Stratified Flow Over Topography and the Generation of Internal Solitary Waves

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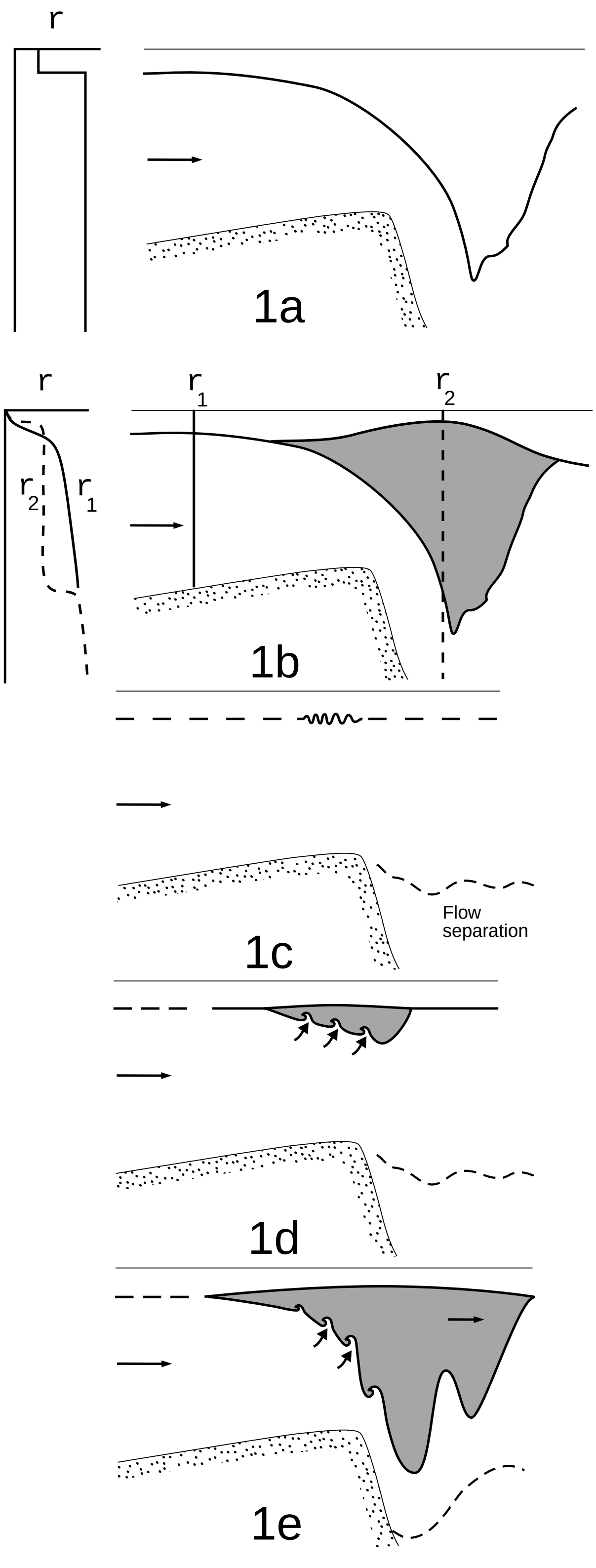
Abstract. Internal solitary waves, which form such a prominent feature in remote sensing images of the coastal zone, are the far field expression of stratified flow over topography. Nonlinear evolution of large amplitude internal tides propagating over the continental shelf is one means by which they might be generated, but detailed observations of wave generation over topography in an inlet illustrate other mechanisms that occur when the topography is abrupt. Tidally forced flow over abrupt topography can lead to streamline splitting. The streamline bifurcation leads to topographic control of the flow beneath the bifurcated layer, even though a simple calculation of the densimetric Froude number using tidal velocities and the unperturbed density field would imply that the flow was subcritical. Its evolution over the changing tidal current is intimately linked to the formation and suppression of boundary layer separation. The split streamlines enclose a slowly moving and weakly stratified layer accompanied by shear flow instability which itself may be a source of nonlinear internal waves. Moreover the collapse of this slowly moving layer as the tidal current slackens generates an internal undular bore. Observations are required to show the generality of these different mechanisms, especially in areas where solitary waves are commonly observed over the continental shelf.

1. Introduction

Internal solitary waves are widespread in the coastal ocean and a prominent feature of remote sensing images of shelf waters, often taking the form of an ordered set of lenticular streaks radiating shoreward. Aside from their fluid dynamical interest as examples of waves of permanent form, internal solitary waves may play a role in cross shelf advection, mixing and the stirring of sediments. Strong currents associated with internal solitary waves may also present a significant hazard to offshore engineering operations. Their prominence in radar and optical images also motivates development of a better understanding of their generation and propagation as remotely sensed signatures of the ocean environment.

The occurrence of internal solitary waves in coastal waters is usually modulated by tidal forcing. It appears that the waves are associated with flow over topographic features such as ridges, canyon walls, and the shelf break; however, the details of the generation process have been investigated in only a few locations. The waves can be generated in the immediate vicinity of the topography due to a strongly nonlinear interaction such as an internal hydraulic transition, or they can evolve as a high frequency far-field manifestation of an internal tide. An example of the latter case which has been explored in great detail, is the soliton train forming over the north west Australian shelf (*Holloway et al.*, 1997, *Holloway et al.*, 1999 and elsewhere in this volume). An internal tide of large amplitude propagating across the relatively shallow shelf steepens, leading to growth of high frequency internal solitary waves. Many aspects of the generation and subsequent behavior of the waves appear explainable with the extended KdV equation, including aspects of the dissipation and the effect of rotation and cubic nonlinearity.

Abrupt topography appears to be particularly effective at nonlinear internal wave generation in the near field. Field studies in Knight Inlet have provided very detailed observations of this process, although the complex and nonlinear pattern of topographic response has thus far resisted comprehensive model analysis. Several mechanisms appear responsible for wave generation, including the collapse of an internal hydraulic response which can lead to intrusive undular bores. Nonlinear internal waves can also be formed as an upstream influence of the topography. Unlike the gradual evolution from a steepening



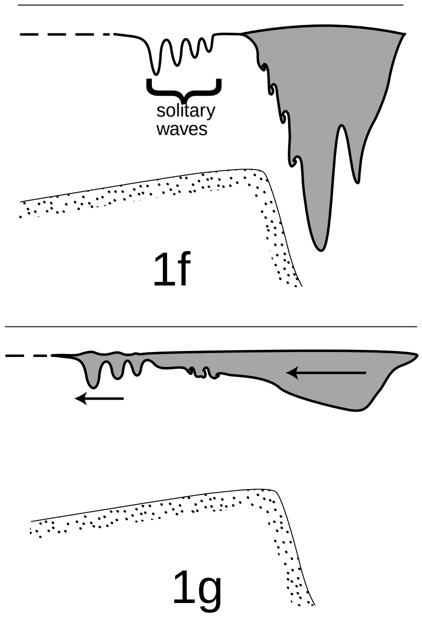


Figure 1. Sketch showing different hydraulic responses discussed in the text, including generation of internal solitary waves.

internal tide, all these processes are associated with strongly nonlinear phenomena in the immediate vicinity of the topography. Therefore we might expect their occurrence to predominate in places and at times where the flow is controlled, i.e., where the flow speed is equal to or greater than the lowest mode internal wave speed. An interesting characteristic of the flows described here is that they can arise even when the Froude number calculated from the tidal current and the equilibrium density field is subcritical. However, as shown below, the flow does develop in such a way that it becomes controlled. The mechanisms associated with this process are conveniently explained in quasi-steady 2-dimensional hydraulic terms. Although the real environment will likely involve other aspects, this simplified description provides a useful starting point for investigating the potential origin of internal solitary waves over abrupt topography, even in places where a Froude number calculation using the unperturbed density field would imply subcritical conditions throughout the tidal cycle.

The evolution of internal solitary waves from flow over abrupt topography discussed here is a coupled time-dependent nonlinear problem with flow bifurcation and boundary layer separation. Despite model analysis of discrete aspects of the flow, there have as yet been no comprehensive models which include all of the essential features. Our purpose here is to illustrate these features with observations acquired in Knight Inlet, British Columbia.

2. Flow separation over abrupt topography

We are concerned with abrupt topography where the flow encounters a sharp increase in total water depth. Unstratified flow past bluff bodies is a widely studied field, with implications for the calculation of drag. Stratified flow over abrupt topography is more complicated and has received relatively less attention (*Huppert and Britter*, 1982), although it has implications for atmospheric flow over mountains. Separation of the boundary layer can occur when there is an adverse pressure gradient caused by high curvature. It results in the formation of a separation 'bubble' downstream of the topography which largely remains isolated from the main flow except for a slow recirculation. The separation has the effect of isolating the main flow from the abrupt feature of the topography and thus reducing the drag. For density stratified conditions development of lee waves or a plunging lower layer can be inhibited by separation. This interaction with the density stratification allows the separation point to move further downslope leading to a progressive transition towards the high drag state.

3. Internal hydraulic response

The steady state internal hydraulic response of stratified flow over topography has been widely studied (*Armi*, 1986). Although the internal hydraulic response to unsteady forcing has been less well studied, some insight into the behavior can be achieved with a quasi-steady model if the time dependence is small enough that the flow can respond locally to the forcing. This will normally be true, for example, if the time required for a density perturbation to propagate across the relevant topographic feature is small compared to the time scale for barotropic forcing.

The steady state hydraulic analysis then identifies the point at which the flow is controlled and describes the asymmetry of the flow about the control characteristic of the nonlinear response. Figure 1a shows this response for a steady two-layer flow, illustrating the asymmetry between the subcritical and supercritical flow. At some point the downstream conditions require that the flow be matched through an internal hydraulic jump. Dissipative effects become important at this location. The jump may be undular, in which case internal waves evolve. These waves can be of large amplitude, comparable to the vertical scale of the topography. In general, coastal flows have more complicated density profiles than the two-layer structure shown in Figure 1a. For example a well defined 3-layer density structure may occur, allowing development of a mode 2 response. This means that an internal hydraulic response can occur, even when a first order calculation of the densimetric Froude number implies that the flow is subcritical. More generally the stratification is normally continuous, for example with a thin strongly stratified layer overlying weaker stratification. Flow establishment may then take place through a bifurcation as shown in Figure 1b.

4. Flow bifurcation

Bifurcation appears to be a very common characteristic of continuously stratified flow over topography. The experiments carried out by *Long* (1955) show streamline bifurcation forming detached, recirculating flows which are trapped downstream of the obstacle. Long's theoretical analysis appears to predict closed streamlined flow which is qualitatively similar to the laboratory results, but the solutions are not strictly valid in this case. If closed streamline flow occurs or, more generally, if a bifurcation in the flow occurs, the question arises as to the origin of the fluid within the bounding streamline. In other words, how is the flow set up?

This problem is implicit in many studies of atmospheric flow over topography where its significance arises because of the drag of mountain ranges which are thought to account for approximately one half of the total drag of the earth on atmospheric circulation (*Baines*, 1995). The evolution of bifurcating flows has been examined with numerical models and these have motivated explanations of the mechanism (i.e., *Peltier and Clark*, 1979). The problem is difficult to model because of Reynolds number limitations and the atmospheric observations lack the density required for detailed comparison. Tidal flow over a sill provides a useful oceanographic analog to the atmospheric case while also illustrating a mechanism that must be widespread in the coastal ocean.

5. Flow establishment: bifurcation and the suppression of flow separation

Quite detailed measurements of the response of stratified flow to tidal forcing over a sill in Knight Inlet (*Farmer and Armi*, 1999a) show the way in which the bifurcation develops. The tidal current starts with small scale instability and entrainment which then allows a weakly stratified and slowly circulating intermediate layer to form. This layer steadily grows, leading to the establishment of the equilibrium steady state hydraulic response. The process is inextricably linked to the flow separation discussed above. The formation of the slowly moving layer downstream of the bifurctaion allows the deeper layer to flow down the lee slope of the topography, suppressing the separation. This transition, however, will occur gradually, since it is coupled to the filling in of the recirculating flow above. Thus the time dependent evolution involves an initial separation from the crest so that the downstream flow leaves as a horizontal jet (Figure 1c).

The strong shear engendered by acceleration over the crest creates instability and the transfer of fluid from the deeper layer up into the weakly stratified flow above. The growing intermediate layer remains trapped above and downstream of the crest (Figure 1d).

As the weakly stratified layer fills in further, the deeper layer starts to descend a little as it passes the sill crest and the separation point moves downstream (Figure 1e). The evolution of this flow therefore represents a coupling between the shear instabilities that transfer fluid into the intermediate layer, the expansion of this intermediate layer, and the suppression of the boundary layer separation.

The location of the bifurcation is determined by the density of the trapped fluid, the surrounding stratification, the shape of the underlying topography and the strength of the barotropic component. If all of these variables are given, the shape of the plunging interface may be derived from the quasi-steady hydraulic equations. The slowly moving intermediate layer is stratified and the resulting variation in the strength of the density step has a significant effect on the resulting shape of the interface. A calculation along these lines is given in *Farmer and Armi* (1999a).

As the barotropic transport continues to increase, the bifurcation will move downstream and may pass over the crest. Analysis of the hydraulic response to a slowly changing tidal current shows that the position of the bifurcation when it is close to the crest is extremely sensitive to the barotropic forcing. A quasi-steady analysis cannot therefore be expected to provide a reliable indication of what happens, but observations confirm that nonlinear internal waves appear just upstream of the bifurcation at the same time as it retreats over the sill crest (Figure 1f and Figure 2). These solitary waves are of first mode and propagate upstream of the bifurcation.

6. Collapse of the intermediate layer and solitary wave generation

As the tidal current slackens, the large mass of nearly stationary fluid that has accumulated over and just downstream of the sill crest collapses. For a short period an exchange flow occurs: the collapsing weakly stratified layer moves upstream while the deeper layer continues downstream. If there is no strong surface stratification, the collapsing upper layer advances upstream as an internal bore which may be undular, thus forming nonlinear internal waves. These have been referred to as 'solibores' (*Henyey*, 1998) and have been widely observed in many coastal environments. If the weakly stratified layer is capped by stronger stratification, the upstream flow advances as an intrusive feature that can be described as a mode 2 internal bore. Again, the bore can be undular and thus exhibit mode 2 nonlinear internal waves (Fig. 1g). If the internal hydraulic jump that had formed during the tidal current downstream of the topography was itself undular, each wave crest can preserve its identity as it moves over the crest, thus producing a train of internal solitary waves that advance upstream (c.f. Figure 9 in *Farmer and Smith*, 1980).

These various outcomes have been well documented and are probably widespread. They depend on control of the flow over the crest. The important point here is that a simple Froude number calculation based on the unperturbed density stratification and predicted tidal current is *in*sufficient to determine whether or not the flow becomes controlled. Bifurcation of the flow allows a single layer 'reduced gravity' hydraulic transition to occur. The formation and relaxation of this flow may then serve as a site for the generation of internal solitary waves.

7. The Generation of internal solitary waves upstream of the control

The formation of mode 1 internal solitary waves upstream of the bifurcation (Figure 1f) represents an example of transcritical wave generation in the sense that the waves are formed upstream of a hydraulic control. It has been observed in both laboratory and numerical models that an instability of the control can generate upstream propagating internal solitary waves, even for steady forcing (*Cummins* 1995, *Melville and Helfrich*, 1987) . In addition to the unsteady mechanism illustrated in Figures 1f and 2, it also appears possible for internal solitary waves to be formed on the unstable shear layer associated with bifurcation.

The bifurcation encloses an intermediate layer that moves slowly relative to the deeper flow. The unstable interface between the layers may form waves that can propagate in either direction. If the scale of the waves is such that the wave speed exceeds the local flow speed, then waves travelling against the flow will propagate upstream. Waves that travel with the flow move quickly downstream. The wave speed depends, of course, not only on the stratification, but also on the shear arising from the larger scale topographic response. These waves will be 'transcritical' in the sense that they are formed and propagate upstream of a flow control. However, the formation of upstream propagating waves in this way requires that the instability be of large enough scale to have a speed exceeding that of the background flow speed. This is the speed of a long interfacial wave at the control; just upstream of this point the flow will be subcritical, but short waves propagating against the current will still be swept downstream.

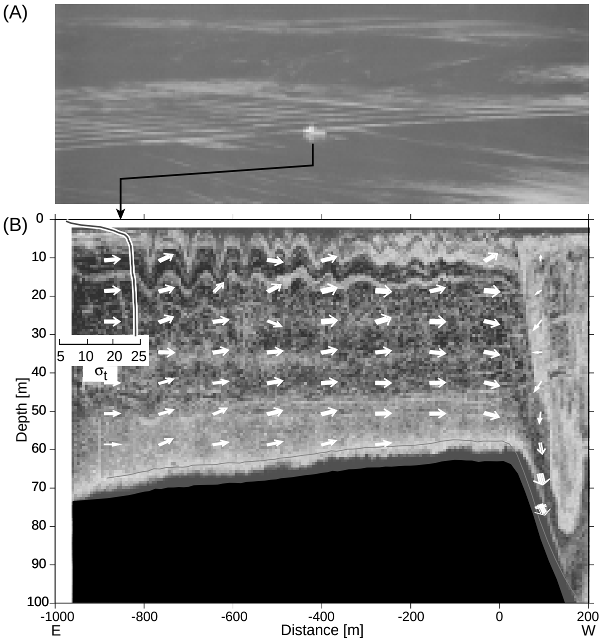


Figure 2. (Above) Air photo of solitary waves in Knight Inlet. (Below) Acoustic image and Doppler velocity field corresponding to the air-photo, showing internal solitary waves. Inset: Upstream density profile.

The scale of the unstable flow is determined by the scale of the fastest growing wave in the unstable shear layer. The resulting wavelength may be too small to produce an upstream propagating wave; however, growing instabilities interact and can therefore generate components having a greater wavelength than the original disturbance. Even if the primary wave is swept downstream, the longer wavelength component, having greater celerity, may still propagate upstream. The stability is found from the dispersion relationship for a stratified flow with shear:

$$c(k)=\left\{P\pm\left(P-\sum R\left[U_1^2k^2R_1+U_3^2k^2R_2-\rho_2 g' k\right]\right)^{\frac{1}{2}}\right\}\Big/ k\sum R$$

where $P = k(U_1R_1 + U_2R_2)$, U_i is the speed, ρ_i the density and h_i the thickness of the upper and lower layers, i=1,2, respectively, k is the wavenumber, $R_1 = \rho_i$ cotan $h(kh_i)$, $\Sigma R=R_1+R_2$, g is gravitational acceleration and $g' = g \times (\rho_2-\rho_1)/\rho_2$ the reduced gravity.

Figure 3a (adapted from *Farmer and Armi* 1999b) shows observations of an unstable shear layer. There is some indication that the wavelength of the instabilities is greater close to the sill crest. Figure 3b shows the same environment 30 minutes later. Nonlinear interfacial waves appear upstream of the obstacle crest. Since the measurements are not continuous we have no direct observation of the origin of the waves. Upstream propagating waves would move along an interface that approaches the surface, thus enhancing the nonlinearity.

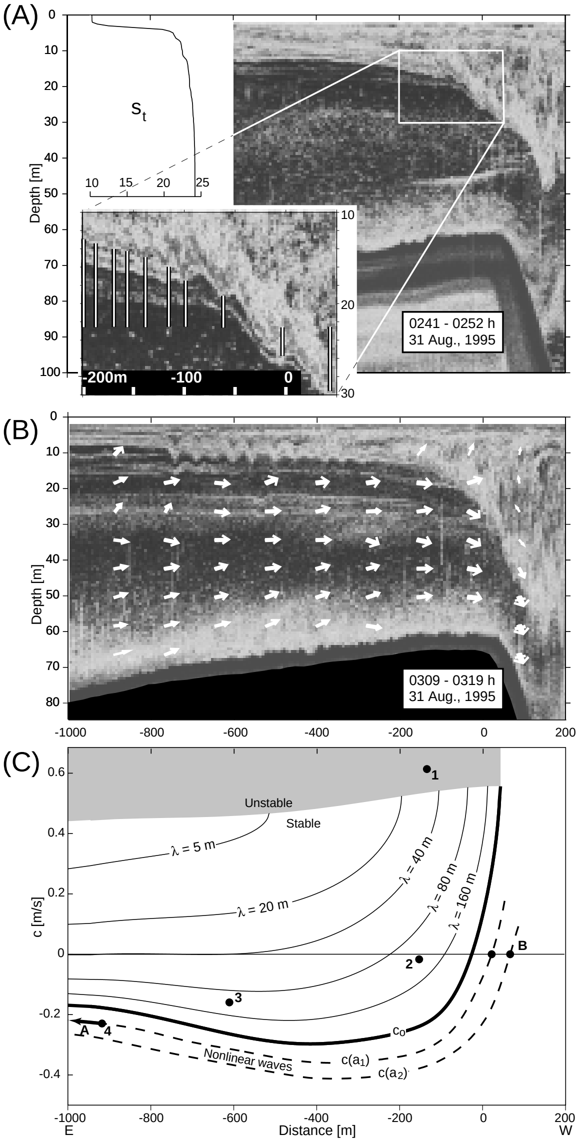


Figure 3. (a) Acoustic image showing bifurcating flow over Knight Inlet and instabilities on the descending shear layer. (Inset: Enlargement to illustrate changing scale of instabilities.) (b*)* As above, but 30 minutes later. (c) Dispersion relation for upstream propagating waves, showing wave speed as a function of wavelength and position. Instability indicated by area of grey tone.

Figure 3c shows the stability and wave speed for different wavelengths and positions calculated from the dispersion equation relative to a fixed frame of reference coincident with the figures above. The wave speeds are only shown for waves that travel against the current. Positive speeds correspond to waves that are advected downstream; only waves with negative speeds can escape upstream. The heavy line identified as c_o identifies the speed of the long interfacial wave.

Figure 3c also illustrates a possible sequence of wave generation. Shear flow instability at (1) leads to waves that interact to produce longer waves (2) which can propagate upstream (3), eventually evolving into solitary waves (4) as they move up the shoaling interface.

Since an inherent property of internal solitary waves is that they have a greater celerity than long infinitesimal waves, it should be possible for a solitary wave to maintain its position against the background flow speed within the supercritical region just downstream of an internal hydraulic transition. This is illustrated in Figure 3c at position B. Solitary waves have been observed to remain trapped at this location for up to 2 h.

9. Conclusions

The observations discussed here illustrate some of the ways in which internal solitary waves can arise from stratified flow over abrupt topography. The mechanisms are involved and evidently depend on an interaction between shear flow instability and mixing, boundary layer separation and the internal hydraulic response. Thus far we lack the observations required to demonstrate the generality of the Knight Inlet results, but it would appear likely that these wave generation mechanisms occur wherever there are strong tidal currents over abrupt topography.

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