

# Differential diffusion in bistable conditions

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**Abstract.** Double diffusive phenomena, due to the difference in molecular conduction coefficients for heat and for ion concentrations in sea water, are widely known. These include salt fingering and diffusive convection which occur when either temperature or salinity is destabilizing while the other (salinity or heat) is stabilizing. Less well known, less dramatic in their occurrence, but far more widespread are circumstances where both temperature and salinity are stabilizing and yet the differences in molecular conduction lead to systematic differences in turbulent transport rates. “Differential diffusion” has been seen in laboratory experiments, in ocean observations and in numerical simulations. Here we examine differential diffusion in terms of turbulence-induced restratifying (up-gradient) fluxes.

## Introduction

Double diffusion occurs in the ocean because heat and salt (ion concentration) experience very different coefficients of molecular conduction. Heat conduction has about  $100 \times$  larger coefficient than ionic conduction. As both heat and salt determine density, circumstances arise where the potential density profile  $\rho(z)$  is stable although the contribution from either temperature  $T(z)$  or salinity  $S(z)$  would, by itself, be gravitationally unstable. Many studies and a large literature have addressed these doubly diffusive phenomena, as *Kelley* (2001) has reviewed at this workshop.

Over the major part of the ocean interior, both  $T(z)$  and  $S(z)$  are stabilizing. While lateral intrusive instabilities may occur, most double diffusive phenomena are not allowed. The prevailing view is that both  $T$  and  $S$  are simply mixed by occasional background turbulence. Turbulent diffusivities for  $T$  and for  $S$  are assumed to be equal. However prevalent this view, it may not be quite so.

Do the differences of molecular conduction cause differences for the turbulent (advective part) fluxes? Laboratory evidence and numerical experiments show that they do. In early experiments, *Turner* (1968) mechanically agitated a fluid stratified with respect to  $T$  and, separately, a fluid stratified with respect to  $S$ , taking care that the two stratifications were initially the same. Under the same mechanical agitation, it was found that  $T$  was mixed more efficiently than  $S$  by an amount greater than could be attributed to molecular conduction. Later, *Altman and Gargett* (1980) performed similar experiments in which a tank

was initially stably stratified with respect to both  $T$  and  $S$ , arranged so that both made the same initial contribution to stratification. Again it was seen that  $T$  mixed more readily than  $S$  by amounts exceeding molecular conduction. Direct observation of differential diffusion in the ocean is hugely technically difficult, and first results are just now being reported by *Nash and Moum* (2001).

Attempting to quantify differential diffusion, one may employ direct numerical simulations. A major challenge to this approach is the necessity to resolve widely disparate scales, with the dissipation scale for  $S$  nearly 100 times smaller than the dissipation scale for  $T$ , itself already 10 times smaller than the dissipation scale for the advecting velocity field. A limited investigation based upon numerical simulations was performed by *Merryfield et al.* (1998) employing the idealization that all fields were independent of one horizontal coordinate, *i.e.*, two-dimensionalizing the problem. By not resolving three dimensions, more computing resource could be applied to wider range of scales in two dimensions.

Forcing with random velocity fields, *Merryfield et al.* found results consistent with laboratory observations, namely that turbulent fluxes (not including explicit diffusive fluxes driven by background gradients) transferred heat more rapidly than salt. Differential transfer rates were explored as dependent upon flow parameters. However, these results were subject to doubt because the two-dimensionalization very much alters the nature of “turbulence”, trapping kinetic energy in large, non-dissipating scales.

As far as computing resources have allowed, we have now performed a suite of experiments in fully three dimen-

sions. In this note we report preliminary results from these experiments and a schematic explanation to account for the occurrence of differential diffusion.

## Direct simulations in 3D

We solve the Boussinesq equations without Earth's rotation:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho_0} \nabla p + \frac{\rho}{\rho_0} \mathbf{g} + \nu \nabla^2 \mathbf{u} \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (2)$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \kappa_T \nabla^2 T \quad (3)$$

$$\frac{\partial S}{\partial t} + (\mathbf{u} \cdot \nabla) S = \kappa_S \nabla^2 S \quad (4)$$

where  $\mathbf{u}=(u,v,w)$  is velocity,  $T$  is temperature,  $S$  is salinity,  $\rho$  is density,  $p$  is pressure,  $\mathbf{g}$  is gravity,  $\nu$  is viscosity,  $\kappa_T$  and  $\kappa_S$  are coefficients of explicit diffusion, and we assume a linear equation of state

$$\rho = \rho_0 [1 - \alpha(T - T_0) + \beta(S - S_0)] \quad (5)$$

$T$  and  $S$  are expanded about a prescribed (non-varying) background

$$T(x, y, z) = \Gamma_T z + T'(x, y, z) \quad (6)$$

$$S(x, y, z) = \Gamma_S z + S'(x, y, z) \quad (7)$$

such that

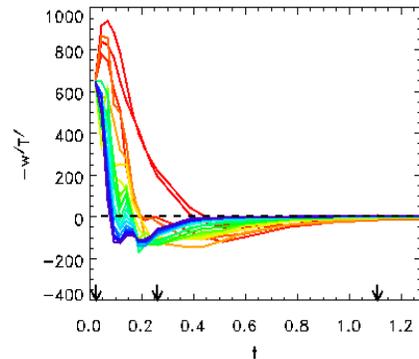
$$R_p = \alpha \Gamma_T / \beta \Gamma_S = -1$$

Equations (1) through (4) are solved pseudo-spectrally, with dealiasing, using filtered leapfrog timestepping. Viscous and diffusive terms are represented by exponential integrating factors in Fourier space. The great difficulty, even in our age of modern computing, is that the requisite range of scales one should wish to resolve in 3-space exceeds computational resource. At our highest resolution, we employ 160 gridpoints in each dimension for velocity and temperature fields while employing twice that (320 gridpoints) for salinity. Even so, for  $\nu = 0.01 \text{ cm}^2/\text{s}$ ,  $\kappa_T = 0.0014 \text{ cm}^2/\text{s}$  and  $\Gamma_T = 10^{-4} \text{ }^\circ\text{C}/\text{cm}$ , we are compelled to work in a cube approximately 17 cm tall with grid spacing about 0.1 cm for  $u$  and  $T$  and 0.05 cm for  $S$ . For salinity dissipation scales to be resolved, we must limit  $\tau = \kappa_S / \kappa_T$  to 0.1 where a more realistic value would be near 0.01.

We then randomly select an isotropic initial  $\mathbf{u}$  with energy spectrum approximately  $k^{-3}$ , while initial  $T$  and  $S$  are nil. Amplitude of the initial  $\mathbf{u}$  are adjusted for experiments in which a turbulent Froude number takes values from 0.4 (“weak”) to 48 (“strong”). During the subsequent decay, we diagnose vertical heat and salt fluxes,  $w'T$  and  $w'S$ , evaluating their cross-spectra at selected times and evaluating time-integrated fluxes. A more complete account will be given by *Gargett et al.* (2001). Here we sketch only key results and an explanation.

## Results

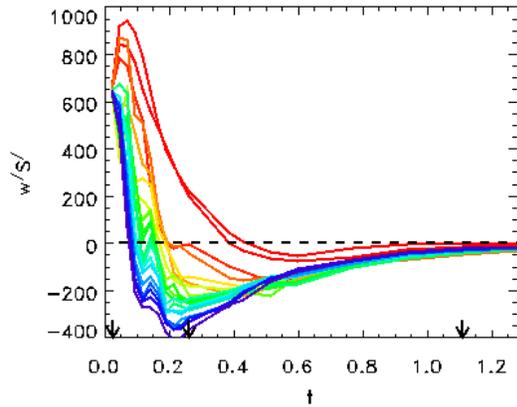
First, we've compared output with previous 2D experiments reported by *Merryfield et al.* (1998). For this comparison, we have rerun the earlier 2D simulations but with  $\tau=0.1$ , consistent with present 3D runs. Results (not shown here) are pleasantly reassuring, tending to confirm the value of earlier 2D simulations and suggesting that our 3D  $\tau=0.1$  results can be extrapolated to  $\tau=0.01$  by analogy with 2D results. For the purpose of this report, we show only a representative case from 3D simulations. Figures 1 and 2 show evolution in time of several spectral bands from cross-spectra  $-w'T$  and  $w'S$ .



**Figure 1.** Timeseries of  $-w'T$  are shown for several vertical wavenumber bands from red=low  $kz$  to blue=high  $kz$ . We show  $w'T$  so that positive values represent downward flux, hence downgradient in  $T$  (usual “mixing”). A half-buoyancy period,  $\pi/N$ , occurs at about  $t=0.8$ . Arrows mark times displayed in Figure 3.

## Why?

Numerical simulations, here in 3D and previously in 2D (*Merryfield et al.*, 1998), are consistent with the sense of laboratory observations. The less diffusive scalar is transported by velocity fluctuations less effectively than the

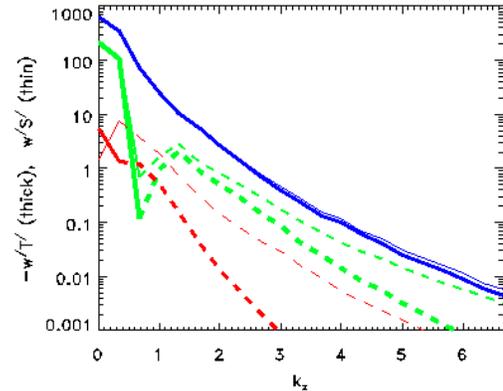


**Figure 2.** Timeseries of  $+w'S'$  are shown as in Figure 1, plotted so that positive is upward flux, hence downgradient in  $S$ . To aid comparison, fluxes in each spectral band have been rescaled just after initiation. While rescaling enables the higher wavenumber bands to be displayed, one is cautioned that this enhances perception of bands dominated by restratifying fluxes.

more diffusive scalar. 3D experiments were performed over a range of Froude numbers 0.4 to 48 and turbulent Reynolds numbers 1 to 80, all with  $\tau=0.1$  (not 0.01). In the higher Fr and Re cases, we obtain flux ratios from 0.7 to 0.9. (At lower Fr and Re, flux ratios can be 0.2 and less approaching the theoretical limit  $\tau=0.1$ . However, in these low amplitude cases, the velocity fluctuation induced transports are smaller than direct conduction.) Taking account that active oceanic turbulence typically occurs with Re greater than we are able to realize in numerical simulations, and that actual salt-heat have nearer 0.01, our results are best described as not inconsistent with the observational inference from *Nash and Moum* who estimate flux ratio  $0.7 \pm 20\%$ . A more complete report will be given by *Gargett et al.* (A.E. Gargett, W.J. Merryfield, and G. Holloway, Differential diffusion of  $T$  and  $S$ : Three-dimensional numerical simulations, in preparation).

We pause to ask why differential diffusion happens at all, and why it has the sense it has. One might have thought *ab initio* that the sense of differential diffusion would be opposite to that observed. If we thought of “usual” turbulence transporting a “usual” (passive) scalar, say  $C$ , then flux cross-spectra  $w'C'$  should remain of downgradient sense at all scales. Diffusivity of the scalar would cut off the high wavenumber range of  $w'C'$  and thus a more diffusive scalar would feel less turbulent transport. Why is this not also the case for active (buoyant) tracer in stratified flow?

The answer is seen in Figures 1, 2 and 3 in the prevalence of upgradient (restratifying) fluxes. Are these fluxes



**Figure 3.** Cross spectra of  $-w'T'$  (thick curves) and  $w'S'$  (thin curves) are plotted versus  $kz$  at the three times marked in Figures 1 and 2 with blue=early, green=mid, red=late. Solid curves indicate downgradient (“mixing”) fluxes while dashed curves represent upgradient (“restratifying”) fluxes.

not also the fluxes reported by *Carnevale* (this proceedings)? If indeed restratifying fluxes are generic, and the restratifying sense dominates the higher wavenumber range, then diffusive suppression of higher wavenumber cross-spectra yields enhanced transport of the more diffusive tracer simply because the more diffusive tracer experiences less restratification. In this case the question would move from “Why differential diffusion?” to “Why restratifying fluxes?” It is a question from ‘Aha Huli’ since more than a decade ago (*Holloway, 1989*), yet unresolved.

## Conclusions

Until a far more confident dynamical basis is established, we are not able to propose a parameterization scheme for differential diffusion in the bistable environment. What we show is that numerical simulations, both in 2D and here in 3D, confirm laboratory and oceanic observations that heat and salt do not mix equally efficiently. This is contrary to modelling practice for which the mixing coefficients for heat and salt are equal except in rare instances where a double-diffusive parameterization has been considered. For the large fraction of world’s oceans where the environment is stable to double diffusion, the assumption of equal mixing of heat and salt is wrong.

## Acknowledgments

We are grateful for support from the Office of Naval Research, grant N00014-99-1-0050.

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