Uncertainty in Oceanic General Circulation Models

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Abstract. The existence of global ocean circulation models has permitted computations of putative future oceanic states running more than 1000 years into the future, but there is little or no understanding of the actual skill of the results. With the parallel appearance of true global ocean data sets, there is real urgency in using the model/data combinations to render oceanography a far more quantitative subject than it has been in the past. Understanding of observational errors is comparatively advanced compared to that for the models. For the latter, two separate error types, systematic and random, must be considered. Systematic errors, even if very small, can accumulate over long integrations and ultimately swamp any forecast skill. In particular, little is known about the accuracy of Lagrangian trajectory products—products which are crucial to climate. The study of the random errors is inhibited by two formidable problems, nonlinearity and dimension. Methods now exist to greatly increase the computational efficiency of model error estimates, including one described here based upon modeling the errors on multiscale trees.

1. Introduction

Oceanography has reached a new stage: we now have global general circulation models (e.g., Semtner and Chervin, 1992; Smith et al., 1992; Marshall et al., 1997) with a considerable degree of qualitative realism. But in addition, largely through the World Ocean Circulation Experiment (WOCE), we have global, or basin scale, ongoing data sets of which altimetry (e.g., Fu et al., 1994) is only the most conspicuous data type among floats, hydrography, and acoustic tomography (ATOC Consortium, 1997) and others. These data sets are being initially used to test the models in an effort to describe the quantitative skill of the latter (e.g., Stammer et al., 1996; Fu and Smith, 1996).

With the arrival of the models — which will improve with time — and with some assurance of continuing observations (less certain), one of the emerging central issues in oceanography is determining and understanding the quantitative skill of the general circulation models as descriptors of the overall ocean circulation. Description and insight into the errors of the models is becoming urgent for a number of reasons, not least among them is their use coupled to atmospheric models (e.g. Houghton et al., 1996, and Manabe and Stouffer, 1994) in attempts to understand future climate states. Such models are now run out to 1000 years and more, and the resulting scenarios are often employed, whether model originators wish it or not, as forecasts of actual future climate. The extent to which 1000 years of integration of an oceanic general circulation model carries any predictive skill at all is almost completely obscure at the present time — despite the immense economic and political stakes.

Modern observations and high resolution models both show that the ocean circulation is, in a basic sense, turbulent. Coarse resolution models represent this turbulent flow as a laminar one. If it is really possible to quantitatively model the turbulent circulation with a laminar representation, the result is a remarkable achievement in fluid dynamics: a serious and extremely important turbulence parameterization problem has been solved. But practitioners of such models do not appear to be claiming a major breakthrough in the theory of turbulence. Perhaps one is extremely fortunate in having a fluid circulation in which the turbulent elements are purely passive both dynamically and kinematically—a remarkable situation in itself if true and whose generalization to other situations should be the focus of intense activity. In the absence of evidence to the contrary, an onlooker is entitled to suspect that the models actually have no demonstrable skill.

On a more immediate basis, many (uncoupled) oceanic models are being combined with data in estimation schemes ("assimilation" is the jargon adopted from meteorology) so as to best-estimate the oceanic state now and in the past (e.g., Ghil and Malanotte-Rizzoli, 1991; Wunsch, 1996). Experience in numerical weather prediction shows that even a nearly perfect model will drift away from the true atmospheric state over a few weeks unless continuously forced back to reality through the systematic use of observations. There is every reason to think that oceanic models will exhibit identical behavior albeit over somewhat longer time scales.

All such estimation schemes are in practice weighted averages of a model estimate of the oceanic state and an equivalent estimate made from the data alone (Wunsch, 1996, p. 342). Let $\mathbf{x}(\mathbf{t})$ be the oceanic "state vector" at discrete time t, defined in such a way that a numerical model can be written as a rule for computing the state vector at unit time step in the future:

$$\mathbf{x}(\mathbf{t}+\mathbf{1}) = \mathcal{L}(\mathbf{x}(\mathbf{t}), \mathbf{B}(\mathbf{t})\mathbf{u}(\mathbf{t}), \mathbf{t}). \tag{1}$$

Here $\mathbf{B}(t)\,\mathbf{u}(\mathbf{t})$ represents, in the notation of Wunsch (1996), known boundary and initial conditions, and $\mathcal{L}$ is a general operator. Suppose at time t, there are observations

$$\mathbf{y}(\mathbf{t}) = \mathbf{E}(\mathbf{t})\,\mathbf{x}(\mathbf{t}) + \mathbf{n}(\mathbf{t}) \tag{2}$$

where $\mathbf{E}(t)$ is the known "observation matrix", and the noise, $\mathbf{n}(\mathbf{t})$ has first and second order moments

$$< \mathbf{n}(\mathbf{t}) >= \mathbf{0},\ \mathbf{R}(\mathbf{t}) =< \mathbf{n}(\mathbf{t})\,\mathbf{n}(\mathbf{t})^{\mathbf{T}} > . \tag{3}$$

The assumption of a linear relationship between observations and state vector is often a good one, but is not strictly necessary.

Suppose that the model has been stepped forward in time from $t = 0$ to a particular time t, producing a state vector $\tilde{\mathbf{x}}_{\mathbf{m}}(\mathbf{t})$. If the model is indeed perfect and driven by perfect boundary conditions from perfect initial conditions, the observations will differ from the model forecast of what the observations should be,

$$\mathbf{E}(\mathbf{t})\,\tilde{\mathbf{x}}_{\mathbf{m}}(\mathbf{t}), \tag{4}$$

only because of the errors in the observations, $\mathbf{n}(\mathbf{t})$. Observations in this situation have nothing to add to what the model produces, and $\tilde{\mathbf{x}}_{\mathbf{m}}(\mathbf{t})$ cannot be improved upon.

More realistically, however, there are errors not only in the boundary and initial conditions, but in the model formulation itself (there are no perfect models of the ocean and surely never will be). Suppose that the error in $\mathbf{x}_{\mathbf{m}}(\mathbf{t})$ were known in the statistical sense through its first and second moments:

$$\begin{aligned} &< \tilde{\mathbf{x}}_{\mathbf{m}}(\mathbf{t}) - \mathbf{x}(\mathbf{t}) >= \mathbf{0}, \mathbf{P}_{\mathbf{m}}(\mathbf{t}) \\ &=< (\tilde{\mathbf{x}}_{\mathbf{m}}(\mathbf{t}) - \mathbf{x}(\mathbf{t}))\,(\tilde{\mathbf{x}}_{\mathbf{m}}(\mathbf{t}) - \mathbf{x}(\mathbf{t}))^T > . \end{aligned} \tag{5}$$

Suppose further that in using the observations we were able to make an estimate of $\mathbf{x}(t)$ from the data alone, written as

$$\tilde{\mathbf{x}}_{\mathbf{d}}(\mathbf{t}) = \mathbf{E}^{+}(\mathbf{t})\,\mathbf{y}(\mathbf{t}) \tag{6}$$

where $\mathbf{E}^{+}(t)$ is a generalized inverse of $\mathbf{E}(t)$. Assume the uncertainty of $\tilde{\mathbf{x}}_{\mathbf{d}}(\mathbf{t})$ is also known statistically as

$$\begin{aligned} &< \tilde{\mathbf{x}}_{\mathbf{d}}(\mathbf{t}) - \mathbf{x}(\mathbf{t}) >= \mathbf{0}, \mathbf{P}_{\mathbf{d}}(\mathbf{t}) \\ &=< (\tilde{\mathbf{x}}_{\mathbf{d}}(\mathbf{t}) - \mathbf{x}(\mathbf{t}))\,(\tilde{\mathbf{x}}_{\mathbf{d}}(\mathbf{t}) - \mathbf{x}(\mathbf{t}))^T > . \end{aligned} \tag{7}$$

Then it makes sense to combine the information available from the model with that obtained from the data. The required estimates are of the form,

$$\tilde{\mathbf{x}}(\mathbf{t}) = \mathbf{W}_{\mathbf{m}}(\mathbf{t})\,\tilde{\mathbf{x}}_{\mathbf{m}}(\mathbf{t}) + \mathbf{W}_{\mathbf{d}}(\mathbf{t})\,\tilde{\mathbf{x}}_{\mathbf{d}}(\mathbf{t}), \tag{8}$$

where $\mathbf{W}_{\mathbf{m}}$, $\mathbf{W}_{\mathbf{d}}$ are weight matrices whose choice is the heart of the estimation scheme and which will govern its skill. A typical and attractive route is to choose $\mathbf{W}_{\mathbf{m}}$, $\mathbf{W}_{\mathbf{d}}$ so as to render $\tilde{\mathbf{x}}(\mathbf{t})$ the minimum variance estimate (e.g., Wunsch, 1996, p. 208):

$$\begin{aligned} \tilde{\mathbf{x}}(\mathbf{t}) \ = \ & \mathbf{P}_{\mathbf{d}}(\mathbf{t})\,[\mathbf{P}_{\mathbf{d}}(\mathbf{t}) + \mathbf{P}_{\mathbf{m}}(\mathbf{t})]^{-1}\,\tilde{\mathbf{x}}_{\mathbf{m}}(\mathbf{t}) + \mathbf{P}_{\mathbf{m}}(\mathbf{t}) \\ \times \ & [\mathbf{P}_{\mathbf{d}}(\mathbf{t}) + \mathbf{P}_{\mathbf{m}}(\mathbf{t})]^{-1}\,\tilde{\mathbf{x}}_{\mathbf{d}}(\mathbf{t}), \end{aligned} \tag{9}$$

whose own uncertainty can be written

$$\mathbf{P}(\mathbf{t}) = \left\{\mathbf{P}_{\mathbf{d}}(\mathbf{t})^{-1} + \mathbf{P}_{\mathbf{m}}(\mathbf{t})^{-1}\right\}^{-1}, \tag{10}$$

or in various algebraically equivalent forms.

$\mathbf{P}_{\mathbf{d}}(\mathbf{t})$, $\mathbf{P}_{\mathbf{m}}(\mathbf{t})$ are in practice very complex, dense, usually non-banded matrices because the error structures of models and data tend to be complicated — with errors being a function not only of position, but also of spatial scale (wavenumber). If $\mathbf{W}_{\mathbf{m}}$, $\mathbf{W}_{\mathbf{d}}$ are chosen arbitrarily (perhaps according to some simple heuristic rule), one runs a grave danger of misweighting high accuracy information relative to low accuracy information in the models and/or in the data.

One often hears firm assertions that "models are too inaccurate" for oceanic data assimilation to be possible. But estimation schemes based upon Eq. (9) show such a statement represents a considerable misunderstanding: *any* model can be used as long as it is properly weighted. If the model has no skill at all, the weight given to it will be vanishingly small and little or nothing is gained by the estimation procedure over one using the data alone. But that is a totally different picture from a claim that the procedure cannot be carried out. If the model skill is small relative to the data, one can always still exploit that information

Understanding the spatial structure of the relative errors is critical to a useful result. Consider, for example, the use of climatological hydrographic data as a set of oceanic observations. Such compilations (e.g. Levitus, 1982) are heavily smoothed temporally and spatially. Under certain restrictive assumptions about the nature of the space/time variability of the ocean, the climatology can be agreed to carry information about the large spatial scales of the true oceanic state (perhaps 1000 km and larger), but contain little or no information on the shorter scales. A number of widely used ad hoc estimation schemes, e.g., "nudging," choose $\mathbf{W}_{\mathbf{m}}$, $\mathbf{W}_{\mathbf{d}}$ so as to