

Internal Wave Breaking at Variable Topographic Slopes

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Abstract. Several recent observational, numerical and theoretical studies indicate that internal tides created by tidal flow over topography have most of their energy in relatively low vertical modes, with insufficient associated shear to lead to significant mixing. Further interactions between the tidally generated internal waves and topography are necessary to transfer energy to smaller vertical scales. Here results of numerical simulations are presented, examining mixing generated by internal wave reflection from variable topographic slopes, including concave and convex slopes near the critical angle, and corrugated slopes.

Introduction

Recent estimates of tidal dissipation (*Egbert and Ray, 2000; Munk and Wunsch, 1998*) suggest that the tides may provide a source of energy for diapycnal mixing of similar magnitude to the wind. Topographic boundaries appear to play an important role in the conversion of barotropic tidal energy into baroclinic energy in the form of internal tides. However several recent studies suggest that internal tides generated by barotropic tidal flow over topography are dominated by low vertical modes, without sufficient shear to lead directly to mixing (*St. Laurent and Garrett, 2002*). Further interactions with topography may be necessary to transfer the baroclinic energy from large to small scales, at which Richardson numbers may be small enough for instability and mixing. Here two examples of reflection of low mode internal tides from sloping topography are considered, both leading to shears on small scales, and diapycnal mixing localized near the topographic boundary.

Internal tide reflection at a near-critical slope

The first scenario we consider is reflection of a low mode internal tide from a sloping boundary near the critical angle (equal to angle of the wave group velocity to the horizontal). This problem has been considered in detail for planar slopes by many authors (*Ivey and Nokes, 1989; Cacchione and Wunsch, 1974; Eriksen, 1982*). Internal waves preserve the angle between the group velocity vector and the horizontal upon reflection from a sloping boundary. For waves normally incident on a slope, traveling from deeper to shallower water, the reflected wave therefore has higher wave-numbers and greater energy density than the incident wave. Labora-

tory experiments and numerical simulations have shown that this increased energy density leads to turbulent mixing when the slope is close to the critical angle (*Ivey et al., 2000; Slinn and Riley, 1996*). Actual topographic slopes in the ocean are of course far from planar, and here internal wave reflection from non-uniform slopes is considered.

Earlier theoretical predictions for small amplitude internal waves (*Gilbert and Garrett, 1989; Müller and Liu, 2000*) suggest that concave critical slopes will have less energy enhancement near the slope than convex or planar slopes, due to cancellation between waves reflected from above and below the critical point. However, numerical simulations (described in detail in *Legg and Adcroft, 2003*) show that for finite amplitude incoming waves, similar levels of mixing can be seen on critical slopes of all shapes. In all cases a bore is generated near the bottom of the slope once per tidal period, shown for three different slope shapes in Figure 1. The bore runs up the slope, and convectively overturns the fluid above the slope as it moves. The net effect is a band of mixed fluid within which stratification is greatly reduced, bounded below by the topography and above by a thin pycnocline of increased stratification, aligned with the wave characteristic.

The apparent contradiction between these fully non-linear numerical simulations and the theoretical predictions can be explained by the finite amplitude of the waves. The bores, with which mixing is associated, are only generated when the reflected wave Froude number exceeds unity. When bores, rather than linear waves, are generated through reflection, the asymmetric nature of the features prevents the exact cancellation of reflected waves predicted for concave slopes in the linear case. Geometric arguments (*Phillips, 1977*) suggest that the reflected Froude number is greater than unity for a range of topographic slopes dh/dx around the crit-

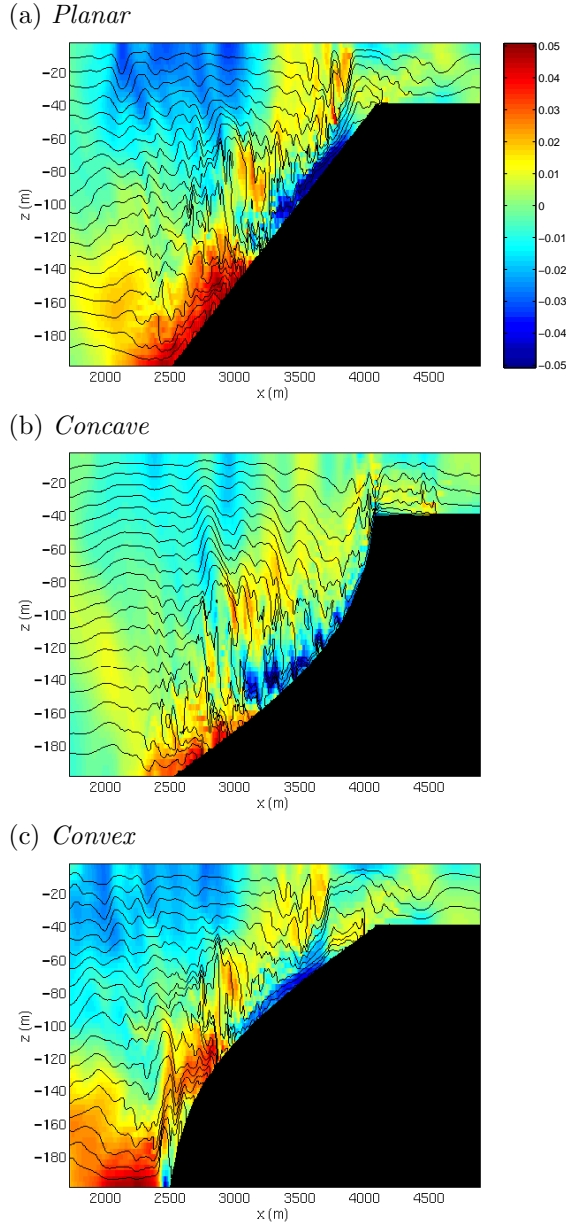


Figure 1. Snapshots of cross-slope velocity (color) and buoyancy at a time $t = 5.39T$ after the beginning of the calculation. The color scale extends between $U = -0.05\text{m/s}$ (blue) $\rightarrow U = 0.05\text{m/s}$ (red) and the contour interval is $\Delta b = 9.81 \times 10^{-6}\text{m/s}^{-2}$, (where g is the gravitational acceleration).

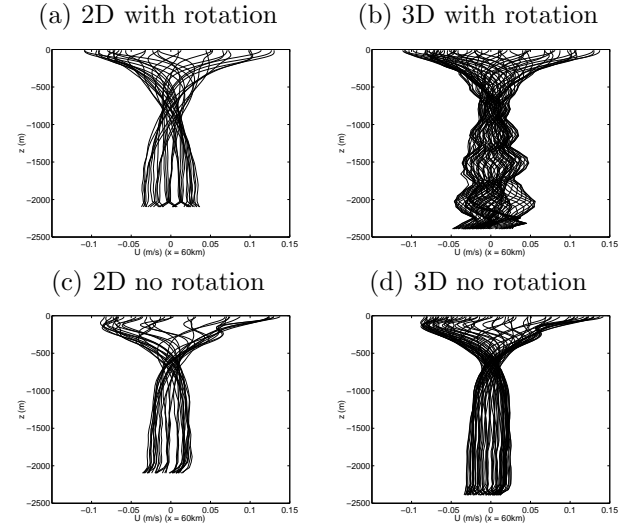


Figure 2. Vertical profiles of cross-slope baroclinic velocity at a location on the continental slope shown over several tidal cycles for (a) 2D simulation without corrugations, including rotation; (b) 3D simulation with corrugated topography, including rotation; (c) 2D simulation with no rotation, (d) 3D simulation with corrugated topography, but without rotation. The simulations are forced by a barotropic M2 tide orientated in the cross-shelf direction. An internal tide is generated at the shelf-break and propagates offshore, reflecting from the continental slope near the location of the profiles.

ical angle given by

$$s \frac{Fr_I^{-1/2} - 1}{Fr_I^{-1/2} + 1} < \frac{dh}{dx} < s \frac{Fr_I^{-1/2} + 1}{Fr_I^{-1/2} - 1} \quad (1)$$

where $s = [(\omega^2 - f^2)/(N^2 - \omega^2)]^{1/2}$, the slope of the wave characteristic, and $Fr_I = U_0 m_I / \omega$, the incoming wave Froude number. ω is the wave frequency, N is the buoyancy frequency, f is the Coriolis frequency, m_I is the vertical mode number of the incoming wave, and U_0 is the velocity amplitude of the incoming wave. The range of slopes agrees with those where mixing is found in the simulations.

Internal tide reflection from a corrugated slope

The second scenario we consider is reflection of a low mode internal tide from a subcritical slope with corrugations running up and down the slope. The scenario is motivated by the continental slope off the SE USA (Nash *et al.*, 2003), but similar features are found on continental slopes around the world and on the mid-ocean ridges. Numerical simulations (described in de-

tail in *Legg* (2003) examine a low mode internal wave traveling in the plane normal to the shelf-break. In this case the internal tide is generated by the barotropic tide at the shelf-break and travels toward the open ocean, but similar behavior would be expected for a wave generated elsewhere and traveling toward the shelf-break). The wave reflects from the subcritical slope dissected by corrugations, and above the corrugations velocity profiles with smaller vertical scales are seen. This small-scale shear is not generated when planetary rotation is not included (Figure 2).

A theoretical interpretation (following *Thorpe*, 2001), extended to include rotation) is that in addition to the primary reflected wave, scattered waves are generated when the rotationally-induced along-slope component of flow associated with the incident wave moves over the corrugations. The scattered waves have a wavenumber in the along-slope direction equal to that of the topographic corrugations. Hence for small-scale corrugations (i.e., $\lambda < 2H/s$, where λ is the topographic wavelength, H is the ocean depth, and s is the wave characteristic slope) the superposition of incident, reflected and scattered waves will lead to velocity profiles with smaller scale shears than in the incident wave alone. These shears can lead to mixing above the corrugations if Richardson numbers are sufficiently small.

Discussion

Two processes involving internal tide reflection from variable slopes - concave and convex critical slopes, and corrugated subcritical slopes - have been described and shown to lead to mixing near the topographic boundaries. Processes like these might form the end of the chain of energy conversion, from barotropic to baroclinic tides, through wave propagation and wave-wave interaction. Hence parameterizations of the mixing induced by wave reflection could form a component of a complete tidal mixing parameterization scheme, such as the framework being explored by *St. Laurent et al.* (2002), (which currently assumes simply that a specified fraction of the baroclinic energy is in the form of propagating waves, and that the waves lead to a uniform background level of mixing) or the Internal Wave Action Model of Peter Müller (private communication). The mixing induced by reflection from near-critical slopes could be parameterized by an enhanced diffusivity in a band bounded by an internal tide characteristic over slopes which satisfy the Froude number criterion, given the amplitude and wave length of the internal tides (from another component model). Similarly the mixing induced by wave scattering from corrugations could be parameterized by increased diffusivity above small wavelength corrugations, again given the amplitude and

wavelength of the incident internal tides. Both suggest the need for detailed knowledge of the topography, particularly on small horizontal scales.

However, we are still a long way from having the necessary information to create a complete parameterization of boundary mixing due to internal tide reflection processes. Especially uncertain is the role of transport of mixed fluid away from the boundary in restratifying the boundary fluid, allowing mixing to continue indefinitely. These export processes cannot be determined from the simulations described here. (Those in the first part were 2-dimensional, and those in the second part were not sufficiently long in duration.) Export processes are likely to be highly three-dimensional, and strongly modified by spatial variations in the amount of mixing. For example laboratory experiments by *Wells* (2003) have shown that spatially localized mixing next to a boundary leads to the generation of secondary circulations which provide a very efficient mechanism for transport of mixing fluid away from the boundary and into the interior. Further studies are needed to examine how the export processes interact with the mixing processes themselves.

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