Flow Separation in the Ocean

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Abstract. Flow separation, which can lead to major modifications of flow patterns and drag, appears in many different guises in the rotating, stratified ocean. Some of these are reviewed, with an emphasis on the role of bottom slope. In particular, this can affect the surfacing of the interface for a coastal current flowing around a bend, and may also enter the criterion for boundary layer separation for low Rossby number flow past a submerged topographic feature.

Introduction

Boundary layer separation is a fluid mechanical process of fundamental importance in a variety of engineering situations. Aerodynamic lift on wings or propellors would be impossible without it, thus ruling out workshops in Hawaii! More typically than for these examples, which involve smooth separation at a sharp trailing edge, boundary layer separation in high Reynolds number flow past a bluff body generally leads to the advection of significant amounts of boundary layer vorticity into the fluid interior, causing a much more turbulent wake and more drag than if the boundary layer remained attached.

Given this potential for separated flow to lead to considerable turbulent stirring and mixing, and for the drag to be very different from that in attached flow, it would thus seem important to investigate the circumstances under which flow separation occurs at topographic features in the ocean. If the flow is homogeneous and at high enough Rossby number for the earth's rotation to be unimportant, then flow past isolated features on the sea floor may be similar to that in familiar engineering configurations, though sometimes complicated by the pre-existence of a bottom boundary layer in the upstream flow (Taylor, 1988). Situations in which the water depth goes to zero, as for shallow water flow past coastal features, are particularly different, in that turbulent bottom friction is more important than lateral friction, giving a criterion for flow separation that is different from the familiar engineering one. This will be reviewed, with particular reference to the instructive study of Signell and Geyer (1991).

The effect of rotation is readily included in the analysis of homogeneous nearshore flows. It also enters into consideration of stratified flow in the ocean and is associated with another type of flow separation: the surfacing, at the shore, of the interface in an inviscid reduced gravity flow around a bend. An analysis of this, involving the competition of Coriolis and centrifugal forces, was conducted by Klinger (1994) for the case of a vertical side wall. This, and the extension to allow for a more realistic bottom slope, will be reviewed, along with a qualitative discussion of viscous boundary layer separation, within the upper layer, even if the interface does not surface. It is also possible to speculate on the secondary cross-stream flows that would be expected in this situation as well as in the previous example of homogeneous flow past a headland.

The flow of a continuously stratified flow past an isolated topographic feature has been discussed in the meteorological literature for high Rossby number, as reviewed by Kaimal and Finnigan (1994). In the ocean the Rossby number may well be small, so that it is necessary to consider the evolution of the bottom Ekman layer as the flow passes the object. This topic, and some surprising possibilities, will be introduced later, but we start with a reminder of the fluid dynamics of boundary layer separation in the flow past a bluff body.

Flow Separation at High Reynolds Number

Kundu (1990) and other fluid dynamics texts describe the basic physics of flow separation in a typical engineering situation. The pressure gradient parallel to a surface is the same at the surface as just outside a boundary layer if this is thin, and, by Bernoulli's theorem, changes sign at the point where the external flow is a maximum (Figure 1). Beyond this point the opposing pressure gradient slows the external flow. It also slows the flow in the boundary layer, but this is already weak and so tends to be reversed, causing separation. This tendency can be offset, however, by the effects of viscosity, since right at the boundary (with x along it and y the normal coordinate)

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \nu \frac{\partial^2 u}{\partial y^2}, \tag{1}$$

where ν is the viscosity, thus only requiring an inflection point in $u(y)$, and not necessarily flow reversal, in a region with $\partial p/\partial x > 0$. In many practical situations separation does occur fairly soon after the point at which the pressure gradient reverses, but it can be delayed by making the change gradual, or by increasing the effective viscosity by triggering turbulence (e.g. Kundu, 1990).

Shallow Water Flow

Signell and Geyer (1991, henceforth SG) contrast this conventional situation with that of the flow past a head-

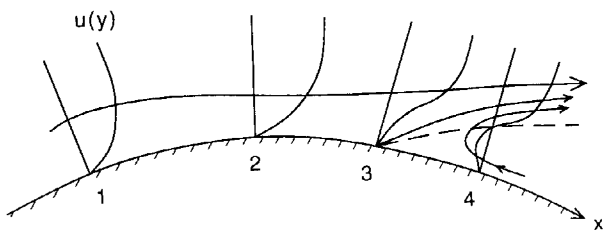

Figure 1. Velocity profiles and streamlines for separating flow past a bluff body. The flow outside the boundary layer increases from 1 to 2 where it is a maximum and the pressure is minimum. Beyond this $\partial^2 u/\partial y^2 > 0$ at the boundary, with reversed flow beyond the point of separation 3. The dashed line has $u = 0$. (Modified from Kundu, 1990).

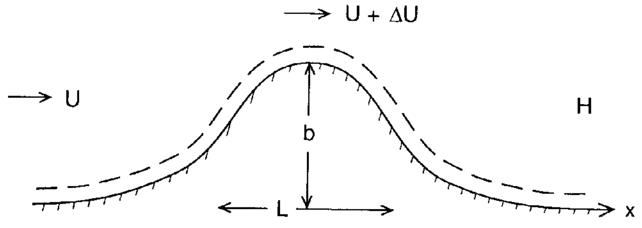

Figure 2. Flow past a coastal feature separated by a narrow sloping region from a sea of constant depth H.

land, in a situation governed by the shallow water equations

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + f\mathbf{k} \times \mathbf{u} = -g\nabla\eta - \frac{C_D \mathbf{u}|\mathbf{u}|}{(h+\eta)} + \nabla \cdot A_H \nabla \mathbf{u} \quad (2)$$

$$\frac{\partial \eta}{\partial t} + \nabla \cdot [(h+\eta)\mathbf{u}] = 0 \quad (3)$$

where $\mathbf{u} = (u, v)$ is the depth averaged horizontal velocity, f the Coriolis parameter, $\mathbf{k}$ a unit vertical vector, g gravity and η the surface elevation in water of depth h at rest. Bottom friction involves a drag coefficient C_D and lateral mixing of momentum is allowed for through a horizontal eddy viscosity A_H. This corresponds to the viscosity of the conventional problem, but the associated term, the last in (2), is small compared to bottom friction near the coast where the water is shallow and A_H must decrease due to limited eddy size.

Close to the coast, then, the alongshore momentum balance is

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial \eta}{\partial x} - \frac{C_D u|u|}{(h+\eta)} \quad (4)$$

where x is the curvilinear longshore coordinate, u the velocity component in that direction and the offshore velocity component is small enough for fv to be negligible. SG make the important point that, from simple scaling of (4), the terms on the left hand side are unimportant right at the shore, so that the balance is between the alongshore pressure gradient and bottom friction, requiring flow reversal as soon as the pressure gradient reverses.

On the other hand, as SG remark and again in contrast to the conventional situation, the influence of friction away from the very nearshore region means that a reversal of the pressure gradient does not occur at the maximum of the alongshore current on a particular streamline, and need not occur at all if the current slows down no more rapidly than would be caused by friction alone.

SG obtain an estimate of the alongshore pressure gradient for the situation in which the water depth is constant outside a narrow region (Figure 2). In this case the external flow is approximately irrotational (in the horizontal), due to the vanishing of the main vorticity generating term involving $C_D \mathbf{u} \times \nabla h$, and so may be estimated and then used in (4) to derive the alongshore pressure gradient which applies also right at the coast if the sloping region is sufficiently narrow.

To avoid a reversal of the pressure gradient for steady flow we require $|u\,\partial u/\partial x| < C_D u^2/H$, where H is the (constant) offshore water depth. SG evaluate u for a headland with the shape of a half ellipse, but in general, for a headland of width L and departure b from a straight coastline, we expect a change $\Delta U \simeq (b/L)U$ in the alongshore flow. Assuming b/L to be small, flow separation is thus avoided if $U\Delta U/L < C_D U^2/H$, or if the headland aspect ratio $b/L \lesssim C_D L/H$. Equivalently, though not expressed in these terms by SG, the radius of curvature, L^2/b, of the coastal feature, must be greater than about H/C_D. This is 10 km if $C_D \simeq 2 \times 10^{-3}$ and $H = 20$ m, with flow separation occurring even for steady flow if the radius of curvature is any less.

SG find that separation is even more likely if b/L is not small, or if the flow is time dependent (as is readily apparent from (4)), and use a two-dimensional numerical model to study the evolution of flows in which separation occurs, carrying significant amounts of vorticity into the flow interior and thus producing vigorous eddy activity.

From this, and earlier work referenced by SG, the physics of two-dimensional flow separation in the coastal ocean is reasonably well understood. A limitation of the SG model is perhaps the assumption of constant water depth outside a narrow strip with a slope. Relaxation of this assumption to permit the slope to continue offshore would invalidate the approximation of two-dimensional potential flow outside a boundary layer, but a numerical model could be effective.

The discussion so far describes only the depth averaged flow. As the frictional forces act at the bottom, rather than throughout the water column, three-dimensional secondary flows, across the depth averaged flow, can be expected. This will be discussed briefly later in the paper.

An Upwelling Density Front

The above discussion has shown the importance of coastline curvature for boundary layer separation in frictional homogeneous shallow water flow in the coastal ocean. Another form of separation that has attracted attention recently is that associated with a two-layer inviscid reduced gravity flow past a curved coastline (Figure 3). The current is held near the straight coast upstream by the Coriolis force, but tends to be pushed offshore by the centrifugal force as it rounds a cape. If the curvature is small enough the interface may surface at the coast, so that the buoyant coastal current detaches itself from the coast and flows into the fluid interior as a jet. This phenomenon was observed in laboratory simulations (Whitehead and Miller, 1979; Bormans and Garrett, 1989) of the initial formation of the Alboran Gyre, in the Western Mediterranean, by Atlantic water flowing through the Strait of Gibraltar. Bormans and Garrett (1989) suggested that separation might require the radius of curvature of the coast to be less than the

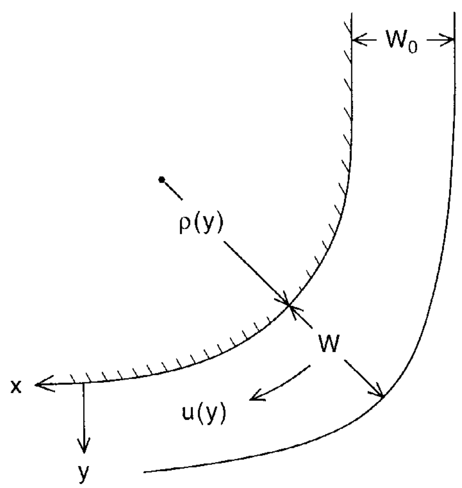

Figure 3. A reduced gravity coastal current flowing past a curved coastline.

inertial radius u/f, where u is the upstream current, and the problem has been addressed theoretically by Klinger (1994). He also considered different configurations in which the coastal current is confined near the coast but the upper layer extends infinitely far offshore. Here I will restrict the discussion to the first case, reviewing Klinger's (1994) analysis for a vertical sidewall and then discussing its extension to allow for a bottom slope.

Vertical Sidewall

Ignoring terms involving derivatives in the downstream, x, direction, the problem involves the cross-stream momentum equation

$$fu + g'\frac{dh}{dy} - \frac{u^2}{\rho + y} = 0 \qquad (5)$$

and the potential vorticity conservation equation

$$h^{-1}\left(f - \frac{du}{dy} - \frac{u}{\rho + y}\right) = \text{constant.} \qquad (6)$$

Here ρ is the local radius of curvature of the coast (Figure 3), $u(y)$ the current and $h(y)$ the interface depth. The sea surface displacement is very small compared with h and has been neglected in the potential vorticity equation. For simplicity, the potential vorticity has been chosen to be uniform across the stream.

The physics of (5) is that the Coriolis force leads to an increase of h as the coast is approached from offshore, but for small enough ρ this can be offset near the coast by the centrifugal term, causing h to decrease.

Klinger (1994) nondimensionalized the current speed with $(g'h_0)^{1/2}$, where h_0 is the upstream depth of the interface at the sidewall (Figure 4), y and ρ with the associated internal Rossby radius of deformation $(g'h_0)^{1/2}/f$, h with h_0 and the potential vorticity with f/h_0.

The governing equations are then

$$u + \frac{dh}{dy} - \frac{u^2}{\rho + y} = 0 \qquad (7)$$

$$1 - \frac{du}{dy} - \frac{u}{\rho + y} = \delta h \qquad (8)$$

where δ is the nondimensionalized, constant, potential vorticity. It and the nondimensionalized W_0 are the only two external dimensionless parameters. The problem may be solved for $u(y)$, $h(y)$, and the current width W, for any choice of δ, W_0 and local radius of curvature ρ. In particular, Klinger (1994) evaluated the critical radius $\rho_c(\delta, W_0)$ for which $h(0) = 0$, with the interface surfacing at the coast. He found that, to a good approximation, and almost independent of the value

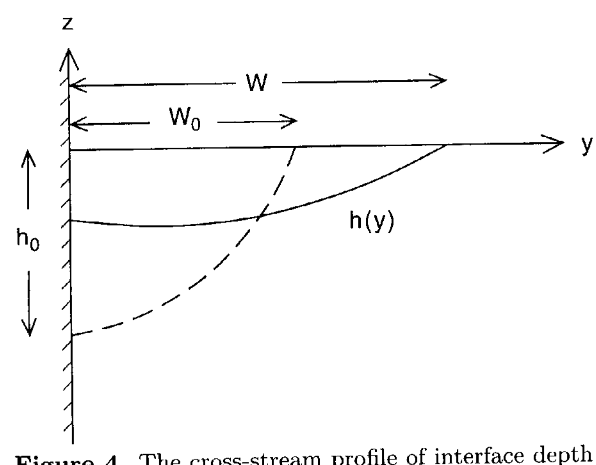

Figure 4. The cross-stream profile of interface depth $h(y)$ upstream (dashed), where the coastline is straight, and past a convex bend. The width of the current changes from W_0 to W.

of the potential vorticity δ, $\rho_c = 0.9W_0^{-1}$. In dimensional terms this becomes $\rho_c = 0.9g'h_0(f^2W_0)^{-1}$, or, using the geostrophic equation upstream, $\rho_c = 0.9\overline{u}/f$, where $\overline{u}$ is the cross-stream average current upstream. This supports the hypothesis of Bormans and Garrett (1989); a typical critical radius, for $\overline{u} = 1$ m s^{-1} and $f = 10^{-4}$ s^{-1}, is 9 km.

Sloping Sidewall

In reality, the sides of the ocean are sloping, not vertical! Jiang (1995) has analyzed the effects of this in the present context by extending Klinger's (1994) study to the configuration of Figure 5. The maximum current depth is still denoted h_0, but W_0 now refers to the upstream width of the current beyond the point where the interface intersects the bottom; the total current width is $s^{-1}h_0 + W_0$, where s is the bottom slope.

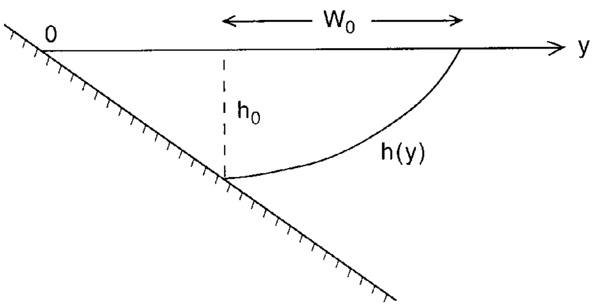

Figure 5. Upstream profile for a linear bottom slope.

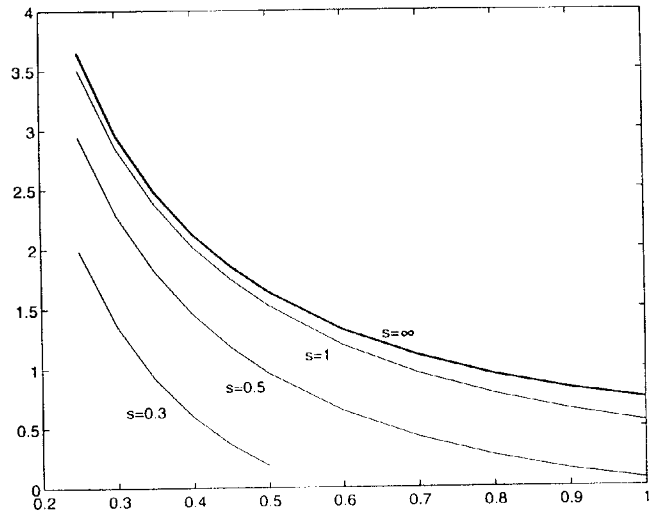

Figure 6. The critical coastal radius of curvature ρ_c, as a function of the upstream current width W_0 for a dimensionless potential vorticity $\delta = 1.1$ and for various values of the scaled slope s.

The governing equations are still (7) and (8) in the "free" region where an interface exists between the upper layer and the stagnant lower layer. In the "wedge" region, inside the point where the interface intersects the bottom, we may still take (7) as the momentum equation, with $h(y)$ now a proxy for the proportional sea surface elevation, or pressure, but the correct depth to use in the potential vorticity equation is now sy rather than h. Thus the right hand side of (8) becomes δsy rather than δh. Here the bottom slope s has been scaled by $fh_0(g'h_0)^{-1/2}$.

As before, the current width, and interface and velocity profiles as functions of y, may be calculated for a given coastal radius of curvature ρ. In particular, the critical radius of curvature ρ_c at which the interface surfaces may be determined, but it is now a function of the three independent dimensional variables δ, W_0 and s. Figure 6 shows ρ_c as a function of W_0 for various slopes s, all for $\delta = 1.1$, and demonstrates that a nondimensional slope less than about 1 is required for any significant difference from the results for a vertical wall. For a given W_0 (and δ), separation requires a smaller radius of curvature if s is small, although the interpretation is more complicated if ρ_c is plotted versus the total upstream current width $W_0 + s^{-1}$. The curves of ρ_c versus W_0 do not vary greatly with δ.

One interesting change for finite bottom slope is that the current can become narrower, rather than wider, as it rounds a bend of critical radius. This is illustrated in Figure 7, showing $u(y)$ and $h(y)$ upstream and at a bend of critical curvature. For a vertical sidewall ($s = \infty$) the current spreads out; it does so less for $s = 1$ and, for $s = 0.3$, it actually narrows. In order to narrow while still conserving the total volume flux, the current speeds up considerably near the coast. This is a consequence of the need for shear vorticity to offset curvature vorticity, much as in the flow of a river around a bend.

For the case of the Alboran Gyre, which was one motivation for these studies, the non-dimensionalized bottom slope seems to be about 4, sufficiently larger than 1 that Klinger's (1994) theory for a vertical wall is appropriate. Thus the suggestion by Bormans and Garrett (1989), that attached flow might be possible if the surface current speed were to drop below about 0.4 m s^{-1}, is not changed, though it remains imprecise due to the complicated nature of the real topography.

The value of the potential vorticity for the example of Figure 7 has been chosen partly to illustrate the flow for a situation in which the upstream current at the coast is very small, as might occur in response to bottom friction. Bottom friction would, of course, oppose the increase in coastal current so that, even for a minimum radius of curvature greater than ρ_c, streamline separation at the coast might be expected at a point downstream of its point of greatest curvature even though the interface did not surface.

Bottom friction would also lead to depth-dependent cross-stream flow within the wedge region, again as for river flow, except that here the near bottom flow would be expected to be offshore in the upstream region with a straight coastline. Offshore bottom flow would depend, in fact, not on having $u^2(\rho + y)^{-1} > fu$, where u is the

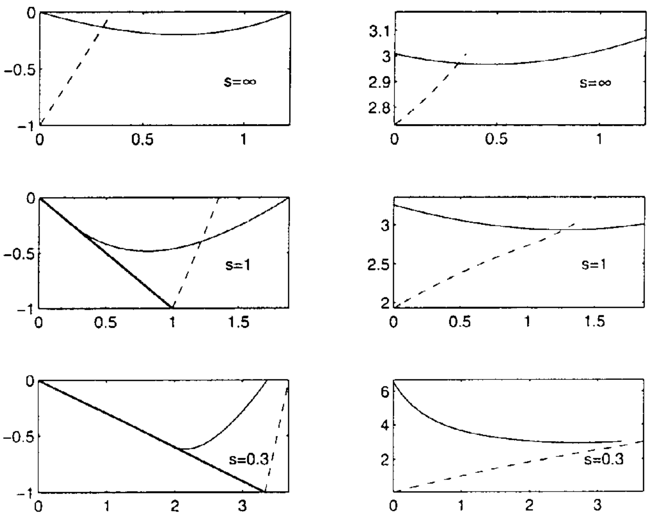

Figure 7. Profiles of interface depth $h(y)$ and current speed $u(y)$ both upstream ($\rho = \infty$) and at the separation point ($\rho = \rho_c$) for $\delta = 0.4$, $W_0 = 0.35$ and three values of bottom slope ($s = \infty, 1, 0.3$).

depth averaged current, but on the u derivative $2u(\rho + y)^{-1} > f$.

All of the discussions in this section have depended on slow downstream variation of the flow, so that x derivatives, and terms involving the velocity component normal to the coast, can be ignored. As pointed out by Klinger (1994), very slow variation of ρ is not possible if the total change in direction is not to exceed, say, 90°. He finds that the ratio of neglected to retained terms is of the order of W/ρ, and so not small for all of the (δ, W_0) parameter space. Further investigation of the problem therefore requires solution of a fuller set of equations.

Stratified Quasigeostrophic Flow Past an Obstacle

Flows past submerged topographic features in the ocean are influenced by bottom slope, stratification and the earth's rotation as well as by bottom friction. Even for homogeneous flow at high Rossby number (so that rotation is unimportant) the simple ideas of boundary layer separation that apply for high Reynolds number flow past objects isolated in three dimensions do not always apply directly because the upstream flow already has the characteristics of a boundary layer (Taylor, 1988). Flow separation then seems not to occur at small bottom slopes, perhaps because, as for the coastal problem of SG, a negative pressure gradient is required just to overcome viscous effects even away from the surface of the feature. Taylor (1988) suggests that separation will occur, however, if the maximum slope is greater than 0.3, at least for two dimensional features. The steepness required for separation appears to be larger for three-dimensional hills, but less so if surface roughness is added (Kaimal and Finnigan, 1994).

If stratification is taken into account, though still ignoring rotation, a wide range of possiblities exists, partly characterized by Froude numbers based on the width and height of the feature (e.g. Kaimal and Finnigan, 1994, and references therein). In particular, the generation of internal gravity lee waves can lead to downslope acceleration in the lee of the obstacle, associated with a negative pressure gradient and thus ruling out separation. In this case, the slowing of the inviscid flow, and hence the possibility of boundary layer separation, occurs upstream, not downstream!

A similar downstream acceleration was found by Richards et al. (1992) to be associated with the non-separation, for sufficiently small Rossby number of rotating, homogeneous, flow past an obstacle. They remark that a predicted positive pressure gradient upstream did not lead to flow separation in their laboratory experiments; perhaps this effect is a function of the Reynolds number.

A general oceanographic situation, involving both stratification and rotation, is thus likely to be very complicated, but it seems clear that a first step is to consider the inviscid flow and then the nature of the bottom boundary layer and the possibility of its direction of flow reversing.

In some situations it may be relevant to invoke the "arrested Ekman layer" concept of MacCready and Rhines (1991, 1993). If the alongslope flow is "upwelling favorable" a bottom Ekman layer will be pushed up the slope, but with a decreasing flux as the buoyancy force becomes more important and eventually balances the pressure gradient which, above the Ekman layer, is in geostrophic balance with the Coriolis force on the alongslope flow. As shown by MacCready and Rhines (1993), this complete arrest of the bottom Ekman layer by buoyancy forces is equivalent to the development of sufficient isopycnal slope for the associated thermal wind to bring the alongslope velocity to zero at the boundary.

If a flow accelerating past a topographic feature were moving slowly enough for the bottom Ekman layer to become arrested, and remain arrested as the flow increased, then the associated thermal wind would reverse the flow near the bottom if the inviscid flow outside the boundary layer decreased downstream. On the other hand, if the water were moving quickly enough that the bottom Ekman layer were only partly arrested, the associated thermal wind might be insufficient to reverse the bottom flow even in a region of deceleration of the inviscid flow.

It thus seems possible that a criterion for the flow not to separate is that the Ekman layer arrest time should be greater than the time for a fluid parcel to flow past the feature. For an upwelling bottom boundary layer, the arrest time is of order $(C_D N/f)^{-1/2} S^{-1} f^{-1}$, where