

Dianeutral Upwelling Without Dissipation of Kinetic Energy

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Abstract. Estimates of the upwelling of bottom and deep water have recently been used to deduce the magnitude of area-averaged mixing activity and the required dissipation of kinetic energy that is input by winds and tides. Here we raise questions about this link between upwelling and the dissipation of energy. Several processes are discussed, each of which lead to the upwelling of water across neutral density surfaces, but none of which have a signature in the dissipation of kinetic energy. Some of these processes seem too large to ignore while others remain to be properly quantified. Until these processes are either quantified or dismissed as unimportant, it is premature to use a mixing efficiency (such as 0.2) to relate global dianeutral upwelling to the dissipation of kinetic energy. The influence of the nonlinear nature of the equation of state on the evolution of gravitational potential energy is also discussed. We raise serious doubts about the suggestion that the contraction-on-mixing feedback effect involving the gravitational potential energy of the whole water column, gives rise to an upper limit on the vertical gradient of potential temperature.

Introduction

Munk and Wunsch (1998) have provided a rather provocative link between the rate of upwelling of dense water across deep isopycnals (which may be assumed known for this purpose) and the dissipation of kinetic energy that seems to be required to cause this upwelling. The rate of dissipation was inferred by assuming that the small-scale turbulent mixing is due to breaking internal waves with a constant mixing efficiency of 0.2 so that the vertical diffusivity was assumed to be $0.2\varepsilon/N^2$ where ε is the rate of dissipation of kinetic energy and N is the buoyancy frequency.

Double-diffusive convection is one process that operates to upset the simple relationship between the vertical buoyancy flux and the dissipation of kinetic energy. In a double-diffusive system the vertical flux of buoyancy is the *negative* of the dissipation of mechanical energy (*McDougall*, 1988). That is, the “mixing efficiency” is *minus five* times the normally assumed value of 0.2. When double diffusive processes operate in the presence of lateral property gradients, interleaving is frequently observed, and the net vertical fluxes of heat, salt and buoyancy are even more complicated, with counterintuitive signs (*Garrett*, 1982, *McDougall*, 1985).

The intermittent nature of turbulent patches means that the appropriate value of the mixing efficiency for the real ocean is significantly less than that measured in continuously turbulent laboratory environments, even when the mixing is due to the breaking of internal gravity waves. Following the work of *Garrett* (1984), *St. Laurent and Schmitt* (1999) and *Arneborg* (2002), it seems clear that a value close to 0.11 is more appropriate than 0.2 for the mixing efficiency of intermittent turbulence. *Arneborg* (2002) states that if a mixing efficiency of 0.11 is adopted in the analysis of *Munk and Wunsch* (1998), the wind’s contribution to the global amount of mechanical energy dissipation rises from 1.2TW to 2.9TW, although it is widely acknowledged that all these numbers are rather uncertain.

This article begins by outlining the complicating effect of the nonlinear nature of the equation of state of seawater on the way the vertical profile of dissipation and the dianeutral upwelling velocity are related. Then the processes of cabbelling and thermobaricity (of the lateral variety) are discussed and it becomes apparent that these processes cause dianeutral advection that is totally independent of the level of the dissipation of kinetic energy. Submesoscale coherent vortices (e.g., Meddies) and the helical nature of neutral trajectories are two more processes that achieve

dianeutral advection even in the limit of vanishingly small dissipation of mechanical energy.

Finally, we examine the “gravitational potential energy local feedback hypothesis” that has been advanced by *Fofonoff* (1961,1998,2001). This hypothesis states that when mixing processes lead to a net decrease of the gravitational potential energy of a whole water column, a feedback process acts to enhance the diapycnal mixing activity at the location of the original mixing, so causing an upper bound on the buoyancy frequency at that depth. This argument is extended to include double-diffusive convection as well as cabbeling and thermobaricity that arise as a consequence of lateral mesoscale mixing. It is concluded that this feedback hypothesis is very unlikely to be active as it has not been possible to imagine a process that would localize any increased mixing activity to exactly the depth and horizontal location where the original mixing occurred.

Relating upwelling to dissipation

By taking an appropriate linear combination of the conservation equations of salinity and potential temperature, small-scale turbulent mixing with diffusivity D is seen to cause dianeutral upwelling at the velocity, e , given by (*McDougall and You*, 1990)

$$e = D_z + gN^{-2}D(\alpha\theta_{zz} - \beta S_{zz}) \quad (1)$$

where α is the thermal expansion coefficient, β is the saline contraction coefficient and the subscripts denote differentiation. *Munk and Wunsch* (1998) wrote the right-hand side of (1) as

$$N^{-2}(DN^2)_z \quad (2)$$

but doing this does not correctly account for the nonlinear nature of the equation of state of seawater. Here we argue that this approximation may have had a material impact on their results.

Taking a fixed mixing efficiency, Γ , the vertical diffusivity of small-scale turbulent mixing is $D = \Gamma \varepsilon / N^2$ and the dianeutral upwelling, (1), can be expressed in terms of the vertical variation of the dissipation of kinetic energy as

$$\frac{e}{D} = \frac{\varepsilon_z}{\varepsilon} - \frac{R_\rho}{(R_\rho - 1)} \left(\frac{\alpha_z}{\alpha} - \frac{1}{R_\rho} \frac{\beta_z}{\beta} \right) \quad (3)$$

where $R_\rho = \alpha\theta_z / \beta S_z$ is the stability ratio of the water column.

When the nonlinear nature of the equation of state is ignored, (3) reduces to the simple relation $\varepsilon_z = \Gamma^{-1} e N^2$ which was used by *Munk and Wunsch* (1998) to estimate ε given estimates of the upwelling velocity, e , and observations of the buoyancy frequency, N . The nonlinear term in (3) typically goes to zero at a depth of about 1500 m in the ocean, where the two dominant contributions to α_z , namely $\alpha_\theta\theta_z$ and $\alpha_p p_z$, tend to cancel each other. *McDougall* (1988) has shown that for an exponential vertical stratification, the nonlinear term in (3) changes the deduced values of the dissipation by a factor of two. This indicates that these terms are too large to ignore when one seeks to relate the dianeutral upwelling of deep and bottom water to the dissipation of mechanical energy and hence to the vertical diffusivity.

Cabbeling and thermobaricity

McDougall (1987a) and *McDougall and You* (1990) have examined the dianeutral consequences of the strong lateral mixing of salinity and potential temperature achieved by mesoscale eddies acting along locally-referenced potential density surfaces. The non-linear nature of the equation of state leads to dianeutral advection given by

$$e^{\text{cabb}} + e^{\text{tb}} = -gN^{-2}K(C_b \nabla_n \theta \cdot \nabla_n \theta + T_b \nabla_n \theta \cdot \nabla_n p) \quad (4)$$

where K is the lateral eddy diffusivity, $C_b = \alpha_\theta + 2\alpha_S(\alpha/\beta) - \beta_S(\alpha/\beta)^2$ is the cabbeling parameter, $T_b = \alpha_p - \beta_p(\alpha/\beta)$ is the thermobaric parameter, and ∇_n denotes the gradient operator along the locally-referenced potential density surface (ie along the neutral tangent plane).

The dianeutral cabbeling velocity e^{cabb} is always negative while the dianeutral thermobaric velocity e^{tb} can take either sign. *McDougall and You* (1990) have plotted $e^{\text{cabb}} + e^{\text{tb}}$ on a variety of density surfaces in the world ocean and find that this combination is generally negative and that in the southern ocean the

dianeutral sinking achieved by these mechanisms is several times larger than the canonical upwelling velocity metric of 10^{-7} ms^{-1} . Only in the North Pacific Ocean is $e^{\text{cabb}} + e^{\text{tb}}$ sufficiently small that it could be ignored for the purposes of estimating the amount of diapycnal mixing required to upwell deep and bottom water throughout the world ocean.

The dianeutral advection achieved by cabbeling and thermobaricity is totally independent of the dissipation of kinetic energy. All that is required for these two dianeutral advection processes to occur is that fluid parcels mix along isopycnals. These epineutral fluxes of heat and salt can occur for any vanishingly small amount of dissipation and so e^{cabb} and e^{tb} have no signature in ε . The way in which $e^{\text{cabb}} + e^{\text{tb}}$ should be included in any followup study of Munk and Wunsch's approach of linking dianeutral upwelling to the dissipation of mechanical energy is through a modification (an increase) to the amount of dianeutral advection that must be supported by the regular processes of diapycnal mixing.

Dianeutral motion of SCVs

It has been argued that sub-mesoscale coherent vortices (SCVs) such as Meddies may account for a substantial fraction of the lateral flux of salt around the North Atlantic ocean (*Armi and Zenk, 1984*). When an isolated body of water such as an SCV migrates laterally around an ocean basin, it does not stay on either a potential density surface or on a neutral surface (*McDougall, 1987b*). Rather, the SCV migrates vertically through both these types of surfaces in a fashion that is essentially a finite amplitude version of the dianeutral motion of lateral thermobaricity. The SCV's dianeutral motion is due to its compressibility being different to that of the surrounding water because of the contrast in temperature (and salinity).

The SCV's dianeutral advection is unrelated to the dissipation of energy in the ocean. *McDougall (1987b)* has examined this process and has shown that whenever SCVs are a significant contributor to the lateral transport of heat and salt, then so is the dianeutral advection of the SCVs significant. If and when we are able to parameterize the lateral fluxes of properties achieved by SCVs, then it will also be possible to parameterize their dianeutral transport.

The helical nature of neutral trajectories

Neutral tangent planes are defined so that an in-situ fluid parcel can be moved adiabatically a small distance in this plane without feeling a buoyant restoring force. The normal to this plane is in the direction $\alpha \nabla \theta - \beta \nabla S$ but just because we have such an expression for the normal to a surface at each point in space does not mean that a surface necessarily exists. That is, the collection of neutral tangent planes, each of which is well defined at each point in space, may not actually join up to form a mathematically well-defined surface. The reason for this pesky behaviour is that in the definition $\alpha \nabla \theta - \beta \nabla S$ of the normal, α and β are functions not only of θ and S but are also functions of a third variable, namely pressure.

As a consequence of the mathematically ill-defined nature of neutral surfaces, it is only possible to find surfaces that are approximately neutral. Such surfaces are well-defined but are only approximately neutral in some least squares sense. Imagine following the mean circulation around the ocean on a path that we assume is closed horizontally, so arriving back at the original latitude and longitude. If we follow a neutral trajectory along this route so that fluid parcels can be laterally mixed along the trajectory without having to do work against gravity, the ill-defined nature of neutral surfaces means that when we arrive back at the starting latitude and longitude, we are not at the starting height. A lengthy excursion around the ocean on a neutral trajectory means that a mean dianeutral advection occurs. *McDougall and Jackett (1988)* estimate that the vertical excursion for a path around the North Atlantic ocean is about 10m, and taking a reasonable recirculation time for this motion results in a mean vertical upwelling that is not completely trivial.

The previous paragraph considered a horizontal path that was closed in latitude and longitude. The real ocean circulation does not perform such closed x - y loops and this serves to complicate the quantification of this effect. Interestingly, the mean dianeutral motion achieved by this effect is proportional to the areal integral of $T_b \nabla_a \theta \times \nabla_a p$, that is, to the vector cross product of the gradients of potential temperature and of pressure in an approximately neutral surface. This contrasts with the dianeutral motion caused by thermobaricity which varies as the scalar product of these same two horizontal vectors, namely to

$$K T_b \nabla_a \theta \cdot \nabla_a p .$$

To date we do not have any believable estimates of the importance of this path-dependence effect for the mean dianeutral motion in the global ocean. Like the dianeutral advection of thermobaricity, cabbeling, and the dianeutral motion of SCVs, the dianeutral advection due to the ill-defined nature of neutral surfaces is totally independent of the dissipation of kinetic energy. In the present case the word “dianeutral” is used to describe the vertical flow through an approximately neutral surface, this being the logical choice of surface since it is mathematically well-defined. If we were able to quantify this effect then the mean dianeutral advection caused by it would affect the estimation of the dissipation of mechanical energy in the same fashion as cabbeling, thermobaricity and the mean dianeutral advection of SCVs, namely by changing our estimate of the amount of dianeutral advection that then needs to be achieved by the dissipation of breaking internal gravity waves.

Some potential energy considerations

Fofonoff (1961, 1998, 2001) has pointed out that when mixing occurs at a particular depth in the ocean, the contraction-on-mixing that occurs at that depth causes the entire water column above this depth to sink a little. For a sufficiently large vertical gradient of potential temperature, θ_z , *Fofonoff* has shown that this reduction of gravitational potential energy by the adiabatic slumping of the water column can be larger than the local increase in gravitational potential energy caused by diapycnal mixing, DN^2 . He then hypothesized that a feedback process might exist which would act to limit θ_z to be no larger than the critical magnitude at which diapycnal mixing leaves the gravitational potential energy of the whole water column unchanged, and some support was found for this idea in ocean data. Here we explore this idea and generalize it to include the effects of epineutral mixing and double-diffusive convection. The discussion will conclude that this feedback mechanism is unlikely to be real and hence that any observational support for this hypothesis may simply be fortuitous.

Following *Fofonoff* (2001), the gravitational potential energy (GPE) of an individual water column with respect to a fixed height z_0 is given by (using the hydrostatic balance and integrating by parts)

$$\text{GPE} = \int_{z_0}^{z_s} \rho g (z - z_0) dz = \int_0^M p / \rho dm \quad (5)$$

where z_s is the height of the free surface and m is the mass of fluid per unit horizontal area above the reference height as a function of height ($dm = \rho dz = -g^{-1} dp$). This derivation has assumed that the pressure at the sea surface is zero, but including the atmospheric pressure there adds only a tiny term that we ignore.

For this analysis of the effects of mixing processes on a single water column we are not interested in any changes in GPE caused by the adiabatic and isohaline rearrangement of water parcels (this is of central concern for the more common calculation of available potential energy). Hence the rate at which local mixing processes affect the GPE of the water column is the material derivative of the integrand in (5) minus the adiabatic and isohaline contribution to that material derivative, namely (using density in the functional form $\rho(S, \theta, p)$)

$$\left. \frac{D(p/\rho)}{Dt} - \frac{\partial(p/\rho)}{\partial p} \right|_{S, \theta} \frac{Dp}{Dt} = \chi \quad (6)$$

where we use the shorthand label χ for this non-advective local contribution to the change of GPE. This can be written as

$$\chi = \frac{p}{\rho} \left(\alpha \frac{D\theta}{Dt} - \beta \frac{DS}{Dt} \right) = \frac{p}{g\rho} e N^2 \quad (7)$$

where e is the dianeutral velocity (*McDougall* (1987a)) which is given by

$$e g^{-1} N^2 = \alpha (D\theta_z)_z - \beta (DS_z)_z - K (C_b \nabla_n \theta \cdot \nabla_n \theta + T_b \nabla_n \theta \cdot \nabla_n p) + (\beta \mathbf{F}_z^S - \alpha \mathbf{F}_z^\theta) \quad (8)$$

where \mathbf{F}^S and \mathbf{F}^θ are the double-diffusive fluxes of salt and potential temperature. The rate of change of GPE is then

$$\frac{D(\text{GPE})}{Dt} = \int_0^M \frac{p}{g\rho} e N^2 dm = \int_0^M \chi dm . \quad (9)$$

In the form (9) we learn that whenever mixing processes conspire to cause the dianeutral advection to be negative, the local contribution to the evolution of vertically integrated GPE is negative.

The contributions to χ from small-scale turbulence and from double-diffusive convection are now written as the vertical divergence of a flux (actually it is a derivative with respect to pressure) and remaining terms so that χ becomes

$$\begin{aligned} \chi = \frac{p}{g\rho} eN^2 = & -(\alpha p)_z \frac{D\theta_z}{\rho} + (\beta p)_z \frac{DS_z}{\rho} \\ & - \frac{p}{\rho} K(C_b \nabla_n \theta \cdot \nabla_n \theta + T_b \nabla_n \theta \cdot \nabla_n p) \\ & + (\alpha p)_z \frac{\mathbf{F}^\theta}{\rho} - (\beta p)_z \frac{\mathbf{F}^S}{\rho} \\ & - \left\{ p D N^2 + p g (\beta \mathbf{F}^S - \alpha \mathbf{F}^\theta) \right\}_p \end{aligned} \quad (10)$$

For any amount of diapycnal and double-diffusive mixing that is localized in space, the last line of (10) will not contribute to the vertical integral of χ in (9) and so will not affect the rate of change of GPE for the water column. Hence, following *Fofonoff* (2001), the last line of (10) can be ignored. We now express (10) as

$$\chi = \chi_D + \chi_K + \chi_{dd} \quad (11)$$

where the contributions of isotropic small-scale turbulence, of lateral mixing, and of double-diffusive fluxes are given respectively by (using the hydrostatic relationship $p_z = -g\rho$)

$$\chi_D = D N^2 - \frac{p}{\rho} D \alpha \theta_z \left(\frac{\alpha_z}{\alpha} - \frac{1}{R_\rho} \frac{\beta_z}{\beta} \right) \quad (12)$$

$$\chi_K = -\frac{p}{\rho} K(C_b \nabla_n \theta \cdot \nabla_n \theta + T_b \nabla_n \theta \cdot \nabla_n p) \quad (13)$$

$$\chi_{dd} = g(\beta \mathbf{F}^S - \alpha \mathbf{F}^\theta) + \frac{p}{\rho} (\alpha_z \mathbf{F}^\theta - \beta_z \mathbf{F}^S). \quad (14)$$

For a linear equation of state (12) confirms that small-scale turbulent mixing increases the GPE of the water column at the rate $D N^2$.

For a linear equation of state, if the mixing efficiency Γ of isotropic small-scale turbulence is say

0.11, then 11% of the dissipation of kinetic energy appears as an increase in the GPE of the water column and 89% appears as a local increase of the internal energy. (The internal energy also changes as a result of the divergence of all manner of *heat fluxes*). When the equation of state is non-linear, mixing processes can cause a net contraction-on-mixing and this has the effect of changing how much of the dissipation of mechanical energy appears as a change in the GPE and how much appears as a local increase in internal energy. If for example there were no net change in GPE so that χ_D were zero, then 100% of the mechanical energy that is dissipated in small-scale turbulent mixing will appear as a local increase in internal energy.

The development above is very clearly based on *Fofonoff's* (2001) work, but it is a little more general because the effects of lateral mixing (cabbeling and thermobaricity) are now included as is double-diffusive convection. The above approach has been developed in terms of diffusivities and so has circumvented an unknown volume fraction of mixing events that appears as an undetermined variable in *Fofonoff* (1998, 2001). Also, if we assume that there is always a background level of small-scale turbulent activity so that D always significantly exceeds the molecular diffusivity of heat, then I believe that we can ignore the process of molecular diffusion of heat that *Fofonoff* discusses. Since the positive feedback of the GPE hypothesis involves *turbulent* mixing that diffuses both potential temperature and salinity, it is not consistent to imagine a feedback that diffuses only potential temperature.

Considering first the small-scale mixing case, (12) can be written as

$$\chi_D = D N^2 \left(1 - \frac{p}{g\rho} \frac{R_\rho}{(R_\rho - 1)} \left(\frac{\alpha_z}{\alpha} - \frac{1}{R_\rho} \frac{\beta_z}{\beta} \right) \right). \quad (15)$$

If we ignore the small variations of the saline contraction coefficient compared with the vertical variation of the thermal expansion coefficient we notice that if

$$\frac{p\alpha_z}{g\rho} \frac{R_\rho}{(R_\rho - 1)} > \alpha \quad (16)$$

then χ_D will be negative so that the reduction of gravitational potential energy by the slumping of the

water column (caused by the contraction on mixing at this depth) will be larger than the local increase in GPE produced by the vertical mixing, DN^2 . The dominant contributions to α_z are $\alpha_\theta \theta_z + \alpha_p p_z$ and given the typical vertical profiles of potential temperature in the ocean, this combination is usually zero at a depth of about 1500m where θ_z is about $2.3 \times 10^{-3} \text{K m}^{-1}$ (McDougall (1988)) with α_z being positive above this depth and negative in the deep ocean where the second term, $\alpha_p p_z$, dominates. Substituting $\alpha_\theta \theta_z + \alpha_p p_z$ for α_z in (16) we find that the vertical gradient of potential temperature must satisfy

$$\left(\theta_z - g\rho \frac{\alpha_p}{\alpha_\theta} \right) \frac{R_\rho}{(R_\rho - 1)} > \frac{g\rho}{p} \frac{\alpha}{\alpha_\theta} \quad (17)$$

if the net effect of diapycnal turbulent mixing is to reduce the GPE. Using typical values of α , α_θ and α_p (17) becomes approximately

$$\left(\theta_z - 0.0023 \text{K m}^{-1} \right) \frac{R_\rho}{(R_\rho - 1)} > \frac{9}{|z|} \quad (18)$$

where $|z|$ is the depth (in m). If, for example, the stability ratio is about 2, then θ_z must exceed $4.5/|z| + 2.3 \times 10^{-3} \text{K m}^{-1}$ in order for χ_D to be negative.

Interestingly, it is mostly the magnitude of θ_z that needs to be large in order for χ_D to be negative; the sign of θ_z is less important. This can be seen by ignoring the generally small term in α_p in (17) so that (17) can be written as $\theta_z^2 > \rho N^2 / (p\alpha_\theta)$. Hence a very negative value of θ_z is as effective as a large positive value in assisting to make χ_D negative. If $\theta_z < 0$, vertical stability ($N^2 > 0$) dictates that $0 < R_\rho < 1$ which is the sense of vertical stratification conducive to the ‘‘diffusive’’ form of double-diffusive convection.

Fofonoff (1998, 2001) has used observed oceanic data to show that there is a tendency for the vertical gradient of potential temperature, θ_z , to rarely ex-

ceed the value that would make χ_D negative. To a good approximation, this means that θ_z is observed to be rarely greater than that given by equality in (18). Fofonoff provides the following tentative explanation for this observation. He hypothesizes that if χ_D were to become negative (essentially due to a large value of $p\alpha_z$, see (16)) then the excess GPE released ‘‘will enhance local mixing’’ so increasing D and further decreasing χ_D (see (15)). This escalation of local vertical mixing will vertically diffuse potential temperature, so reducing both θ_z and $p\alpha_z$ and hence increasing the value of χ_D towards zero.

This hypothesis describes a feedback mechanism whereby the GPE that is released due to the local contraction-on-mixing somehow causes the ocean circulation to alter in just such a way as to provide the prevailing conditions necessary for an increase in the small-scale turbulent mixing at just the depth where it can be most effective at further reducing the GPE. This feedback process is then thought to continue until θ_z and $p\alpha_z$ are reduced by the enhanced vertical diffusion and χ_D becomes zero. Fofonoff (2001) recognized that this feedback mechanism is an unproven hypothesis that is lacking a dynamical basis to date.

The enhanced vertical turbulent mixing of this feedback hypothesis diffuses not only potential temperature but also salinity, while not changing the value of the buoyancy ratio. If the stability ratio, $R_\rho = \alpha\theta_z / \beta S_z$, is regarded as given, then it must be recognized that this proposed mechanism is not so much a hypothesis for limiting θ_z but rather is a hypothesis for limiting the buoyancy frequency. Since N^2 is equal to $g\alpha\theta_z(1 - R_\rho^{-1})$, we find that the criterion for χ_D to be negative can be written as (from (17))

$$N^2 > \frac{g^2 \rho \alpha}{\alpha_\theta} \left(\frac{\alpha}{p} \left(\frac{R_\rho - 1}{R_\rho} \right)^2 + \alpha_p \frac{R_\rho - 1}{R_\rho} \right) \quad (19)$$

where we remember that this expression, like (16) (17) and (18), is approximate in the sense that the variations of β have been ignored in comparison with those of α (including ignoring two terms in β_θ).

It is clear that the reduction in GPE that is caused

by the contraction-on-mixing at depth must be an important part of the energy budget, and to quote *Fofonoff* (2001), this process “may serve as an energy source to supplement the tidal, internal wave, and other processes that mix the ocean”, but I have difficulty accepting that this mechanism operates to limit the vertical gradients of potential temperature, salinity and buoyancy in the ocean. In order to make this connection one has to believe that having done a little mixing at a particular location in space where χ_D is negative, the ocean has some way of being able to create the conditions required for small-scale turbulent mixing (read breaking internal gravity waves) at just the correct height and at the exact horizontal location where the original mixing occurred. This seems implausible. I prefer to think that the limiting value of N^2 , namely (19), is rarely exceeded simply because the buoyancy frequency does tend to decrease with increasing pressure in the ocean for reasons unrelated to this energy feedback hypothesis.

Consider now the effect of epineutral mixing processes on the evolution of GPE. Since the sum of cabbeling and thermobaricity is usually in the sense of a downward dianeutral velocity it is clear from (4) and (13) that χ_K is almost always negative so that the effect of lateral mixing is almost always to reduce the GPE. Should we then hypothesize that this energy source for mixing will somehow feed back to reduce the magnitude of χ_K ? The most effective way to reduce the magnitude of χ_K would be to reduce the lateral heat flux carried by this epineutral mixing. Such a feedback mechanism would then need to act to *reduce* the amount of mesoscale eddy activity. Surely it is difficult to believe that this alteration in the intensity of mesoscale mixing could occur simply as a consequence of the rather subtle nonlinear nature of the equation of state. It is especially difficult to accept because the ocean would need to pinpoint its response to just the location where the initial cabbeling and thermobaricity occurred. But perhaps a closer analogy with the original hypothesis would be that the negative value of χ_K would lead to an increase in the amount of *diapycnal* mixing at the location of the cabbeling until eventually $\chi_D + \chi_K$ was increased to be non-negative. The difficulty in imagining a series of physical processes that would focus the ocean’s response back to the original location of cabbeling still remains.

Finally, consider the influence of double-diffusive

convection on the evolution of GPE. From (14) we see that if the equation of state is linear then $\chi_{dd} = g(\beta \mathbf{F}^S - \alpha \mathbf{F}^\theta)$ and it is a characteristic of double-diffusive convection that this density flux is negative. That is, even for a linear equation of state, double-diffusive convection acts to reduce the GPE of a water column. If we extend the GPE contraction-on-mixing feedback hypothesis to this situation, then it would seem that double-diffusive convection would always carry the seeds of its own destruction. That is, a little bit of double diffusive convection would first cause a negative χ_{dd} and a reduction in GPE. Then the above hypothesis would imply that this reduction in GPE arranges for enhanced isotropic small-scale mixing at just the location where the double-diffusive convection occurred. The turbulent diapycnal mixing then would do battle with the double-diffusion, smoothing out the vertical property gradients, presumably until $\chi_D + \chi_{dd}$ became non-negative.

If we take the depth-integrated GPE feedback process at face value, it would be expected to operate whenever the dianeutral velocity is negative since the contribution of local processes to the rate of change of GPE is (see (7)) $\chi = peN^2/g\rho$. The GPE feedback hypothesis would suggest that whenever the ocean detects that e is negative at some location, it would somehow increase the vertical diffusivity, D , at that location, so increasing χ_D until the negative downwelling was arrested. In this way it would seem that the presence of lateral mesoscale eddy diffusion and cabbeling would lower the bar at which this regulating mechanism would come into play since χ_D would still be positive when χ becomes negative and the feedback mechanism kicks in.

If we were to take this localized GPE feedback mechanism seriously as a source of mixing activity, would it be expected to occur as a sudden onset whenever χ became negative? Two possibilities come to mind. One suggestion might be that the ocean is not 100% efficient at focusing its response at the exact location where the mixing activity first occurred and that one should perhaps expect that χ should be negative by some significant margin before the feedback process was in fact effective. Another suggestion might be that well before χ actually becomes negative, the fact that the dissipation of kinetic energy does not in fact give up its regular fraction $\Gamma \approx 0.11$ to increase the GPE of the water column

might mean that the efficiency of local mixing is increased to be larger than 0.11 so as to have the effect of still delivering the same fraction of the dissipation to depth-integrated GPE. These two different possibilities are simply offered as additional speculation on this GPE feedback process.

The conservation equation of total energy can be written as

$$(\rho E)_t + \nabla \cdot (\rho \mathbf{u} E) = -\nabla \cdot (p \mathbf{u}) - \nabla \cdot \mathbf{F}_Q + \nabla \cdot \left(\mu \nabla \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} \right) \right) \quad (20)$$

where

$$E = I + \Phi + \frac{1}{2} \mathbf{u} \cdot \mathbf{u} \quad (21)$$

is the sum of the specific internal energy, I , the geopotential, $\Phi = gz$, and the kinetic energy per unit mass, \mathbf{F}_Q is the molecular and radiative fluxes of heat, and μ is the dynamic viscosity. On volume integrating (20) over the whole ocean it is apparent that when kinetic energy is dissipated it must appear as an increase of the volume integral of $I + \Phi$. Similarly, if the nonlinear nature of the equation of state causes a reduction in the volume integrated GPE, this could appear as either an increase in the volume integrated internal energy or as an increase in the volume integrated kinetic energy. But there is no requirement that a decrease in the depth-integrated GPE on one particular cast should appear as an increase in kinetic energy on that cast, nor indeed at the depth of the original mixing. Even if there was a rather localized increase in kinetic energy, there is no guarantee that all of this increased kinetic energy would be dissipated locally and so cause the increased local diapycnal mixing that is part of the assumed feedback mechanism.

In summary it is difficult to imagine why the hypothesized feedback process would alter the mixing activity just at the specific location where the initial burst of mixing occurred, especially as the slumping of the water column and therefore any change to the horizontal pressure gradient occurs at all heights above the mixing location.

Conclusions

The link between vertical mixing and the dissipation of mechanical energy has been examined in this

paper. The nonlinear nature of the equation of state dictates that the relationship between dianeutral upwelling and the vertical diffusivity is more complicated than is normally assumed. To be specific, the vertical gradient of the dissipation, ε_z , is normally assumed to be equal to $\Gamma^{-1} e N^2$ but the second term in (3) which involves the vertical gradient of the thermal expansion coefficient (and of the saline contraction coefficient) must also be included.

Several special mixing processes have been discussed that each allow fluid to dianeutrally upwell independently of the dissipation of kinetic energy. These special processes are lateral cabbeling, lateral thermobaricity, the dianeutral advection of sub-mesoscale coherent vortices, and the helical nature of neutral trajectories. This last process is the hardest to understand and also the hardest to quantify.

The last half of this article has explored the implications of the ‘‘gravitational potential energy local feedback hypothesis’’ that has been advanced by *Fofonoff* (1961, 1998, 2001). This is a rather intriguing hypothesis and it can be summarized as follows. When mixing processes occur at any depth in the ocean, the non-linear nature of the equation of state (and also double-diffusive convection) usually cause a net contraction in the vicinity of the mixing and consequently, the gravitational potential energy of the fluid above this depth decreases. The argument then notes that in some circumstances the loss of depth-integrated gravitational potential energy by the slumping of this water column can exceed the local increase in gravitational potential energy that is associated with the mixing. So far so good. Then the hypothesis says that when there is a net reduction in the gravitational potential energy of the whole water column, this net energy will feed back and cause extra diapycnal mixing activity at exactly the location of the original mixing activity. This seems unlikely and in the absence of a physical mechanism to provide this pin-pointed feedback, I am inclined to discount the hypothesis.

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