ) of wave groups in coastal and open ocean waters. They found compelling evidence of a strong relationship between wave breaking and group structure, observing that the

mean fraction of groups containing a breaking wave close to its center was 0.69. They also showed that it was difficult to separate breaking and non-breaking waves on the basis of local wave steepness geometry alone.

## 2D wave packets

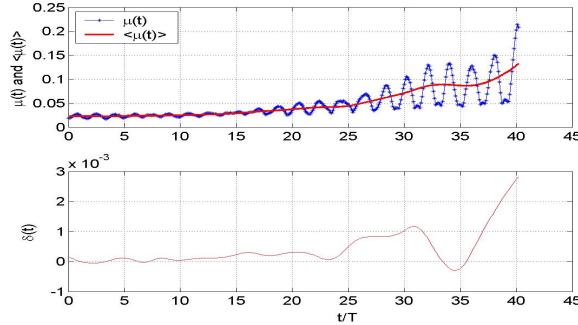
The generic mechanism of nonlinear energy focusing in unidirectional wave groups has been known for many years (e.g., *Dold and Peregrine*, 1986). More recently it has been proposed as a key mechanism for wave breaking onset (e.g., *Tulin and Li*, 1992; SB02). The interesting behavior of the SB02 growth rate of the energy at the envelope maximum, where a rogue wave is formed generically, offers a surrogate for the complex energy focusing within the wave group evident in the evolution of three model ‘narrow-band’ initial wave groups, as seen in Figure 2.



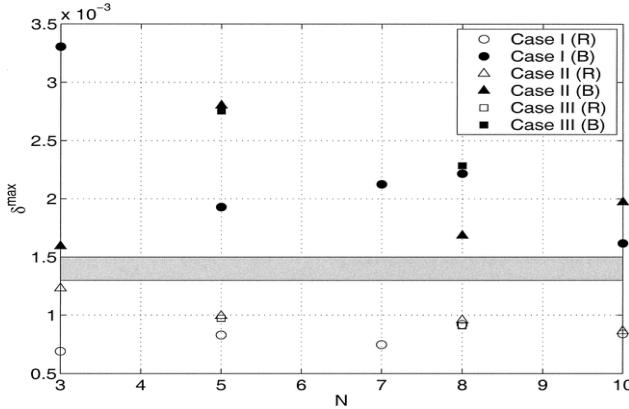
**Figure 2.** The evolution of typical 5-wave groups (after SB02). Cases I, II and III are narrow-band initial spectra. I: central main component + two symmetric 10% sidebands, II: bimodal spectrum and III: chirped packets. Each left sub-panel shows the surface profile at four different times in the recurrence cycle. The energy density distribution is in each right sub-panel. Full details are in SB02.

In such calculations, the characteristic growth rate depends on the initial degree of groupiness, with well-formed groups evolving more rapidly to a sharp crested wave at the center of the group. Typical growth rates can be based on the evolution of the dimensionless energy  $\mu = Ek^2$ , where  $E$  is the local energy density at the envelope maximum, and  $k$  is the local wavenumber, or mean carrier wavenumber, assuming a narrow-band wave group. The corresponding non-dimensional growth rate  $\delta(t)$  is the growth rate of  $\mu(t)$  normalized by the centre packet frequency. Figure (3) shows a typical evolution of  $\mu(t)$  and  $\delta(t)$  to breaking of a bi-modal 5 wave group.

SB02 investigated the nondimensional growth rate  $\delta(t)$  for a range of unidirectional narrow-banded initial packet geometries and found (Figure 4) a common breaking rate threshold of 0.0013 - 0.0015. All recurrence cases fell below this threshold and all breaking cases exceeded it once, independent of the initial packet geometry and number of carrier waves.



**Figure 3.** Typical evolution to breaking of the non-dimensional energy  $\mu(t)$  (upper panel) and its growth rate  $\delta(t)$  for a 5 wave bimodal wave group.

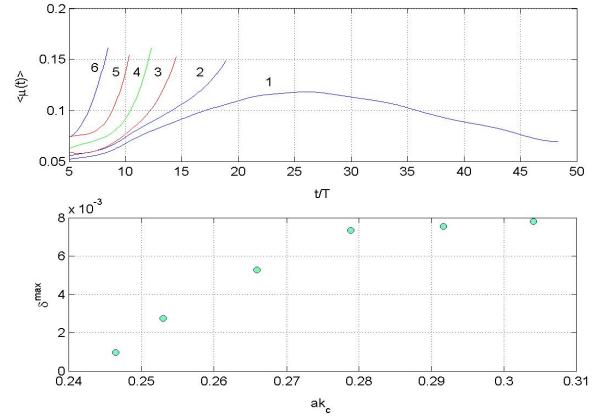


**Figure 4.** Maximum values of the nondimensional growth rate  $\delta(t)$  for a range of unidirectional narrow-banded initial packet geometries.  $N$  is the number of carrier waves in the group.

SB02 also investigated how the maximum growth rate  $\delta^{\max}$  depended on the initial mean steepness of the group and found that this growth rate increased progressively. The range of maximum growth rate at *recurrence* was bounded above by a value of 0.0015, but increased to  $O(0.01)$  just prior to breaking. Figure 5 shows this behaviour for case III 5-wave groups as a function of mean initial packet steepness. This order of magnitude increase of  $\delta^{\max}$  with mean packet steepness was typical of the cases investigated.

It should also be noted that the wave groups investigated in SB02 were of relatively narrow spectral band-

width. However, Peregrine confirmed (this volume) that the same qualitative behavior was also found for broad bandwidth unidirectional cases.



**Figure 5.** Typical dependence of the maximum growth rate on the initial mean steepness (nonlinearity) of the wave group. The case shown is a chirped wave packet (III) with  $N=5$

### 3D wave packets

For interacting directional wave packets, there are no comparable direct measurements yet available for the corresponding growth rate of the envelope maximum. Indications from observations are very interesting, and complement the findings from the HOS phase-resolving simulations of Wu, Liu and Yue (this volume). Laboratory observations confirm that directional convergence promotes steeper, shorter-crested waves at breaking inception, with maximum steepness exceeding the corresponding unidirectional limiting steepness and even the Stokes (2D) limiting steepness (e.g., Kolaini and Tulin, 1995; Wu and Nepf, 2002). However, the underlying nonlinear wave energy fluxes have not been measured, so how these compare with 2D breaking energy convergence rates remains to be determined.

### Breaking and rogue wave energy flux rates

In the absence of direct measurements, initial insight on this key issue, highlighted in the preceding section, is provided by the computational results of Wu, Liu and Yue (2005, this workshop) of energy convergence rates  $\delta^{\max}$  for cases of rogue wave formation in a 3D wave field. These growth rates were significantly higher than the SB02 *breaking threshold* growth rate for 2D wave groups. However, they were comparable with the 2D growth rates *just prior to breaking onset* reported by

SB02. Thus in a sense, their growth rates associated with rogue wave formation were possibly ‘exceptional’, flagging potentially different physical processes associated with rogue wave formation compared with breaking onset. However, before this issue can be settled, it remains to determine how reliably HOS phase-resolving models such as *Wu, Liu and Yue* (this volume) have implemented are able to describe the details of the waveform evolution *near breaking onset*. Resolution of this issue appears central to a holistic understanding of the rogue wave phenomenon, and needs to be investigated carefully.

## Conclusions

In the open ocean, the onset of wave breaking and the formation of rogue waves appear to be underpinned by the same generic mechanism – nonlinear interactions within wave group structures. For unidirectional cases, this is very clearly the case. In directional sea states, this association is still evident. However, the increased inherent complexity of wave group structures in directional wind seas presents a greater challenge to understanding their evolution and the associated formation of very steep and breaking individual carrier waves. Further study of the link between rogue wave formation and breaking is needed to allow phase resolving models to provide reliable calculations of occurrence rates of such steep waves, and whether or not they will break. This will also bridge the present knowledge gap regarding the relative occurrence statistics of rogue waves and breaking dominant sea waves.

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