Workshop Assesses Monte Carlo Simulations in Oceanography

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Oceanographers enthusiastically integrate global ocean circulation models in conjunction with atmospheric models over periods of thousands of years in order to assess future climate states--without actually knowing the skill of their ocean models. Oceanic circulation models represent the turbulent ocean as a laminar flow. This representation, except in the most fortuitous of circumstances, involves some grave errors whose effects need to be understood and quantified. Usually, models are validated and/or calibrated by comparing their predictions with observations. This has worked well for weather prediction models, but we cannot wait out climate changes. An alternative way to estimate some of the errors in circulation models is to perform Monte Carlo, or ensemble integrations, with them. This and other applications of Monte Carlo simulations in oceanography were reviewed at the ninth 'Aha Huliko'a Hawaiian Winter Workshop, held January 14-17, 1997, at the University of Hawaii in Honolulu.

Monte Carlo methods approximate solutions to a variety of problems by performing statistical sampling experiments on a computer. They can be applied to problems with and without an inherent probabilistic structure. Monte Carlo methods become computationally more efficient than straightforward *n*-point evaluations as the dimensionality of the problem increases. In fields like physics and Bayesian statistics, Monte Carlo methods have made it possible to attack problems that would have previously been intractable.

The workshop brought together oceanographers, climatologists, physicists, statisticians, and probabilists to look at data assimilation, turbulent transport, population dynamics, and propagation through random media. All these problems are described by model equations whose parameter and/or initial and boundary conditions are assumed to be random. Monte Carlo becomes the method of choice when the nonlinearity and dimensionality of the problem renders traditional moment-based methods either conceptually intractable or computationally inefficient.

A Case for Random Modeling

For most systems it is quite natural to assume that some parameter in the dynamical equations or some of the initial and/or boundary conditions are uncertain or random. For oceanic general circulation models, randomness is generally assumed to come in only through the initial and /or boundary conditions. However, the Langevin approach of eliminating subgrid scale motions is to introduce both a dissipative term, which draws energy from the resolved scales, and a randomly fluctuating term, which feeds energy from the subgrid motions to the resolved motions. The case was made at the workshop that some such randomness be included in the dynamical equations. More specifically, the dynamical equations of an oceanic circulation model are obtained by averaging and approximating the Navier-Stokes equations. The resulting equations contain fluxes induced by the subgrid scale motions, which need to be parameterized. No valid parameterization scheme is known. It must be expected that these fluxes, especially internal wave-carried radiative fluxes, cannot be parameterized in terms of local mean flow quantities. In addition to this nonlocal aspect, the subgrid scales may respond to phenomena (such as storms) not directly related to the large scale flow. These internal wave fluxes can in principle be handled by including an additional internal wave radiation field obeying a transport equation, just as light from the sun is handled as an additional radiative flux. Until this is done, however, the errors or uncertainties in the parameterization of internal waves and other subgrid motions should be modeled by a random process, in concordance with the Bayesian credo that every uncertainty in a quantity should be treated as random. Thus the oceanic general circulation should be regarded as a stochastic problem described by a set of stochastic partial differential equations.

Data Assimilation

Model errors are an important ingredient in data assimilation. Ocean model data are very different from ocean observational data. Model results are complete in that they exist throughout space and time, yet they have large uncertainties. Observational data, in contrast, are relatively more accurate, but they are very sparse in space and time. In spite of this profound difference, existing assimilation schemes incorporate the observation data by using a weighted average of an estimate from a model and an estimate from data. The weighting factors contain the model and data covariance matrices. For oceanic general circulation models, one runs into two problems: the high dimensionality ($N=10^5$ and larger) and the nonlinearity of the system.

The high dimensional covariance matrices of a general circulation model are complex, dense, and non-banded, and we do not have the means to describe and interpret such matrices. Furthermore, in a linear sequential data assimilation scheme (Kalman filter) the calculation of the covariance matrix represents an exorbitantly heavy computational load. One approach is to model the error fields by a stochastic process that will give the desired covariance matrices. A particularly promising approach is to model the error on a multiscale tree so as to account for different physics on different scales. A second approach is to perform an ensemble integration of the circulation model, in order to calculate the statistical moments needed in the assimilation scheme. This method also has the advantage of being applicable to nonlinear systems where extensions of the Kalman filter face closure problems.

The vast majority of the data assimilation schemes in use or proposed for oceanographic application were derived and validated for linear systems with Gaussian noise. We need to gain intuition for stochastically perturbed nonlinear systems. The nonlinearity might actually lessen the dimensionality problem since the motion of the system might become confined, as shown in dynamical systems theory, to some lower-dimensional subset of the full state space.

Turbulent Transport

The simulation of particle paths in a random velocity field has long been a tool to understand the transport and mixing of a passive tracer in a turbulent flow. This random mixing is supposed to model the chaotic mixing that occurs in deterministic but chaotic flow fields. Random mixing studies usually assume the flow field to be incompressible, isotropic, Gaussian and time-independent, and described by a wave number spectrum, usually of the Kolmogorov type. The Gaussian assumption is the most crucial one. Velocity fields modeled as Gaussian processes in an Eulerian coordinate system lead to some unphysical aspects of tracer transports. That is because in reality, but not in the Gaussian models, the correlations of small scale motions are advected by larger scale flows. Nevertheless, the Gaussian models still offer considerable insight into transport processes. Much effort has gone into the computational techniques to generate random velocity fields for a given wavenumber spectrum. New wavelet-based representations have been developed that in many circumstances are superior to the traditional Fourier and moving average methods. Once time-dependence is introduced, new phenomena occur; in two-dimensional flows particles can escape closed streamlines, and new simulation algorithms have to be employed.

Gaussian velocity fields lead in their simplest implementation to Brownian motions with sample paths that are continuous but not smooth. Brownian motions are a limiting case of a general class of self-similar processes called Levy α-stable flights ($0<\alpha\leq 2$). Except for the Brownian motion limit $\underline{\alpha}=2$, their distributions have "fat" tail probabilities and infinite variance. Their sample paths become discontinuous, with jumps increasing in size as $\underline{\alpha}$ decreases. Levy flights lead to anomalous diffusion, which means that spatial sizes scale as a power of time, where the power is not equal to 1/2 as in true diffusion. Richardson, observing smoke emanating from smokestacks, deduced such an anomalous diffusion scaling law in the atmosphere. In two dimensions, Levy flights exhibit clusters where "trapping" occurs. Levy flights might thus be the appropriate tool to model particle transports in turbulent flows consisting of coherent vortices and jets, e.g., where particles are caught in a vortex for a period of time (trapping event) and then ejected into a jet where they travel at high velocity (flight event) only to be trapped in another vortex. "Smart" numerical algorithms are being developed to model these Levy flights.

Population Dynamics

The workshop considered, as a third model class, models of population dynamics: a food web model, a fish population model to aid in the design of a tuna tagging experiment, and a coupled climate-economic cost model to find optimal CO_2 emission paths that minimize the sum of climate damage and abatement costs related to anthropogenic climate change. These models include a whole new array of model processes such as birth and death processes, prey and predator behavior, fishing strategies and, in the case of the coupled climate economic cost model, political and economic actors that may or may not cooperate and may or may not trade. It is clear that our knowledge about these processes is even more uncertain and that these uncertainties must be included in the modeling, stochastically or otherwise, in order to obtain meaningful results. Here, as in the other modeling problems, it is imperative to give a great deal of thought to how the randomness or uncertainty ought to be modeled. Consider the food web model. One can add a "superprocess," which is a Brownian motion intermingled with random birth and death processes. Because of the randomness in the birth and death process, superprocesses exhibit clustering into disconnected communities and other interesting non-Gaussian behavior. Just adding a Gaussian white noise process leads to much less interesting behavior and might even cause population densities to become negative.