Turbulent Properties in a Wave-Energized Benthic Boundary Layer on a Slope

Kraig B. Winters[1,2] and Gregory N. Ivey[2]

[1]Applied Physics Laboratory, University of Washington, Seattle WA
[2]Centre for Water Research, University of Western Australia, Australia

Abstract. Laboratory experiments were conducted in which a beam of steadily forced monochromatic internal waves encounters a planar, sloping boundary. Measurements of the rate of dissipation of turbulent kinetic energy ε and density flux ρw were made at several locations within the turbulent benthic boundary layer. The observations show that ε is approximately constant within a distance h of the bottom and that both h and the average dissipation rate $<\varepsilon>$ are well predicted by simple scaling arguments. Within the turbulent boundary layer, the time averaged density flux $<\rho w>$ is negative. The observed correlation results primarily from motions at frequencies near the forced wave frequency and shows no obvious change in character outside the turbulent boundary layer but within the interaction region of incident and reflecting waves. Motions at frequencies higher than N contribute negligibly to the total correlation. The implications of these results for estimating mixing rates are discussed and a new scaling of the diapycnal diffusivity K_ρ in terms of ε, the fluid viscosity ν and the background stratification N^2 is proposed and applied to the experimental results.

1. Introduction

A series of laboratory experiments was conducted to investigate the properties of boundary layer turbulence generated in a uniformly stratified fluid as a wave beam of finite width forward-reflects from a sloping boundary. The problem is of interest both because sloping boundaries are thought to be preferential locations of wave driven mixing and because such mixing can have important implications for nutrient fluxes in oceans and lakes. The present work is an extension of earlier work (*Ivey and Nokes*, 1989, *Ivey et al.* 1995) in that turbulent properties of the wave-driven boundary layer are directly measured. In this note we focus in particular on the implications of the measurements for estimating diapycnal mixing. A more complete discussion of these experiments can be found in *Ivey et ala* (1998).

Experiments

2.1. Facility

The experiments were conducted in a glass walled tank with a working section 3 m long by 54 cm wide with a depth of 60 cm. A plexiglass sheet was used to form a uniform sloping bed at angle β as shown in Figure 1.

The tank was filled with linearly stratified salt water to a depth of 50 cm. A folding paddle consisting of eight hinged blades, each 5 cm wide by 53.5 cm long, was oscillated at a constant frequency ω_f to excite an internal wave beam propagating toward the sloping bed. The wavelength in the plane of the paddle was 20 cm and the entire paddle assembly was oriented at an adjustable angle α as indicated in the schematic.

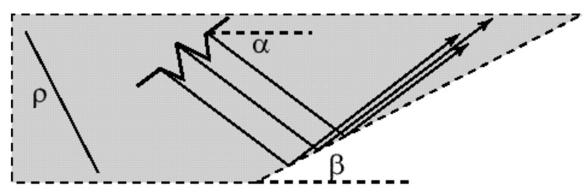

Figure 1. Geometry of experiments. Internal waves propagate at an angle α towards a smooth bed inclined at angle β. The experiments were conducted for forward reflecting waves under steady forcing in uniformly stratified salt water.

The geometric parameter $\gamma = \sin\alpha/\sin\beta$ determines the direction of wave reflection at the sloping boundary. The experiments reported here are for forward-reflecting waves with $1.25 < \gamma < 3.76$.

2.2. Procedure

The experiments were initiated by oscillating the paddle blades at the forcing frequency ω_f. Typical experimental parameters were $N \approx 0.71\ s^{-1}$, $\omega_f \approx 0.5\ s^{-1}$ with characteristic wave amplitudes a of 1 cm just above the turbulent boundary layer. The experiments were run under steady forcing conditions for approximately 5 minutes. Data were recorded throughout each run and analyzed only after initial transients had decayed and quasi-steady conditions had been established.

2.3. Stratification Measurements

Vertical density profiling was accomplished using a fast-response, four-electrode PME microscale conductivity probe, dynamically calibrated via an auxiliary siphoning probe, and a Thermometrics FP07 thermistor. Sensor spacing for the three probes was less than 4 mm. The probes were mounted on a computer-controlled assembly and lowered at a speed of 10 cm/s. Direct and differentiated output from each sensor was sampled and recorded at 100 Hz using a 16-bit A/D converter.

Vertical profiling was carried out continuously throughout the experiments. Differences in N^2 estimated by averaging monotonized profiles taken during the experiments and averages of restratified profiles taken before and after individual experiments were negligible.

2.4. Dissipation Measurements

Estimates of the rate of dissipation of turbulent kinetic energy were made from measurements of temperature gradients along horizontal traverses at fixed distances from the sloping bottom. Horizontal profiling takes advantage of the two-dimensional nature of the wave forcing and results in much longer record lengths than would be possible with vertical profiling. FP07 thermistors were traversed at 10 cm/s and sampled at 100 Hz. Interior segments of 256 mm, sampled over 2.56 seconds, thus yielded 256 data points for spectral analysis. These data were then fit to the theoretical Batchelor spectrum to produce estimates of the dissipation rate ε. From the work of *Luketina and Imberger* (1998) we estimate the accuracy of the dissipation estimates to be about 10%.

Measured dissipation rates are plotted as a function of perpendicular distance from the bottom in Figure 2 for several experiments. The distance coordinate is scaled by $h = 0.1\,\lambda_p$, where λ_p is the wavelength measured perpendicular to the sloping bottom as discussed in *Ivey et al.* 1995. The dissipation rates are scaled by the quantity $\varepsilon_* = E_{in}/hl$ where E_{in} is the incident energy flux and l is the along slope mixing length determined by the intersection region of incident and linearly reflecting wave rays. This scaling implicitly assumes a reflection coefficient of zero. A somewhat better result was obtained for a reflection coefficient of 0.3 but a zero value was used here for simplicity. Scaled dissipation rates are approximately uniform for $z/h < 1$ and decay rapidly with distance for heights $z/h > 1$. The scale h can thus be taken as the defining scale of the turbulent boundary layer. The values

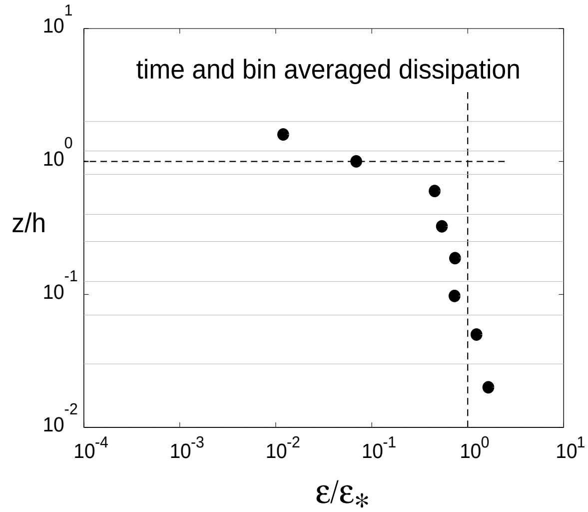

Figure 2. Measured dissipation rates as a function of perpendicular distance from the sloping boundary. Heights are scaled by the estimated boundary layer scale h. Dissipation rates scaled by the scale ε_* (see text). The figure summarizes all experiments conducted in which dissipation measurements were made.

shown have been averaged within the vertical bins marked in the figure.

Variability within individual experiments (not shown) is most likely due to a dependence on the phase of the incident waves. To first order, the simple scaling with h and ε_* appears to collapse the time-averaged dissipation rates within the boundary layer over a moderate range of the criticality parameter γ. The magnitudes of the unscaled dissipation rates within the boundary layer were generally between 5×10^{-7} and 10^{-6} m^2/s^3. These values correspond to $\varepsilon/\nu N^2$ of approximately 1 for the range of stratifications used in the experiments.

2.5. Flux Measurements

Time series measurements were made at fixed positions using the Portable Flux Profiler (PFP) (*Imberger and Head*, 1994). The PFP is equipped with duplicate pairs of thermistors and fast response conductivity sensors. A laser Doppler anemometer (LDA) measures two independent components of velocity. The temperature and conductivity sensors were separated by 2.5 mm in the horizontal, the time response and resolution of the thermistors was 12 ms and 0.001 C and those of the conductivity probes 4 ms and 0.0004 S/m. The LDA measured two velocity components along and across the (inclined) probe axis with a resolution of 0.001 m/s. Temperature and conductivity signals were recursively filtered to match the difference in time response. The probe itself was mounted up-slope of the interaction region, with sensors protruding into the region of interest, to minimize the effects of flow around the probe casing. Fluxes of both mass and momentum were determined from time series of two velocity components, temperature and conductivity over sampling periods of approximately five minutes after the paddle motion was initiated.

Figure 3 shows the co-spectrum of density and vertical velocity $Co_{\rho w}$, in area-preserving form at four different heights for a typical experiment. The heights are normalized by the boundary layer scale h determined by the dissipation measurements. A negative value of the co-spectrum indicates a release of available potential energy and a transport leading to restratification. The clearest feature for all four heights is the large negative contribution at the forcing frequency ω_f. This feature dominates the total flux with the result that the total flux is negative at all heights measured. Note that even for $z/h = 1.31$, the time series measurements were still made close to the bottom boundary, i.e., within the region where incident and reflected waves can interact according to linear theory (see Figure 1). The magnitude of the flux near ω_f is comparable both inside and outside the boundary layer but decays significantly very close to the bed. The co-spectrum is small in magnitude and can be of either sign at frequencies near and greater than N.

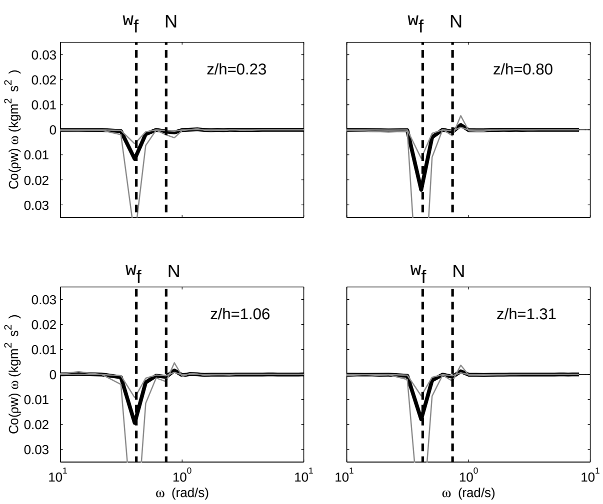

Figure 3. Co-spectrum of vertical velocity and density at four heights for a typical experiment. Spectral analysis was conducted for time series sampled at 100 Hz for approximately 5 minutes. The forced wave and the buoyancy frequencies are indicated on the figure.

3. Interpretation

The benthic boundary layer is turbulent. Typical cycle-averaged dissipation rates within the boundary layer are on the order of 10^{-6} m^2/s^3. This rate is *much* greater than the laminar dissipation rate $\varepsilon_{wave} \approx \nu(ak\omega_f)^2 \approx 10^{-9}$ m^2/s^3 characteristic of the incident wave motion. Average dissipation rates are approximately uniform within a distance h of the boundary and decay rapidly to laminar rates at distances greater than h.

Though the enhanced dissipation rates indicate that the flow is turbulent, the parameter $\varepsilon/\nu N^2$ is only of order 1. When $\varepsilon/\nu N^2$ is equal to 1, the Ozmidov scale $L_O = (\varepsilon/N^3)^{1/2}$ is equal to the Kolmogorov scale $L_K = (\nu^3/\varepsilon)^{1/4}$. This is usually taken to indicate that there is no bandwidth available for a turbulent cascade. In these experiments however, there is an additional scale ($\approx a$), the scale at which waves overturn and break. Energy is continuously supplied to the boundary layer at this scale. Thus, though $\varepsilon/\nu N^2 = 1$, there is a finite range of scales, from L_K to a, within which a nonlinear cascade can occur.

The mass fluxes measured within the boundary layer are signatures of nonlinear interactions between the incident and reflecting waves. The correlation is dominated by frequencies close to the incident wave frequency with little contribution from significantly higher frequency motions. The observed mass fluxes are not spatially correlated with the dissipation rates, which are clear signatures of turbulence intensity. Since the flux is carried primarily by wavelike motions near the forcing frequency, and these motions are not confined to the zone of enhanced dissipation but extend throughout region where incident and reflected waves can interact, the buoyancy flux measured within the turbulent layer is difficult to interpret. It would appear that global rather than local measurements of buoyancy flux are required to remove the wavelike signature, even after long-time averaging,

Inference of diapycnal mixing from the local flux measurements is thus problematic. The observed local correlation $\langle \rho w \rangle$ is negative within the quasi-steady turbulent mixing zone. If we *define* a diapycnal diffusivity K_ρ in terms of the observed flux, i.e., as $\langle \rho w \rangle / |d\rho/dz|$, then we must conclude that $K_\rho < 0$, which is clearly not sensible. It would appear that the (unmeasured) positive fluxes well outside the boundary layer should be included. Unfortunately, this is not possible. It is tempting however to attempt a local analysis based only on the high frequency motions, say for frequencies greater than N. If only these frequencies are included however, the experiments suggest that K_ρ is negligible. A negligible diffusivity implies that there is a negligible cascade of scalar variance to small scales where molecular diffusion occurs. It is not at all clear how a turbulent flow could sustain a velocity cascade sufficient to enhance laminar dissipation rates by three orders of magnitude but not support a corresponding scalar cascade. We suggest that a properly defined diffusivity should be non-negligible for these experiments. The question is how should we define the diffusivity?

These observations lead to a rather fundamental question: should we always expect *local* measurements of buoyancy flux to be highly correlated with turbulent dissipation and/or mixing when the turbulence is localized in space? After all, in these experiments the waves that carry the flux are clearly not confined to the mixing region. If not, is it reasonable to infer mixing rates from formulae based fundamentally on a *definition* of K_ρ in terms of the local buoyancy flux $<\rho w>$?

These experiments provide an opportunity to re-evaluate some of our fundamental assumptions regarding the link between dissipation, buoyancy flux and irreversible mixing in situations where turbulence is spatially localized and results from the collapse of relatively energetic internal waves. We suggest that these characteristics are not confined to turbulence at sloping boundaries but may also be characteristic of much of the mixing in the interior of stratified basins.

Diapycnal diffusivities K_ρ are frequently inferred from dissipation rates ε using the formula $K_\rho = 0.2\ \varepsilon/N^2$. It is worthwhile to remind ourselves that this expression is derived by assuming a local steady balance of turbulent kinetic energy (TKE), with divergence terms neglected and the buoyancy flux acting as a sink for TKE. *Defining* the diffusivity in terms of the local mass flux, i.e., via $K_\rho = -<\rho w>/(d\rho/dz)$ and substituting into the assumed TKE balance leads to an expression which is often simplified to the familiar result $K_\rho = 0.2\varepsilon/N^2$. It would appear that this result can be no better than the assumption that the diffusivity is reasonably defined in terms of the local buoyancy flux. Since we have already argued that using the measured flux to estimate the diffusivity is problematic in these experiments, it is difficult to justify estimating the diffusivity from $0.2\varepsilon/N^2$.

4. Proposed Scaling of K_ρ

Many of these conceptual difficulties can be alleviated by returning to first principles and defining a diapycnal diffusivity in terms of irreversible diffusive flux at molecular scales. The average diffusive or diapycnal flux F across an isopycnal surface S is given by (*Winters and D'Asaro*, 1996)

$$F = \frac{1}{A}\int \kappa |\nabla\rho|\ dS = \kappa \frac{\langle|\nabla\rho|\rangle^2}{\left|{d\rho}/{dz_*}\right|} \tag{1}$$

where the quantity z_* is an isopycnal coordinate with dimensions of length, the angled brackets represent a spatial average over the isopycnal surface S, κ is the thermal diffusivity and A is the geometrical projection of S onto a horizontal plane. If we define the diapycnal diffusivity K_ρ as the average flux F divided by the thermodynamically relevant gradient, i.e., as

$$K_\rho = \frac{F}{\left|{d\rho}/{dz_*}\right|} = \kappa \frac{\langle|\nabla\rho|\rangle^2}{\left|{d\rho}/{dz_*}\right|^2} \quad (2) \tag{2}$$

where $\rho(z_*)$ is the adiabatically resorted reference gradient. Scaling estimates can be obtained for K_ρ defined in this way by considering an idealized model of turbulent eddies under the influence of buoyancy.

We consider stably stratified fluids with $\Pr = (\nu/\kappa) > 1$, an ambient N^2 given by $(-g/\rho_0)(d\rho/dz_*)$ and turbulent motions characterized by eddies of scale L. The maximum density difference $\Delta\rho$ between fluid parcels within an eddy is thus of order $L\ |\ d\rho/\ dz_*|$. Straining or wrapping motions result in a convergence of isopycnal surfaces with the characteristic density difference $\Delta\rho$. The separation distance between two such surfaces will decrease until it becomes comparable to the Batchelor scale L_B at which point molecular diffusion will prevent further enhancement of the gradients. Since the gradient spectra will be concentrated at small scales, this idealized model implies that the characteristic magnitude of the density gradient will be given by

$$\langle|\nabla\rho|\rangle \sim \frac{\Delta\rho}{L_B} \sim \frac{\left|{d\rho}/{dz_*}\right| L}{L_B} \tag{3}$$

Substituting (3) into (2) yields

$$K_\rho \sim \kappa \left(\frac{L}{L_B}\right)^2 \tag{4}$$

The characteristic displacement scale L, i.e., the scale that supplies the density contrast in (3) will depend on both fluid and flow properties. We now outline three distinct flow regimes and give scaling relations for the diapycnal diffusivity K_ρ.

4.1 Laminar flow

In this regime the gradients induced by eddies are not appreciably greater than the background gradient and so $L \approx L_B$ and K_ρ reduces to the molecular value κ as required.

4.2 Weak density-stratified turbulence

We wish to estimate a scale L proportional to the scalar contrast when the turbulence is weak enough that the fluid viscosity is dynamically important. If the scalar were passive, then the scale could be determined by assuming that $L = L(\varepsilon,\nu)$ yielding $L = L_K = (\nu^3/\varepsilon)^{1/4}$ as discussed in *Batchelor* (1959). For density-stratified fluids, even small scale motions require work to be done against gravity and we thus expect the buoyancy N to be important and to reduce the characteristic scale L. Assuming that $L = L(\varepsilon,\nu,N)$ gives $L = (\nu\varepsilon)^{1/4}/N^2$. N will be important when the buoyancy time-scale $1/N$ is less than (or of the same order as) the straining time scale $(\nu/\varepsilon)^{1/2}$. This is equivalent to the condition $(\varepsilon/\nu N^2)$ is less than or equal to order 1. We note that this scale L approaches L_B as $\varepsilon \to \kappa N^2$ from above and $L \to L_K$ as $\varepsilon \to \nu N^2$ from below.

Substituting this result into equation (4) yields

$$K_\rho \sim \frac{\varepsilon}{N^2} = \nu\left(\frac{\varepsilon}{\nu N^2}\right) \qquad (5)$$

This expression is equivalent in form to the familiar expression $K_\rho = 0.2\varepsilon/N^2$ but required no assumptions about the buoyancy flux or the balance of turbulent kinetic energy. The scaling analysis offers no information on the constant of proportionality.

4.3 Energetic density-stratified turbulence

For energetic density-stratified turbulence, the fluid viscosity ν is no longer dynamically relevant. We expect L to be larger than for weak density-stratified turbulence and to depend only on ε and N. In this regime buoyancy will act to suppress scales larger than $L_O = (\varepsilon/N^3)^{1/2}$ and so an upper bound on the length scale L will thus be the Ozmidov scale L_O. Substitution into (4) gives

$$K_\rho \sim \nu\left(\frac{\varepsilon}{\nu N^2}\right)^{3/2} \qquad (6)$$

The requirement that L_O be greater than the maximum L in the weakly turbulent regime is simply that $L_O > L_K$ or $\varepsilon/\nu N^2$ is order 1 or greater. In the limit of vanishing N, not only does L_O become unbounded but our simple conceptual picture of the characteristic density contrast in an eddy being proportional to the ambient gradient becomes increasingly inappropriate. We thus expect equation (6) to be an upper bound and to overestimate the diffusivity in the unstratified limit.

5 Application of Scaling to Experiments

The time averaged dissipation rates are in the range $(\varepsilon/\nu N^2) \approx 1$ for the experiments under discussion. This regime sits at the upper limit of weak density-stratified turbulence and the lower limit of energetic density-stratified turbulence. Thus, either of equations (5) or (6) can be used to estimate the magnitude of the diapycnal diffusivity K_ρ. Both formulas are in agreement and give $K_\rho \approx O(\nu)$ as $\varepsilon/\nu N^2 \to 1$.

Note that ν is equal to Pr times the molecular diffusivity κ and for a salt-stratified fluid Pr $\approx$ 700. The scaling results thus predict an enhancement of diffusivity by nearly three orders of magnitude over molecular levels. This result seems reasonable in light of our earlier observations that measured ε values were approximately 1000 times the laminar rate ε_{wave}. We note however that this interpretation is not consistent with the notion that when $\varepsilon/\nu N^2 < 15$-25, turbulence is too weak to support turbulent mixing.

6. Discussion

Experiments were conducted in which nearly monochromatic wave beams in a uniformly stratified fluid forward reflect at a sloping boundary. Under conditions of steady forcing, measurements of the dissipation rate of turbulent kinetic energy revealed the presence of a turbulent benthic boundary layer with nearly uniform time-averaged properties. Despite the simplicity of the experimental set-up, the observations are not readily explained using existing models of stratified turbulence.

The observations underscore the difficulties inherent in separating "wavelike" motions from "turbulent" motions, a notion implicit in existing models. The measured turbulence has $\varepsilon/\nu N^2$ of order 1 which implies $L_O = L_K$ and thus (apparently) a vanishing bandwidth for a "turbulent" cascade. In these experiments however, energy is supplied to the boundary layer at the wave scale a which is much larger than L_K. Apparently, the dynamics of breaking waves supports a cascade from a to L_K and thus a greatly enhanced dissipation rate. Are the motions responsible for this cascade "wavelike" or "turbulent"?

The question is not entirely semantic because defining K_ρ in terms of the buoyancy flux requires that these motions, which are characterized by $\langle\rho w\rangle < 0$, be regarded as "wavelike" and somehow separated out from the "turbulent" motions before examining the local TKE balance. If such a separation is difficult or impossible in controlled laboratory settings, how useful is the concept when applied to time dependent mixing events in the field? How much faith should we have in the resulting formula $K_\rho = 0.2\varepsilon/N^2$? Despite the importance of this issue there are no controlled experiments that we are aware of where an appropriately defined diapycnal diffusivity, ε and N^2 have been independently measured. What is the relationship between these quantities and how does it depend on flow and fluid properties?

We have attempted to address these difficulties by avoiding the necessity of a wave/turbulence decomposition and defining K_ρ directly in terms of the average diapycnal flux (2). The resulting formula is diagnostic; it requires the measurement of small-scale density gradients and averaging on isopycnal surfaces. The expression is positive definite, greater than or equal to the molecular diffusivity κ and independent of the sign of the buoyancy flux, all properties that would appear intuitively desirable.

Introducing a simple model for the estimation of the density gradients in (2) results in scaling laws for K_ρ in terms of ε and two fluid properties ν and *N*. The model, and hence the resulting scaling, is based on two related ideas. The first is that at scales large compared to turbulent eddies, the fluid exists in a state not far from its adiabatic equilibrium and so the characteristic density contrast within eddies is proportional to the product of the eddy scale and the ambient density gradient. The second is that the eddy scale is influenced by buoyant restoring forces and so *N* must be included in the scaling. Rather than splitting waves and turbulence, these ideas suggest a fundamentally wavelike nature of density-stratified turbulence, i.e., even small-scale vertical motions induce a restoring force. Clearly, these ideas break down in the limit of vanishing *N*. No insight is gleaned from the analysis regarding an upper limit on the range of validity.

An experimental program in which ε, *N*, and changes in background potential energy were measured in a controlled environment would aid in the clarification of many of these issues.

Acknowledgments. We would like to acknowledge the contributions of Prabath DeSilva who carried out much of the laboratory work. Nicky Grigg also assisted with the preliminary experiments. The flux measurements were made possible by Jorg Imberger who made the PFP available to us. This work was supported by the Office of Naval Research (Code 322 PO) , the National Science Foundation and the Australian Research Council.

References

Batchelor, G. K., Small-scale variation of convected quantities like temperature in a turbulent fluid. Part 1: General discussion and the case of small conductivity, *J. Fluid Mech., 5*, 113-133, 1959.

Imberger J., and R. Head, Measurements of turbulent properties in a natural system, in *Fundamentals and advancements in hydraulic measurements and experimentation*, pp. 1-20, ASCE, Buffalo, 1994.

Ivey, G. N., and R. I. Nokes, Vertical mixing due to the breaking of critical internal waves on sloping boundaries, *J. Fluid Mech., 204*, 479-500, 1989.

Ivey, G. N., I. P. D. DeSilva, and J. Imberger, Internal waves, bottom slopes and mixing. *1995 Aha Hulikoa Hawaiian Winter Workshop on Topographic Effects in the Ocean*, pp. 199-205, Honolulu, Hawaii.

Ivey, G. N., K. B. Winters, and I. P. D. DeSilva, Turbulent mixing in an internal wave energised benthic boundary layer on a slope, *J. Fluid Mech., submitted*, 1998.

Luketina, D. and J. Imberger, Determining turbulent kinetic energy dissipation from Batchelor curve fitting, *J. Atmosph. And Oceanic Technology*, (submitted) 1998.

Winters, K. B. and E. A. D'Asaro, Diascalar flux and the rate of fluid mixing, *J. Fluid Mech., 317*, 179-193, 1996.