Representing the effects of mesoscale eddies in coarse-resolution ocean models

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Abstract. Mesoscale eddies in the ocean mix fluid parcels in a way that is highly constrained by the stratified nature of the fluid—so much so that much of our intuition about ocean mixing comes from thinking in density coordinates. Temporal-residual-mean (TRM) theory provides the link between the different views that are apparent from averaging turbulent flow in height coordinates and in density coordinates. The TRM theory reduces the parameterization problem from three dimensions to two dimensions and it shows how the divergent part of the relevant eddy density flux is skew-symmetric in height coordinates and that the total advection velocity can be adiabatic. The Gent-McWilliams scheme is best interpreted as a scheme for implementing the temporal residual mean. The TRM theory has very specific things to say about how the tracers in coarse-resolution models should be interpreted and how the skew diffusion streamfunction should approach zero at boundaries. Here it is emphasized that the extra advection in the TRM theory is not the bolus transport because the extra TRM advection is non-divergent and diapycnal in character while the bolus velocity is divergent and adiabatic. When using eddy-permitting numerical model results to deduce a parameterization for mesoscale eddies, it is very important to use the full TRM theory, and in so doing, the parameterization task is simplified somewhat.

Introduction

Gent and McWilliams (1990) realized that when averaging in density coordinates, the thickness-weighted average velocity involves the bolus velocity, defined as the flow along density surfaces caused by the correlation between the horizontal velocity and the thickness between adjacent density surfaces. This motivated them to recommend additional terms in the tracer conservation equations. For several years these extra terms were interpreted as being undesirably diapycnal in character; for example it was emphasized that while the flow at any point in space was forced to be diapycnal by the mixing scheme, the area-averaged flow was not diapycnal.

The paper of *Gent et al.* (1995) pointed out the important property that the total flow (the sum of the resolved-scale and the eddy-induced velocity) did not have a diapycnal component. *Gent et al.* (1995) also cast the scheme in terms of a two-dimensional streamfunction and they pointed out the sign-definite sink of domain-averaged gravitational potential energy that the scheme provides. While the motivation of both *Gent and McWilliams* (1990) and *Gent et al.* (1995) came from the bolus velocity of density coordinates, it is argued below that the *Gent et al.* (1995) mixing scheme is not a parameterization for the bolus velocity but rather is a parameterization for the extra streamfunction of the temporal-residual-mean (TRM) theory. The TRM theory says unambiguously how one must interpret the variables that are carried by coarse-resolution ocean models and it also provides the physical justification for the tapering of the extra streamfunction to zero at ocean boundaries. We begin with a quick revision of the theory and then discuss some common misconceptions about the *Gent et al*. (1995) parameterization scheme.

The temporal-residual-mean approach

The averaging operator (the overbar) here is defined to be a low-pass temporal average and primed quantities are the deviations from this low-passed value. For simplicity the equation of state is taken to be linear *(McDougall and McIntosh,* 2001) have derived the TRM theory for a nonlinear equation of state). The usual conservation equations for Eulerian-averaged density and for half the density variance are

$$\bar{\gamma}_t + \nabla\cdot\left(\bar{\mathbf{U}}\bar{\gamma}\right) = \bar{Q} - \nabla\cdot\left(\overline{\mathbf{U}'\gamma'}\right) \qquad (1)$$

$$\bar{D}_t\,\bar{\phi} = \overline{Q'\gamma'} - \overline{\mathbf{U}'\gamma'}\cdot\nabla\bar{\gamma} + O\left(\alpha^3\right). \qquad (2)$$

Here $\bar{\phi} \equiv 1/2\,\overline{\gamma'^2}$ is half the density variance measured at a fixed point in space, and the terminology $O\left(\alpha^3\right)$ indicates terms that are of cubic or higher order in perturbation amplitude. Q represents the effects of molecular diffusion and small-scale mixing processes.

In order to develop residual-mean conservation equations that apply to unsteady flows we need to admit the possibility that the Eulerian-mean density may not be the most appropriate mean density to appear in the mean density conservation equation. For example, the Eulerian-mean density, $\bar{\gamma}(x,y,z,t)$, describes a density surface whose average height is not that of the original Eulerian averaging, z. The appropriate mean density is the one whose surface is, on average, at the height of the averaging. This density can be expressed in terms of $\bar{\gamma}$ and $\bar{\phi}$ by

$$\tilde{\gamma} = \bar{\gamma} - \left(\bar{\phi}/\bar{\gamma}_z\right)_z + O\left(\alpha^3\right). \qquad (3)$$

The distinction here is between averaging density at a given height, z, and averaging the height of a given density surface $\gamma = \tilde{\gamma}$. McDougall and McIntosh (2001) used the density variance equation (2) to rewrite the mean density conservation equation (1) as

$$\begin{aligned}\tilde{\gamma}_t + \nabla\cdot\left(\bar{\mathbf{U}}\tilde{\gamma}\right) &= \bar{Q}^{\#} - \nabla\cdot\left(\mathbf{U}^{+}\tilde{\gamma}\right) - \nabla\cdot\mathbf{M} + O\left(\alpha^3\right)\\ &= \bar{Q}^{\#} - \nabla\cdot\left(\mathbf{A}\nabla\tilde{\gamma}\right) - \nabla\cdot\mathbf{N} + O\left(\alpha^3\right)\end{aligned} \qquad (4)$$

where

$$\mathbf{U}^{+} \equiv \nabla\times\left(\Psi\times\mathbf{k}\right) = \Psi_z - \mathbf{k}\left(\nabla_H\cdot\Psi\right), \qquad (5)$$

$$\bar{Q}^{\#} \equiv \bar{Q} + \left[-\frac{\overline{Q'\gamma'}}{\bar{\gamma}_z} + \frac{\bar{Q}_z}{\bar{\gamma}_z}\left(\frac{\bar{\phi}}{\bar{\gamma}_z}\right)\right]_z, \qquad (6)$$

$$\mathbf{\Psi} = -\frac{\overline{\mathbf{V}'\gamma'}}{\bar{\gamma}_z} + \frac{\bar{\mathbf{V}}_z}{\bar{\gamma}_z}\left(\frac{\bar{\phi}}{\bar{\gamma}_z}\right) + O\left(\alpha^3\right), \qquad (7)$$

and $\mathbf{M}$ and $\mathbf{N}$ are non-divergent fluxes, expressions for which are given in *McDougall and McIntosh* (2001). The anti-symmetric tensor $\mathbf{A}$ is defined in terms of the two components of the quasi-Stokes streamfunction, $\mathbf{\Psi} = \left(\Psi^x, \Psi^y\right)$, as

$$\mathbf{A} \equiv \begin{bmatrix} 0 & 0 & \Psi^x \\ 0 & 0 & \Psi^y \\ -\Psi^x & -\Psi^y & 0 \end{bmatrix}. \qquad (8)$$

Isopycnal Interpretation The upper line of equation (4) shows that the modified density, $\tilde{\gamma}$, is advected with the TRM velocity, $\bar{\mathbf{U}}^{\#} = \bar{\mathbf{U}} + \mathbf{U}^{+}$, where both the Eulerian-mean velocity, $\bar{\mathbf{U}}$, and the quasi-Stokes velocity, $\mathbf{U}^{+}$, are non-divergent. This equation shows that the TRM velocity only has a component through the modified density surfaces because of the modified diabatic source term, $\bar{Q}^{\#}$. This is in contrast to the Eulerian-mean velocity, because (see (1)) the Eulerian-mean velocity, $\bar{\mathbf{U}}$, flows through the Eulerian-mean density surfaces due not only to the diabatic source term, $\bar{Q}$, but also due to the mesoscale eddy flux of density.

A Taylor series expansion shows that the horizontal component of the TRM velocity, $\bar{\mathbf{V}}^{\#}$, is the same as the thickness-weighted horizontal velocity that is found by averaging in density coordinates, $\hat{\mathbf{V}}$. That is

$$\hat{\mathbf{V}} \equiv \tilde{\mathbf{V}} + \mathbf{V}^{B} = \bar{\mathbf{V}} + \Psi_z + O\left(\alpha^3\right) = \bar{\mathbf{V}}^{\#} + O\left(\alpha^3\right) \qquad (9)$$

where $\tilde{\mathbf{V}}$ is the horizontal velocity averaged on a density surface and the bolus velocity, $\mathbf{V}^{B} = \overline{\mathbf{V}'|_{\tilde{\gamma}}\, z'_z}$, is due to correlation between the horizontal velocity evaluated on the density surface and the thickness between density surfaces. Because of (9) it can be shown (McDougall and McIntosh, 2001) that the TRM conservation statement for density in the upper line of (4) is the same as is found by thickness-weighted averaging in density coordinates. The key to making this connection is to simply ignore the non-divergent flux, $\mathbf{M}$, in (4). The same Taylor series approach shows that the modified source term, $\bar{Q}^{\#}$, is the thickness-weighted source term of density coordinates.

The skew diffusion approach in the second line of (4) is preferable numerically because the quasi-Stokes streamfunction is less spatially differentiated than in the advective approach of the upper line of (4) (*Griffies*, 1998). The skew diffusion of density is given by

$$-\mathbf{A}\nabla\tilde{\gamma} = -\tilde{\gamma}_z\,\mathbf{\Psi} + \mathbf{k}\left(\mathbf{\Psi}\cdot\nabla_H\tilde{\gamma}\right)$$

and if it takes the *Gent et al*. (1995) form of the quasi-Stokes streamfunction, then the parameterization of mesoscale mixing that was used before GM90 is equivalent to retaining the horizontal component of this skew flux but ignoring the vertical component.

The Taylor series approach also furnishes a physical explanation of the quasi-Stokes streamfunction, $\mathbf{\Psi}$; at any height, z, $\mathbf{\Psi}$ is the contribution of temporal perturbations to the horizontal transport of water that is denser than $\tilde{\gamma}(z)$, the density of the surface having time-mean height z.

The TRM analysis procedure can also be applied to the tracer conservation equations (see *McDougall and McIntosh*, 2001) and one finds that the terms required to account for the mixing by mesoscale eddies are (i) the epineutral mixing of passive substance along the local neutral tangent plane (*Griffies et al.,* 1998) and (ii) the skew diffusion of tracer with the same skew diffusion tensor as for density, namely (8). It is this second term that corresponds to the *Gent and McWilliams* (1990) scheme. The extra mixing term can be regarded as an extra advection by the quasi-Stokes velocity, $\mathbf{U}^{+}$, or as the skew diffusion term, $-\nabla \cdot \left(\mathbf{A}\nabla \hat{\tau}\right)$. An important new finding of the TRM theory is that the tracer conservation equation is written in terms of the thickness-weighted tracer of density coordinates, $\hat{\tau}$. This finding completes the correspondence between each of the terms in the TRM conservation equations and those that arise in density coordinates.

Cubic order terms and exact TRM equations. All of the above results are accurate to cubic order in perturbation amplitude. Recent unpublished work has demonstrated that away from solid boundaries the cubic terms that appear in the above expressions are quite small in comparison with the quadratic order terms that are to be parameterized. It has also been shown by *McDougall and McIntosh* (2001) that the conservation equations are exact if the quasi-Stokes streamfunction is in fact as described in the physical explanation above, rather than as defined in (7). The exact form of the quasi-Stokes streamfunction which leads to exact tracer conservation equations is

$$\mathbf{\Psi} \equiv \overline{\int_{z}^{z+z'} \mathbf{V} dz''} \tag{10}$$

which is the temporal average of the vertical integral of the horizontal velocity integrated between the fixed height, z, and the instantaneous height of the density surface, $z+z'$. If we were ever clever enough to parameterize this form of the quasi-Stokes streamfunction, the job of parameterizing the skew diffusion by mesoscale eddies would be complete and the analogy with the equations found by averaging in density coordinates would be exact. The parameterization task for coarse-resolution ocean models is then to parameterize (10).

In summary, I believe the important practical features of the TRM theory are that

(i) it demonstrates exactly how one must interpret the tracers that are carried by eddyless ocean models,

(ii) it shows that the job of parameterizing the dynamical effects of mesoscale eddies is reduced to the task of parameterizing the quasi-Stokes streamfunction, (10) (or (7) in the approximate form), and

(iii) the physical interpretation of the quasi-Stokes streamfunction demands that it be tapered smoothly to zero at ocean boundaries (as opposed to other suggestions of having delta functions at the boundaries or of allowing flow through the boundaries).

The rest of this paper will attempt to correct some common misconceptions about the *Gent et al.* (1995) scheme, and then to make some remarks about the task of deducing eddy parameterizations from eddy-permitting models. But first, some comments are in order about other ways of averaging the conservation equations.

Comparison with other averaging approaches. There are other ways of averaging the conservation equations, for example the ‘effective’ approach following *Plumb and Mahlman* (1987), the “stochastic” approach of *Dukowicz and Smith* (1997), and the Lagrangian-mean approach of *Andrews and McIntyre* (1978). To date these approaches have proved less useful than the TRM theory for the following reasons. When the “effective” approach is extended to temporal averaging in three dimensions (as opposed to zonal averaging) one finds that the density which is advected by the “effective” velocity is devoid of a physical interpretation and, in particular, does not have a connection to averaging in density coordinates. Similarly, the extra two-dimensional streamfunction of the “effective” approach is written in terms of the first order approximations to Lagrangian displacements of fluid parcels and is similarly not amenable to an isopycnal interpretation.

In the stochastic approach both the eddy diffusion coefficient and the mean velocity are written in terms of probability density functions of the eddy motions, but what is missing is a connection to the Eulerian-mean velocity that is naturally carried by the momentum equations in a coarse-resolution model. Consequently, various authors have made different assumptions as to what the mean velocity in the “stochastic” approach might be, with correspondingly different interpretations of the mean variables and of the things that need to be parameterized. For example, the papers of *Dukowicz and Smith* (1997) and *Dukowicz and Greatbatch* (1999) both use the same ‘stochastic’ theory but have different conclusions because of different assumptions about the physical meaning of the ‘stochastic’ velocity.

The Lagrangian-mean theory of *Andrews and McIntyre* (1978) is beautifully general but the Lagrangian-mean velocity is divergent at leading order and the Lagrangian-mean tracer values are not readily observable. The beauty of the TRM theory is that it provides a one-to-one match between every variable (tracer, density and velocity) that arises in the TRM approach with the same variable that arises when averaging in density coordinates. Since the rate of oceanic diapycnal mixing is so small in relation to the lateral fluxes of mesoscale eddies, it is imperative to cleanly separate these two types of mixing processes, and all of our intuition in this regard comes from thinking and averaging in density coordinates. It is this direct link to averaging in density coordinates that makes the TRM approach attractive for *z*-coordinate models.

Neither GM90 nor the quasi-Stokes velocity is the bolus velocity

The stated aim of *Gent and McWilliams* (1990) and of *Gent et al.* (1995) was to provide a parameterization for the bolus velocity and it was argued that this should be directed down the isopycnal gradient of thickness so that the extra horizontal velocity would be $\kappa\left(\nabla_H \tilde{\gamma}/\tilde{\gamma}_z\right)_z$. However, in order to make the streamfunction a locally determined quantity, and particularly to ensure a sign-definite sink of gravitational potential energy, the diffusivity was moved inside the vertical derivative so that the streamfunction of *Gent et al.* (1995) became $\kappa\nabla_H \tilde{\gamma}/\tilde{\gamma}_z$. This form also allowed the boundary conditions at the top and bottom of the ocean to be easily satisfied by having the diffusivity go to zero.

If one makes the most elementary assumption for the first term in (7) that the horizontal density flux is directed down the horizontal gradient of $\tilde{\gamma}$ with diffusivity κ (that is, $\overline{\mathbf{V}'\gamma'} = -\kappa\nabla_H \tilde{\gamma}$) and if one ignores the second term in (7), the quasi-Stokes streamfunction becomes $\mathbf{\Psi} = \kappa\nabla_H \tilde{\gamma}/\tilde{\gamma}_z$, exactly as we advocated in *Gent et al.* (1995) and as used by many subsequent authors. In this way, this simple Fickian assumption on the horizontal flux of density has enabled us to interpret the *Gent et al.* (1995) scheme as a particular parameterization for the quasi-Stokes streamfunction of the TRM theory. But from (9) it immediately follows that the GM90 and *Gent et al.* (1995) schemes are not parameterizations for the bolus velocity because the horizontal component of the bolus velocity is different from the horizontal quasi-Stokes velocity, $\mathbf{\Psi}_z$, by the amount $\bar{\mathbf{V}} - \tilde{\mathbf{V}}$ which is the difference in the horizontal velocity averaged at constant depth and at constant density.

The movement of the diffusivity inside the vertical derivative in the *Gent et al.* (1995) expression for the eddy-induced horizontal velocity, $\left(\kappa\nabla_H \tilde{\gamma}/\tilde{\gamma}_z\right)_z$, far from being a weakness of *Gent et al*. (1995) because of the weaker connection to the bolus transport, is actually a very important strength of the GM90 scheme. Consider for example the situation where the slopes of density surfaces are independent of height so that the original idea of down-gradient thickness flux would mean that the eddy-induced horizontal velocity was zero at all heights except for delta functions of eddy-induced horizontal velocity at the top and bottom. Finite-amplitude baroclinic instability operating on such an initial condition will lead to the simultaneous relaxation (in the mean) of isopycnals throughout the entire water column, not merely at the boundaries of the domain. The TRM approach can achieve this property by allowing the diffusivity, κ, to be a function of height. In this way the quasi-Stokes velocity of the TRM theory can achieve a more realistic parameterization of baroclinic instability than is possible with a down-gradient scheme. This type of depth-dependent diffusivity is being pursued by *Killworth* (2001). The same criticism applies to an up-gradient potential vorticity mixing scheme which has often been suggested as a parameterization for $\mathbf{\Psi}_z$. When there is no epineutral gradient of potential vorticity in the whole water column, we do not expect baroclinic instability to cause mean motion and relaxation of isopycnals only in delta function boundary layers as would occur with a potential vorticity parameterization.

In addition to the issue of having the diffusivity inside the vertical derivative, there are two other compelling reasons why the eddy-induced velocity of *Gent et al.* (1995) (which we are interpreting as the quasi-Stokes velocity of the TRM theory) is neither the bolus velocity nor the down-gradient thickness flux. The first of these is that the quasi-Stokes velocity is three-dimensionally non-divergent whereas the epineutral flux of thickness is divergent (see *McDougall and McIntosh*, 2001). The second reason is that the bolus velocity and the epineutral thickness flux have no diapycnal component, whereas the quasi-Stokes velocity has a large diapycnal component (see for example, Figure 6b of *Hirst and McDougall*, 1998). These differences arise through the use of the streamfunction (that is, the continuity equation) to construct the vertical component of both the quasi-Stokes velocity and of the eddy-induced velocity in the *Gent et al.* (1995) scheme.

The task of parameterizing the effects of mesoscale eddies is very different for isopycnal models than for height-coordinate models. In an isopycnal model the extra horizontal velocity that is needed is indeed the bolus velocity. The sum of the bolus velocity and the resolved-scale horizontal velocity of an isopycnal model, $\tilde{\mathbf{V}}$, gives the thickness-weighted velocity of isopycnal coordinates, $\hat{\mathbf{V}}$. In sharp contrast to the large diapycnal component of the quasi-Stokes velocity of height-coordinate models, the bolus advection that is needed in isopycnal models has zero diapycnal component (see *McDougall and McIntosh*, 2001).

Some issues in parameterizing Ψ

It is argued above that the effects of mesoscale eddies enter the tracer conservation equations through (i) the symmetric diffusion tensor which represents the passive epineutral mixing of tracer, and (ii) the skew diffusion tensor (8) whose elements are the quasi-Stokes streamfunction, (10), of TRM theory. Some authors have suggested that $\mathbf{\Psi}_z$ should be parameterized as being up the gradient of potential vorticity (measured along the local neutral direction). Apart from the small meridional change in the Coriolis parameter, this suggestion is the same as the down-gradient thickness idea. *McDougall and McIntosh* (2001) have argued that irrespective of whether or not these suggestions turn out to be a good parameterization for $\mathbf{\Psi}_z$ they leave open the question of the vertical component of the quasi-Stokes velocity. Equivalently, this down-potential vorticity parameterization does not address the issue of how $\mathbf{\Psi}_z$ behaves at the top and bottom boundaries of the ocean. *McDougall and McIntosh* (2001) have shown that this uncertainty means that the contribution of this parameterization to the meridional flux of heat is quite uncertain. This uncertainty is equivalent to the practical difficulty that arises when parameterizing the bolus velocity in coarse-resolution layered ocean models where, in practice, the interface height is smoothed rather than actually introducing a parameterized bolus velocity to the model. The key to overcoming this considerable uncertainty in the meridional heat flux is to parameterize the quasi-Stokes streamfunction, $\mathbf{\Psi}_z$, itself, rather than its vertical derivative.

There are other rather general problems with the down-gradient potential vorticity parameterization. *Cummins* (2000) has pointed out that such a parameterization can lead to a fictitious torque which spontaneously generates angular momentum while *Adcock and Marshall* (2000) have shown that energy conservation is violated as the scheme attempts to homogenize potential vorticity (or thickness) along neutral density surfaces.

While our skill in parameterizing the quasi-Stokes streamfunction is clearly in its infancy, some eddy-permitting models are showing intriguing results. For example, *Treguier* (1998) has analyzed a primitive equation model for the zonally averaged bolus velocity and has found a small diffusivity for use in the quasi-Stokes diffusivity. Such a result would be expected to have some benefits so long as the models remain stable with these smaller diffusivities. The two benefits that come to mind are avoiding the slowing of the horizontal circulation of the subtropical gyres that occurs with the larger values of the quasi-Stokes diffusivity, and reducing the intrusion of Antarctic Bottom Water into the North Atlantic which is too strong with present values of the quasi-Stokes diffusivity (*Hirst and McDougall*, 1998).

Given the above problems with both the down-gradient thickness and the up-gradient potential vorticity schemes, it seems that the way forward for the foreseeable future will be to stay with the general *Gent et al.* (1995) form of the quasi-Stokes streamfunction, $\mathbf{\Psi} = \kappa \nabla_H \tilde{\gamma} / \tilde{\gamma}_z$, but to allow the diffusivity, κ, to be a function of space.

It must be pointed out (following *Tandon and Garrett*, 1996) that the adiabatic nature of the TRM velocity implies that the eddy kinetic energy of mesoscale eddies cannot be dissipated in the ocean interior but rather must be dissipated near the upper and/or lower boundaries. This seems a rather special restriction on the energy budget.

When seeking to determine a parameterization for the quasi-Stokes streamfunction from the output of eddy-permitting ocean models, it is important to use the correct TRM conservation equations. For example, *Gille and Davis* (1999) have analyzed an eddy-permitting primitive equation model of a zonal channel with respect to the conservation of the Eulerian-mean density, $\bar{\gamma}$, rather than the modified density of (4), and they also considered only the first term in the expression, (7), for the quasi-Stokes streamfunction. Analyzing model data in this way leaves the extra forcing term,

$$\left(-\overline{\mathbf{U}'\gamma'} \cdot \nabla \bar{\gamma} / \bar{\gamma}_z\right)_z \quad , \qquad (11)$$

in the density conservation equation and *Gille and Davis* (1999) conclude that this term is too large to ignore and needs to be parameterized. *McDougall and McIntosh* (2001) have shown that this extra source term does not arise when the full residual-mean transformation is performed. Rather, all that needs to be parameterized is the quasi-Stokes streamfunction.

Similarly, *Roberts and Marshall* (2000) have examined the divergent part of the horizontal density flux and found that it was directed as much perpendicular to the mean density gradient as down the mean gradient. Unpublished work has shown (at least for the Southern Ocean) that while this is true of the first term in the expression, (7), when the second term is also included, the full quasi-Stokes streamfunction is much closer to being directed down the mean density gradient. This supports the contention that it is very important in efforts aimed at finding parameterizations for mesoscale turbulence that the conservation equations are carefully and accurately derived. When all the terms are kept, as has been done in the above TRM equation set, the parameterization task seems to be more promising than when leading order terms are ignored.

Conclusions

The intimate relationship between averaging in density coordinates and the TRM conservation equations is what provides the satisfying physical interpretations for the various quantities that arise in the TRM approach. For example, the quasi-Stokes streamfunction, (10), provides a compelling link between the coordinate systems since it is the contribution of temporal perturbations to the horizontal transport of water that is denser than $\tilde{\gamma}(z)$, the density of the surface having time-mean height *z*. I have argued that the parameterization task we face is to parameterize the quasi-Stokes streamfunction; not the horizontal bolus velocity, not the down-gradient flux of thickness and not the up-gradient flux of potential vorticity. While it might seem tedious to keep track of all the leading order terms (second order in perturbation quantities) in the conservation equations, when seeking to parameterize mesoscale eddies, it is becoming more obvious that we cannot afford to ignore these leading order terms.

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