

Meeting report: From stirring to mixing in a stratified ocean

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Over fifty years ago, Carl Eckart described the sequence of processes as a passive scalar is stirred and mixed in a turbulent flow (Eckart, 1948). At first, during the stirring phase, the variance of the scalar gradient is greatly increased, but later, during the mixing phase, the gradients become sufficiently sharp that molecular diffusion becomes important and the gradient variance rapidly decreases. The process is of great practical importance in many engineering situations as well as being familiar from adding cream to coffee.

The interplay of stirring and mixing is also important in the ocean for temperature, salinity, chlorophyll and other naturally occurring tracers as well as for introduced material. The situation is complicated by the numerous physical processes that can cause stirring and by the interplay of processes that act along isopycnals (surfaces of constant mean density, where “mean” requires careful definition) with “diapycnal” processes acting across the density surfaces.

The Twelfth ‘Aha Huliko’ a Hawaiian Winter Workshop was convened from 15 to 19 January 2001 to review this theme, with the support of the Office of Naval Research and the participation of 25 invited speakers. The general topics included in presentations and discussions were (1) mesoscale eddy parameterization, (2) general approaches to stirring, including the application of ideas from dynamical systems theory, (3) inertial instability, submesoscale motions and vortical motions, (4) the interplay of isopycnal and diapycnal processes, (5) processes in the surface mixed layer, (6) the stirring and mixing of active, particularly biological, tracers, (7) mixing efficiency, i.e. the fraction of energy lost from the mean flow which produces a vertical buoyancy flux, and (8) differential mixing of heat and salt in doubly-stable environments as well as those which permit double diffusion. Before summarizing the discussions on these topics, we first review the general approach to parameterizing unresolved processes.

The need for parameterization

Ocean general circulation models used in climate studies lack the resolution to treat small-scale processes explicitly and so must parameterize their effects in terms of the

average properties that are resolved. If the eddy flux of a scalar is related by a tensor to the local gradient of the mean concentration of the scalar, as for a mixing length argument, the symmetric part of the tensor is diagonalizable and may be thought of as anisotropic diffusion. The general assumption is that two principal directions are along the “neutral” surfaces of constant mean potential density, with very much greater diffusivity in these directions than in the direction normal to the surfaces. The isopycnal diffusion along the neutral surfaces is largely associated with mesoscale eddies, whereas the diapycnal mixing across them is caused by breaking internal waves and, in some locations, double-diffusive processes.

The antisymmetric part of the tensor describes a “skew flux” normal to the mean gradient of the scalar. Part of this flux is non-divergent and does not affect the mean scalar concentration. The remaining, divergent, part may be expressed as simple advection of the scalar by an additional mean flow, the “skew velocity”, over and above that which would be recorded by current meters at a fixed location. To some extent the skew flux can be thought of as representing an eddy flux of thickness, or spacing between neighbouring isopycnals, but it is actually a more subtle representation, in a coordinate system using z as the vertical coordinate, of a flux that would appear in a coordinate system using the potential density instead.

These results are purely kinematic. Specific numerical values for the diffusivities and skew velocities could be obtained from suitable measurements in the present ocean. Any model that aspires to predictive capability, however, requires an understanding of the dynamical processes responsible for the transports to which the model output is sensitive. The parameterizations must then be specified as formulae rather than just numerical values. Consider diapycnal mixing by internal wave breaking. If it is to be characterized not by a number but by a formula then the origin of the wave field becomes important. Vastly different feedback loops can be obtained in climate studies depending on whether the intensity of the wave field and hence diapycnal mixing is determined by the ocean circulation, cascading energy via eddies to the waves, or by the atmospheric wind field generating waves at the sea surface, or by the surface tide generating waves at the sea floor.

The need to parameterize subgrid processes is not idiosyncratic to oceanic general circulation models but generic to all models. There are always unresolved processes.

Mesoscale eddies

So-called mesoscale eddies, with a spatial scale of the order of 100 km and a time scale of the order of a month, are a dominant feature of ocean variability. They are dynamically analogous to synoptic-scale eddies in the atmosphere. The deformation of an air layer by such eddies is depicted in Pierre Welander's (1955) now famous check-board figure, here reproduced as Figure 1. The figure also depicts the stirring along oceanic isopycnals if time and space are appropriately rescaled.

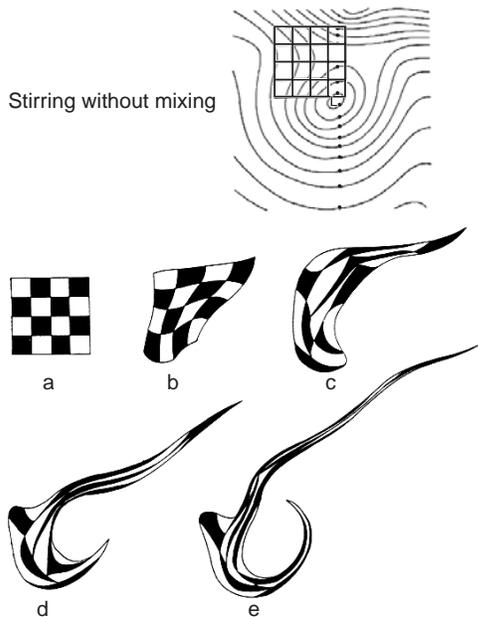


Figure 1. Stirring without mixing of passive tracer. The initial streamlines of the two-dimensional turbulent flow are shown on top. The flow deforms the checkerboard pattern into elongated filaments. In the atmospheric case, the length of the checkerboard is 1200 km and the deformation is shown after 6, 12, 24 and 36 h. In the oceanic case, the length of the checkerboard is 100 km and the deformation is shown after 3, 6, 12 and 18 d. (After Welander, 1955).

Determining the magnitudes of the isopycnal diffusivity and skew velocity associated with oceanic mesoscale eddies requires an understanding derived from a combination of general principles, dynamical studies of processes such as baroclinic instability, and numerical models that resolve the eddies in various idealized situations. There are no simple answers. Invoking general principles, such as mixing potential vorticity, can imply unphysical sources of

momentum and energy; eddy parameterizations should not do what eddies cannot do! To some extent the eddies serve to release available potential energy from the mean state, as in the popular scheme of Gent and McWilliams (1990), but this will lead to an ocean at rest in a situation, without forcing or dissipation, where a steady, stable, circulation is more likely. Moreover, idealized eddy-resolving runs have mostly been used to produce numerical values for the free parameter in the Gent and McWilliams scheme, rather than the formula in terms of resolved variables which we really need for predictive models.

The extant parameterizations of mesoscale eddies really just treat the stirring effect of the eddies. One unanswered question is whether this stirring connects directly to molecular mixing at very fine scales, or whether there is an intermediate mechanism, in series with the eddy stirring, which accomplishes the transfer to mixing. Another question concerns the ultimate fate of the energy released from the mean state. Parameterization schemes essentially assume that this is dissipated adiabatically, but this may not be true. We return to both these issues later.

Another very serious question is whether the effect of mesoscale eddies can be represented as a purely local effect, with eddy fluxes just proportional to local mean gradients. This may be reasonable in some regions, such as the Antarctic Circumpolar Current, but inappropriate for non-zonal flows such as the Gulf Stream from which radiating Rossby waves and coherent rings may carry properties over large enough distances to invalidate any local mixing length approach. In such situations eddies may drive, rather than dissipate, the mean flow.

Ocean models may remain inadequate until mesoscale eddies are resolved explicitly. Quite apart from the difficulties associated with mesoscale eddies, high resolution also seems to be required to produce a sufficiently high effective Reynolds number for the correct representation of phenomena such as the separation, variability, and penetration of the Gulf Stream. Although results stabilize with increasing resolution, they remain sensitive to choices in forcing and viscosity parameterization, even at the finest grid spacing used today.

General approaches to stirring

Turbulent stirring leads to the distortion of a tracer field. The exact distortion is determined by the complete Lagrangian evolution of the flow field, but much can be inferred from the velocity gradient tensor. If the relative motion near a point is dominated by vorticity, then tracer gradients tend to align with the shear and not increase. If, on the other hand, the rate of strain dominates the relative

motion, then a tracer tends to become elongated along the principal axis having the largest rate of strain and the tracer gradient grows exponentially.

Simulations of two-dimensional flows show additional phenomena such as barriers to mixing and areas of filament production. These barriers and areas can be identified when additional information is used. In one approach, the vorticity (from the velocity field) is combined with the rotation of the rate of strain tensor (from the acceleration field) into an effective rotation to characterize the flow field. Another approach uses concepts from dynamical systems theory. It identifies hyperbolic regions in the flow field (characterized by strong deformations in the velocity gradient tensor) to calculate special material curves, or manifolds, that determine the stirring and mixing. A blob of tracer can advect without much distortion, but then be ripped apart rather abruptly when it encounters these hyperbolic regions in a flow. Realistic simulations of the surface currents in the Gulf of Mexico illustrate this process (Figure 2).

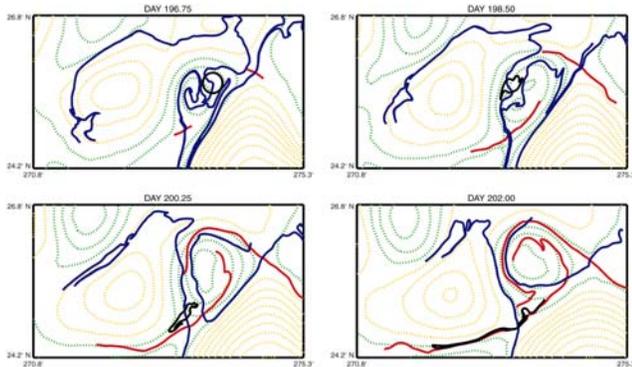


Figure 2. Evolution of a blob of tracer in a simulation of the surface currents in the Gulf of Mexico. The Loop Current is located in the lower right corner and a cyclone to the northwest. Height anomalies are shown as dashed lines. The red and blue lines are inflowing and outflowing manifolds. There are two hyperbolic regions associated with the cyclone and the Loop Current. On day 196.75 a circular blob (black) straddles an inflowing manifold (blue). This manifold delineates which portion of the blob will travel west into the central Gulf and which portion will travel east through the Straits of Florida. The blob is stirred with modest deformation for the next 3.5 days as it rotates around the cyclone. On day 200.25, however, the blob approaches the hyperbolic point created by the intersecting inflowing and outflowing (blue) manifolds to the southwest of the cyclone. Here it begins to show strong compression along the inflowing manifold direction and strong distortion along the outflowing manifold direction. Day 202.00 shows the fate of the blob when it reaches this intersection. It has collapsed along the outflowing manifold. (Courtesy of A.D. Kirwan and L. Kantha).

Inertial instability, submesoscale motions, and vortical motions

Mesoscale motions may arise from baroclinic instability of large-scale flows in the ocean and may, in turn, be unstable to submesoscale eddies an order of magnitude smaller. Near the sea surface it seems that these eddies, often visible at the sea surface via beautiful spiral patterns (Figure 3), are mainly cyclonic, quite possibly because anticyclonic ones would break down rapidly via an inertial, or barotropic, instability. These small eddies may be a link in the pathway from stirring to mixing, but this has not been quantified. Similarly, in the ocean interior there seems to be a family of “vortical modes”, of largely unknown scale range and intensity, which play a role in lateral stirring. They can be generated by frictional effects in flow past topographic features and, possibly, by the collapse and geostrophic adjustment of regions mixed by breaking internal waves.

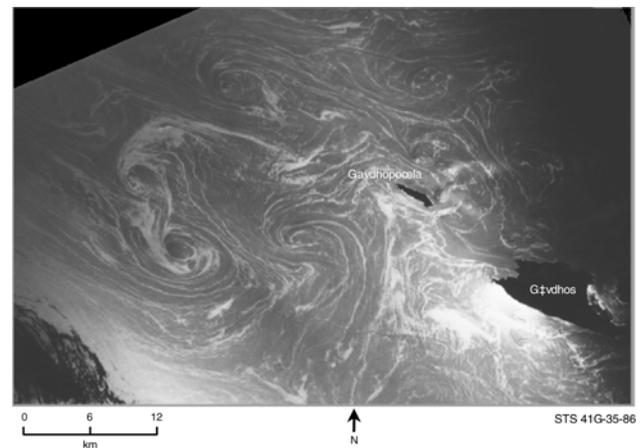


Figure 3. A pair of interconnected spirals in the Mediterranean Sea south of Crete. The vortex pair has a clearly visible stagnation point between the two spirals, the cores of which are aligned with the preconditioning wind field. Picture taken on October 7, 1984. (Courtesy of W. Munk).

A general question about the links between low frequency eddies and higher frequency motions concerns the extent to which the mesoscale or submesoscale eddies generate internal waves, particularly in situations where the eddies generate sharp fronts. Theoretically, the mesoscale and submesoscale eddies are assumed to satisfy some sort of balanced dynamics, such as geostrophy. These balanced motions are also called the “slow manifold”. Balanced turbulent flows exhibit an upscale energy cascade and are very inefficient in dissipating energy. Internal gravity waves and other more nearly isotropic flows, on the other hand, exhibit a downscale cascade with effective dissipation. The structure of the equations governing bal-

anced flows is such that they can be integrated forward in time only if certain conditions are met. Analysis of several classical instability problems shows that the fluid-dynamical transition occurs in the neighborhood of these integrability conditions. It is hypothesized that violation of these conditions characterizes in general the flow regimes where energy is transferred from balanced to unbalanced motions.

The interplay of isopycnal and diapycnal processes

When a scalar is stirred by high Reynolds number turbulence in homogeneous water, the eddy flux is mainly determined by the turbulent stirring and is largely independent of the small value of molecular diffusivity. Reducing the latter would merely lead to finer scales in the tracer field, without significantly affecting the turbulent transport. The situation is more subtle in the ocean because of the need to quantify both isopycnal and diapycnal transports. One issue, already raised, is the extent to which energy released by mesoscale eddies is dissipated in the ocean interior, perhaps by breaking internal waves which have drawn some of that energy from the eddies. This could partly tie diapycnal mixing rates to the lateral stirring rates.

Another issue concerns the ultimate mechanism for the destruction of fine-scale lateral gradients of temperature and salinity created by stirring on isopycnals. One possibility is that vertical shear of the stirring process leads to very strongly tilted frontal regions, so that pre-existing diapycnal mixing can remove the strong gradients. This could be described as a “passive” destruction of the strong lateral gradients. A more active process is possible: thermohaline fronts caused by lateral stirring on isopycnals could become unstable to a double-diffusive interleaving process. This would introduce further diapycnal mixing at the same time as it destroyed lateral gradients.

The relative importance of these scenarios has not been established. Preliminary estimates favor the latter process, the signature of which should be internal double-diffusive interleaving regions in the ocean. These are certainly known to occur, but a systematic catalog of their locations and characteristics has not been developed. If the process is important it may provide significant diapycnal mixing in regions with strong lateral gradients of potential temperature, and compensating salinity, on isopycnals. Moreover, the diapycnal mixing is upgradient for density, as is necessary in any double-diffusive process.

A significant sub-theme which emerged on this and other issues concerns the extent to which different stirring and mixing processes are in parallel or in series. If two

processes are in parallel, their strengths may be compared to decide which is more important. If processes are in series, one might need to know which is the rate-controlling one. In the usual case of the stirring of a tracer in a homogeneous fluid, molecular mixing clearly operates in series, but with the ultimate mixing rate being controlled by the turbulent stirring. In the ocean, where there might be some intermediate process between the stirring agents and the final molecular diffusion, the answer may be more complicated. For example, the flux of warm water across the Gulf Stream may be controlled by the amount of air–sea interaction that occurs in warm-core rings before they are reabsorbed by the Stream; the flux is not necessarily controlled just by the ring formation rate. The general issue of series or parallel processes needs to be borne in mind in any attempt to focus on processes that are important, and may be aided by a triple decomposition into mean, eddies and turbulence (Figure 4).

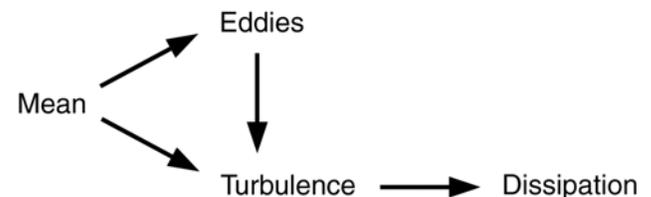


Figure 4. A simple schematic of the path of scalar concentration variance from mean to dissipation via eddies and turbulence. (Courtesy of C. Garrett).

Processes in the surface mixed layer

Because of intense vertical mixing there, the surface mixed layer of the ocean provides many examples of the interplay of stirring and mixing processes. One such process relies on the fact that the density of sea water is determined by both temperature and salinity.

Typical of the surface mixed layer are compensated fronts that are warm and salty on one side and cold and fresh on the other side such that the density contrast is small. Figure 5 shows an example along a longitude line in the subtropical North Pacific. The existence of the fronts has been rationalized as follows (Young, 1994): Nonhomogeneous atmospheric forcing and entrainment of thermocline waters generate horizontal gradients in temperature, salinity, and density. Density fronts slump by the action of horizontal density-driven shear currents which are eventually arrested by vertical mixing. This process can be

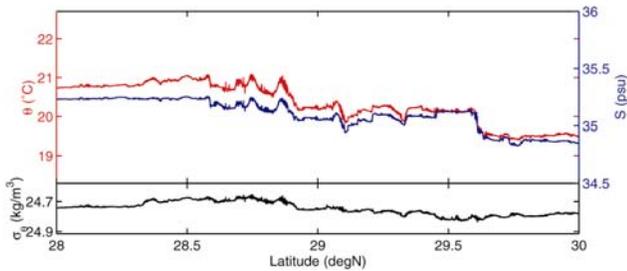


Figure 5. Potential temperature (red line), salinity (blue line) and potential density (black line) from a horizontal Sea Soar tow at 50 m in the subtropical North Pacific, at 140°W, between 28 and 30°N. The depth is in the middle of the local mixed layer. The vertical axes for temperature and salinity are scaled by the expansion coefficients so that excursions of temperature and salinity show the change they imply for density. (Courtesy of R. Ferrari).

thought of as shear dispersion. Compensated fronts do not experience such shear dispersion. The result is that density fronts diffuse whereas compensated fronts persist. The ubiquitous existence of compensated fronts suggest that horizontal mixing in the surface mixed layer is very much larger for density than for “spice”, the combination of temperature and salinity that is “orthogonal” to density. Indeed, eddy-resolving models that diffuse temperature and salinity and hence density and spice with the same coefficient are unable to reproduce the observed tendency towards thermohaline compensation.

Air–sea fluxes and the behavior of the surface mixed layer are also involved in a thermodynamic approach to water-mass formation. Basically, water must flow across isopycnals to balance surface buoyancy loss or gain, and the convergence of this volume flux provides an estimate of the subduction rate. This rate may be compared with a dynamically-based prediction from Ekman convergence. Both the thermodynamic and dynamic approaches, however, require allowance for the role of eddies and diurnal and seasonal cycling of the mixed layer depth. The problem is thus complicated by the interplay of various stirring and mixing processes which need to be understood.

The stirring and mixing of non-conservative tracers

Remote sensing of the sea surface from space has made us aware of beautiful and complex patterns in biologically active variables such as chlorophyll (Figure 6). One question is whether the observed patchiness is a consequence of biological processes or merely of the passive stirring of large-scale gradients. Some clues come from comparing the shapes of the wavenumber spectra of active and pas-

sive scalars, such as chlorophyll and temperature respectively. If the former shows comparatively more variance at small scales, this is evidence for the importance of scalar growth processes.

Accounting for such patchiness in a model is particularly difficult if the non-conservative processes are nonlinear. In that case it is clearly inadequate to proceed with a model that deals only with an ensemble average variable; a Lagrangian approach which follows the development of individual patches is necessary. A very basic model of this kind is the “Brownian bug model”. It describes an ensemble of bugs that perform random walks and experience random birth and death processes along the way. Mathematically, this is a “superprocess.” A homogeneous initial distribution spontaneously develops patches and voids.

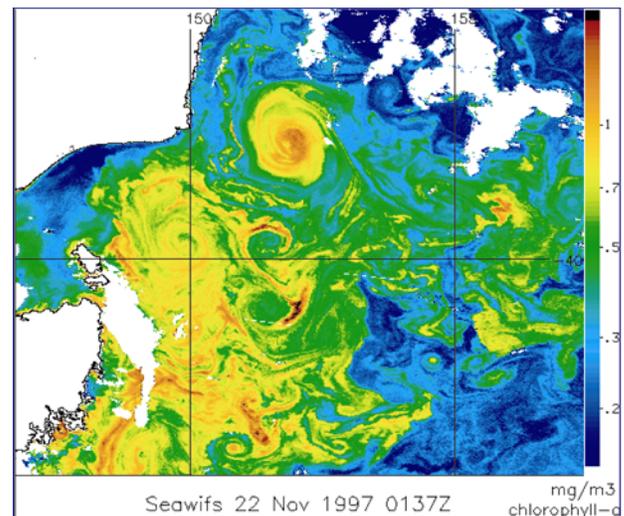


Figure 6. Chlorophyll concentration in the Tasmanian Sea as observed by SeaWiFS. (Courtesy of B.L. Hua).

Examining data for evidence of patchiness or intermittency can be aided by examination of high order structure functions. For a variable $C(\mathbf{x}, t)$ the q th order structure function is defined as $\langle \delta(C(l))^q \rangle$, where $\delta C(l)$ is the difference in C over a separation l and $\langle \rangle$ denotes an ensemble average. The structure function exponent $\zeta(q)$, defined by fitting a power law $l^{\zeta(q)}$ to the structure function, characterizes the degree of intermittency for multifractal processes. It is a linear function of q for non-intermittent processes and becomes a nonlinear concave function for intermittent processes. Calculation of this exponent from oceanic observations suggests that, in the scale range of nearly isotropic turbulence, passive scalars such as temperature are more intermittent than the advective velocity field. This is a result also obtained in numerical simulations of 3D homogeneous turbulence (e.g., Jiménez, 2000). Biologically

active tracers show intermittency beyond that associated with turbulent stirring. The excess increases with decreasing turbulence intensity (Figure 7). These observational results need to be corroborated and understood. Further to this, the possibility of a relationship between biological and physical variables can be studied through examination of the behavior of joint structure functions, though caution is required to avoid over-interpretation of artifacts introduced by particular sampling schemes or noise.

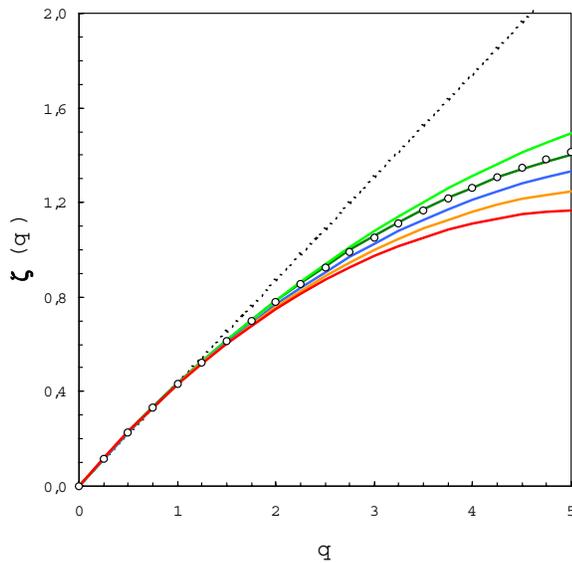


Figure 7. Empirical values of the structure function exponent $\zeta(q)$ for temperature (circles) and in vivo fluorescence for different values of the turbulent dissipation rate ε (red $\varepsilon = 10^{-6} \text{ m}^2 \text{ s}^{-3}$; orange $\varepsilon = 5 \times 10^{-6} \text{ m}^2 \text{ s}^{-3}$; blue $\varepsilon = 10^{-5} \text{ m}^2 \text{ s}^{-3}$; dark green $\varepsilon = 5 \times 10^{-5} \text{ m}^2 \text{ s}^{-3}$; light green $\varepsilon = 10^{-4} \text{ m}^2 \text{ s}^{-3}$). The linear curve (dotted line) is expected for non-intermittent turbulence. The data were collected in the eastern English Channel. (Courtesy of L. Seuront).

Mixing efficiency

Much of the diapycnal mixing in the ocean is associated with the shear instability of internal waves. The subsequent overturning and stirring leads to finescale patterns of density which then disappear through the action of molecular thermal diffusivity or the molecular diffusion of salt. The turbulent eddies are being dissipated by molecular viscosity at the same time as they are working against buoyancy forces; a major question concerns the relative fractions of turbulent kinetic energy lost to dissipation at a rate ε and converted to mean potential energy at a rate $K_v N^2$, where K_v is the eddy diffusivity and N the buoyancy frequency. The ratio $\Gamma = K_v N^2 / \varepsilon$ is generally thought to be about 0.2, largely based on calibration against other methods of esti-

imating K_v . There is no reason to suppose that Γ is a universal constant, however, but it has not been established what environmental parameters might determine its value.

Some evidence reported at the meeting, based largely on a laboratory experiment but with some field support, suggests that Γ is less than the value of about 0.2 quoted above if the turbulence is weak with a “turbulent Reynolds number” $Re_T = \varepsilon / \nu N^2$ less than about 100. This is not surprising: weak turbulence dies away before it has accomplished much buoyancy flux. What was a surprise was a claim that Γ also falls off again for values of Re_T greater than about 1,000. This dependence is hard to accept; it may transpire that Re_T in these experiments was actually a proxy for an input variable that does not involve the viscosity ν . One variable that might lead to reduced Γ in the ocean is the duration of high shear compared with the buoyancy time N^{-1} . A long-lived mixing event might produce all its buoyancy flux at first and subsequently stir water that is already mixed, dissipating energy but with little further buoyancy flux. More study is required, with high resolution 3D numerical simulations of breaking internal wave fields becoming increasingly feasible and informative.

The question of mixing efficiency is important in any assessment of the amount of mixing that might be associated with, for example, the energy loss from the barotropic tide to internal tides. There are also implications for the circulation that is diagnosed from measured dissipation rates ε and the observed mean density structure. The mean diapycnal velocity w_* may be obtained from the mean buoyancy equation (e.g., *St. Laurent et al.*, 2001)

$$w_* N^2 = \partial(\Gamma \varepsilon) / \partial z$$

so that the value of w_* can depend on gradients of Γ . Vortex stretching given by $f \partial w_* / \partial z$ then affects the mean flow along isopycnals.

Differential mixing

The rapid decay of turbulent kinetic energy at small values of the turbulent Reynolds number $Re_T = \varepsilon / \nu N^2$ can also lead to less turbulent transport of salt than heat, given the very small molecular diffusivity of the former. Although the effect is clearly established in 2D and 3D numerical simulations and in laboratory experiments that mimic the breaking of internal waves in a stratified ocean, it is somewhat counterintuitive at first sight. If the turbulent fluxes were downgradient at all scales then the more diffusive tracer would lose its higher wavenumber fluctuations and would experience less turbulent transport. It seems, however, that the fluxes are upgradient at high wavenumbers, as a simple consequence of the secondary

instability and collapse of an overturning billow. The more diffusive tracer thus loses some upgradient flux and therefore experiences more downgradient transport.

The importance of differential mixing for the ocean is not yet known. It may turn out that in regions where mixing matters it is at high enough Re_T that the eddy diffusivities of heat and salt are the same. This is particularly likely to be true given the intermittent nature of mixing in the ocean compared with the rather steady stirring in the numerical experiments; Re_T based on ϵ during a mixing event in the ocean may be much larger than that based on the average ϵ .

Different rates of mixing for heat and salt are, however, firmly established for regions of the ocean where double-diffusive processes are significant. These regions include places, such as the C-SALT region near Barbados (Schmitt, 1994), where the vertical profiles of temperature and salinity are nearly compensating in density, and places such as the Arctic Ocean where internal wave activity is low. Even in these regions there are significant puzzles, partly because laboratory results seem not to be readily applicable, and partly because we lack adequate theoretical guidance. One obvious problem is to explain the step size in thermohaline staircases. In the diffusive regime (with warm salty water below cooler fresher water), some useful predictions come from a local water column model, whereas in the salt fingering regime (with the opposite stratification) it seems that the step size might be set by lateral intrusive processes. The issue is certainly not fully resolved, and the interfaces in the fingering regime also seem more complex in the ocean than in the laboratory.

Importantly, many regions of the ocean which might support double-diffusive processes seem to have these disrupted by things like the shear of internal waves, with the breaking of these waves then dominating diapycnal transports. Even these regions can show some evidence for double diffusive transports, however, so that the phenomena warrant continued attention.

Summary

There are typically nearly 10^{20} spatial degrees of freedom between the smallest scales being resolved by numerical models of ocean circulation and the smallest scales of variability. The meeting was therefore being ambitious in attempting to survey a vast range of scales and phenomena. Some general, if rather platitudinous, principles do seem clear: (1) Studying the processes at work in the ocean is worthwhile science. (2) There is a need not just to understand these processes but also to parameterize them in a form that can be implemented in models for the large-scale behavior of the ocean. (3) These models should be

used in sensitivity tests to determine which parameterizations need improvement.

It is not really clear what issues have the highest priority. After a sometimes tumultuous discussion the participants arrived at the following ranking by vote:

1. What are the constraints on eddy parameterization and how should they be implemented?
2. What nonlocal effects matter and how can they be parameterized?
3. How uniform are stirring and mixing processes? Are there any "hotspots," particularly in association with topographic features?
4. What are the effects of diapycnal boundary processes on eddy parameterization?
5. What is the parameter dependence of the mixing efficiency or flux Richardson number?
6. How does energy leak from the slow manifold?
7. What aspects of stirring and mixing affect biological processes? What do patterns of biological tracers tell us about fluid dynamics?
8. Under what circumstances do we need different diffusivities for different tracers?
9. What are the effects of adiabatic stirring on diapycnal mixing?
10. Do subgridscale processes provide stochastic forcing?

The last question addresses a very fundamental aspect of the Reynolds decomposition. A second fundamental aspect is that the simple Reynolds decomposition ideally requires a spectral gap, in time or space, separating the unresolved processes to be parameterized from the slowly changing "mean." Both these issues were discussed at the meeting, but without any profound new insights.

Stirring and mixing in a stratified ocean is the physics that needs to be parameterized in ocean circulation models. Challenging open problems remain at all levels, from very fundamental to highly applied aspects.

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