

STATISTICAL METHODS IN PHYSICAL OCEANOGRAPHY: MEETING REPORT

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Physical oceanographers deal with randomness and uncertainties when analyzing ocean data or formulating ocean models. Concepts and results from probability theory, statistical inference, and stochastic processes are applied: space-time averages are interpreted as ensemble averages, variances and spectra estimated, and stochastic terms added to dynamical equations. Special aspects arise when the huge amount of real or model ocean data and the complexities of ocean physics are considered. Efficient data representation and analysis algorithms are sought, and idealized dynamics that incorporate statistical and chaotic tendencies are explored. Progress on such statistical methods was discussed at the seventh 'Aha Huliko'a Hawaiian Winter Workshop, held January 12-15, 1993 in Honolulu. Specifically, the participants considered the variety of oceanographic observations, methods for efficient flow and data representation, frequentist versus Bayesian inference, data assimilation, and idealized dynamics. The size and complexity of oceanographic problems often prevent the application of standard methods and physical oceanographers are faced with the task of inventing methods that deal with the peculiarities of their problems in a sensible manner. These special methods are discussed in more detail in this article. Names in parentheses refer to the authors of lectures given at the meeting and chapters published in the proceedings.

OCEANIC OBSERVATIONS

Oceanographic phenomena cover many space and time scales and the questions asked, probability assumptions made, and inferences drawn differ widely. A few examples were considered.

The smallest scales of variability are important for mixing. One of the oldest unsolved problems is how the strength of ocean mixing varies with location and over time. Traditional measurement methods have depended upon making estimates of dissipation rates from direct observations at scales of centimeters. Fluxes are inferred from

dissipation, after further uncertain assumptions. As instruments for direct observation of the fluxes become available, the relation of flux to dissipation should become clearer. However, the difficulty remains that direct observation at centimeter scales is a time-consuming (hence expensive) operation, limiting its applicability for larger scale ocean surveying. A possibility is that larger scale observation of vertical velocity by a modified acoustic Doppler current profiler may provide a basis for estimation of dissipation and mixing while enabling rapid surveying (A. Gargett). The value of this proposed method will depend upon extensive intercomparison with more traditional measurements, using statistics to calibrate, and to estimate the confidence of, inferences from acoustic Doppler surveys.

Over vertical scales from meters to tens of meters, it is natural to describe the variability in the ocean interior in terms of vertical displacements of isopycnals. The frequency of chosen isopycnals in a given vertical bin size may be fitted against a Poisson distribution while the separation between isopycnals may approximate a gamma distribution, constrained to have unit mean (R. Pinkel). The mean, variance, and skewness at many scales is thus described by a single (dimensional) parameter, whose physical significance remains elusive.

On larger scales, underway acoustic Doppler profiling together with accurate global positioning has made possible the surveying of upper ocean currents along ship tracks (E. Firing). Length scales of velocity variations, both in the horizontal and vertical, are observed to change with latitude and depth, including strong signatures when crossing the equator. No comprehensive statistical descriptions of the variability and its changes have been established.

Satellite altimeter data have allowed the mapping of sea level variance (D. Chelton). A close relation between the variance of transient eddies and the intensity of mean flows and the bathymetry is found. Wavenumber spectra show a break of slope near the first internal Rossby radius, then are descending steeply towards higher wavenumbers. At the crossovers of ascending and descending satellite ground tracks, the two components of surface geostrophic velocity can be determined. This has been used to investigate the anisotropy of velocity variance and lateral transfers of momentum by Reynolds stresses on a dense global grid. Further inferences are complicated by the unique space-time sampling characteristics of satellite observations. The choice of orbit parameters for satellite missions can be discussed in terms of these sampling characteristics and their implied filters, transfer functions, and biases.

Interest in oceanography is often focussed on the larger scales, and considerable ingenuity goes into averaging local observations to remove the "noise" and obtain the "signal." Noise can be suppressed by making observations that are inherently of an integrating type, such as acoustic thermometry, reciprocal tomography, electric field and bottom pressure, inverted echo soundings, cable voltages, polar motion, and length of day. Such integrating

measurements often provide cleaner signatures of underlying dynamical processes. For example, bottom pressure and electric voltage are excellent proxies for the barotropic flow component and are found to be well correlated with the atmospheric windstress curl, suggesting that the subinertial barotropic variability in the ocean is atmospherically forced (D. Luther).

FLOW REPRESENTATION

The proper choice of basis functions on which to decompose a flow field becomes crucial when one seeks to reduce the system space. Reduced bases should optimally resolve the underlying dynamics, representing the more significant flow patterns. Wavelet transforms might be such an optimal choice. Traditional analyses of turbulence have resorted either to Fourier spectra or to grid point (Dirac) treatment, suggesting superposition of waves or interaction of isolated vortices. These are extreme views, emphasizing either perfect wavenumber resolution with no spatial resolution or the converse. Wavelets make a compromise, offering limited wavenumber resolution with limited spatial resolution, suggesting 'coherent structures.' Applied to analyses from numerical two-dimensional turbulence (M. Farge), wavelets were shown to be efficient at retaining the full range of spectral information while permitting spatially local examination of regions dominated by rotation compared with regions dominated by straining.

Another generalization of the traditional Fourier transform is based on the solutions of exactly integrable nonlinear wave equations. The method, known as inverse scattering transform, decomposes a time series into a superposition of nonlinear oscillation modes, which include ordinary sine waves, cnoidal waves, solitary waves, and other special wave forms (A. Osborne). Applications of the method, based on the solutions of the Korteweg-de-Vries equation, have been efficient and insightful in the analysis of field and lab data.

Bases for representing actual data are often chosen in an empirical way by methods known as 'empirical orthogonal functions' (EOFs), 'principal component analyses' (PCAs), or 'factor analyses.' Eigenvalues from the data covariance matrix permit ranking the empirical eigenvectors according to what fraction of total data variance is represented by each eigenvector. This allows retention of a relatively few eigenvectors in lieu of the much larger dataset. Where data are seen to be "clumped" EOFs are rotated to more nearly recognize the "clumps." A major open question is the identification of the significant EOFs. Selection rules that claim to perform a significance test may lack a thorough statistical basis and the often applied rules-of-thumb are just what they claim to be—rules-of-thumb. A promising approach is the testing against artificial data (G. Mitchum). Data generated by a red noise process are particularly relevant to oceanographic fields such as sea level.

Time series from coefficients of EOFs may be subject to further analyses. By best fitting such time series to a first order vector Markov process, one identifies linear combinations

of EOFs that exhibit oscillation or propagation, termed 'principal oscillation patterns' (POPs) (H. von Storch). A POP analysis can be performed both on actual data and on the output from large numerical models. It can be a tool in identifying linear subsystems when these linear subsystems control a significant portion of the variability.

FREQUENTIST VERSUS BAYESIAN INFERENCE

Statistical inferences depend on the probability assumptions made. One interpretation of probability is from a 'frequentist' viewpoint. If an experiment can be regarded as being repeated many times, previous outcomes may be collected to estimate the probability for subsequent outcomes. In contrast, a Bayesian approach may be taken in a case where only one experiment is possible. Then one expresses one's prior beliefs concerning uncertain model parameters as a probability distribution, observes data, then computes a posterior distribution about those parameters. The frequentist's method assumes that the process and all its parameter values remain constant over the (often imagined) series of experiments. The Bayesian approach has to assume a prior distribution. Both methodologies have their advantages, and the appropriate statistical approach depends on the type of problem considered and the type of inference desired (G. Casella).

The Bayesian approach emphasizes a clear statement of prior beliefs. This may be valuable in bringing out 'hidden' assumptions (J. Kadane). How one formulates a null hypothesis can be crucial. If one formulates the null hypothesis as a 'sharp' statement, then collecting sufficient data will lead to rejection, unless the hypothesis is exactly true. Another danger is that formulating a null hypothesis after observing some data, for example in the case of Earth's climate record, may confuse the testing of such a hypothesis (H. von Storch). Bayesian methods have successfully been applied to the quality control of data used for assimilation in numerical weather prediction models where the prior knowledge about error distribution and background fields need to be properly weighted (A. Lorenc).

DATA ASSIMILATION

Combining data with models serves various purposes. The model may help to complete a data set, 'dynamically interpolating' to fill data gaps. An example is the reconstruction of Gulf Stream paths (M. Chin).

One of the ways that oceanography may differ from weather forecasting is that there is less emphasis upon ocean 'forecasting.' To a considerable extent, the role of data assimilation in the ocean may be more as a means to obtain information about uncertain parameters in the ocean physics. Among the more rigorous methods, oceanographers employ adjoint methods, integrating backward in time to obtain the gradient of the cost function. An example is the estimation of vertical eddy viscosity and surface drag coefficient from data in a wind forced Ekman layer (J. O'Brien). For more complex systems, simpler algorithms are being developed that seek to reduce a chosen cost

function toward a smaller value which nonetheless might not be a minimum. This can be seen in a time-dependent, three-dimensional circulation model of the North Atlantic, including an embedded mixed layer with uncertain parameters (J. Schröter). Repeated forward runs of the model are made using different choices of parameters, with outcome evaluated by its cost function.

Model errors, such as caused by imprecisely known physics or forcing fields, often represent a major uncertainty in the problem and need to be accounted for in the cost function. This is the case for the heat flux uncertainties in a model of the tropical sea surface temperature (C. Frankignoul). Since the heat flux uncertainties have poorly known correlation scales, an adaptive method is applied where the model being tuned is also used to determine the uncertainties in the heat flux field. Parameters are found that reduce the warm sea surface temperature bias and that might eliminate the need for a flux-correction term in climate studies.

There are various methods to seek extrema of a function. The adjoint equation technique yields estimates of the gradient of the cost function about a state of model variables, permitting use of some descent algorithm to seek a minimum cost. However, when the cost function has a complicated dependence on model variables, there is danger that descent algorithms may not converge or may locate only a local, rather than a global, minimum. Two alternatives were described (N. Frazer). Simulated annealing (by analogy to cooling from a melt), employs random perturbations of parameters while seeking to reduce a cost function (characterized as a Gibb's free energy, U). When a particular random perturbation reduces U , that perturbation is accepted and a subsequent perturbation is applied. When a perturbation increases U , the perturbation may be accepted with probability given by a Gibb's distribution $\exp(-U/T)$, where parameter T has the role of temperature. The 'art' is to reduce T according to a 'cooling schedule' which is efficient yet avoids 'freezing' into a local minimum of U . The second alternative, termed genetic algorithms, bears similarity to simulated annealing. A population of possible choices of sets of parameters is generated. Members are paired, and randomly chosen portions of the parameter set are exchanged, including some 'mutations.' Members are evaluated by a 'fitness' function (which might be the previous Gibb's distribution) to determine probable representation in a next generation.

CHAOS

Concepts from chaotic dynamics have been applied to account for fluid behavior that appears to be random. Laboratory experiments show aspects of how mixing comes about. A theoretical precept about turbulent mixing is that line elements advected with a fluid should tend to grow exponentially in length. It is argued that this is impossible in a two-dimensional steady flow. Beginning from this simple case, what further dynamics are needed before exponential line stretching occurs? Experiments (J. Ottino) show that a simple (periodic) time dependence is already sufficient to introduce chaotic regions,

yielding exponential line growth. Low order fixed points of the time-periodic mapping dominate the regions of chaotic mixing, where it is seen that hyperbolic fixed points are surrounded by elliptic islands. Unmixed regions stretch and contract, but form coherent islands in the midst of chaos. In three dimensional systems these regions might appear as tubes. These results have implications regarding the character of velocity fields inferred via flow visualization.

The study of chaotic dynamics may contribute to an understanding of several problems in ocean dynamics. If a system is chaotic, perturbation expansions are misleading and predictability is limited. It has been suggested that the El Niño/Southern Oscillation system can be modeled as a low order chaotic system. A phase-space reconstruction analysis (M. Brown) performed using a measured time series of eastern tropical Pacific sea surface temperature provides some support for this hypothesis. "Spaghetti plots" of float trajectories often suggest chaotic behavior. However, floats that have been seeded in an isolated eddy may undergo many rotations of the eddy as it translates without dispersing. It is suggested that such coherent eddies are essential to the observation of anomalous diffusion (rates of particle separation greater than would result from random walks). On the theoretical side, WKB ray paths of surface waves propagating through an ocean of periodically varying depth exhibit chaotic behavior when the amplitude of the topographic variation exceeds a critical value.

A major problem in the application of ideas relating to chaos is the determination of whether an ocean phenomenon is chaotic. Estimates of measures of chaos (such as dimension or the Lyapunov exponent) from oceanographic data require an enormous number of degrees of freedom for any reasonable degree of confidence and are often counter-intuitive. For example, filtering for the purpose of suppressing noise can sometimes be seen to increase the Lyapunov exponent (E. Carter). A related issue is the characterization of the roughness of seafloor topography. Spectra and fractal dimensions have been used. A novel approach considers the 'geometric temperature' of any curve, calculated from the number of intersections of the test curve with randomly selected straight lines (W. Woyczynski). From geometric temperature, one may proceed to an analogous thermodynamics of curves and surfaces.

STATISTICAL DYNAMICS

The evolving statistics of flows have long been an object of turbulence theory. Renormalization group (RG) techniques have been applied to beta-plane turbulence, seeking an efficient non-eddy-resolving parameterization of small-scale processes to enable more cost-effective large scale modeling. Both viscosity and beta (Coriolis gradient) have to be renormalized or rescaled. Such renormalization re-interprets and quantifies previously obtained results and yields new insights. In particular, the RG-derived spectral energy transfer shows that beta-plane turbulence naturally tends to self-organize into zonal-jetlike flows and westward propagating Rossby waves. Two-parametric eddy viscosity and beta

coefficients account for the effect of unresolved turbulence and waves on resolved scales and are suggested for use in non-eddy-resolving simulations of beta-plane turbulence (B. Galperin). A further statistical study of beta-plane turbulence, utilizing Lagrangian-based second order closure, shows that the westward phase speed of Rossby waves is significantly enhanced by random nonlinear interaction (Y. Kaneda).

Outcome of many random interactions within a flow can sometimes be expressed in terms of a large scale statistical dynamics. Numerical experiments with inviscid geostrophic dynamics show random initial conditions forming basin-scale mean flows. When viscosity is included, regions of homogenized vorticity form in the basin interior. Inclusion of topography modifies the resulting mean flows. Sufficiently steep topographic slopes will cause jet-like flows along isobaths, corresponding to the zonal jets in Rossby wave turbulence (G. Vallis). A tendency for random interactions to yield large scale mean flows suggests a method to parameterize eddies by directly introducing the statistical dynamical tendencies at large scale. Experiments with a coarsely resolved global ocean model show that various observed large scale flows, thought to be due to eddy interactions, can be obtained by such parameterization (G. Holloway).

CONCLUSIONS

Oceanographic phenomena differ widely in the characteristics of data, the governing physics, the underlying probability space, and the inferences sought. The statistical methods reflect this diversity. They range from signal detection, via parameter estimation and data assimilation, to stochastic or chaotic models. A common challenge in most applications is the huge amount of data and the complexities of the physics. The system space or the number of degrees of freedom must be reduced to a manageable size. EOF and POP analyses are used to reduce large and complex data sets; wavelet and inverse scattering transforms are applied to represent efficiently the underlying physics; idealized dynamics are employed to understand chaotic and statistical tendencies. In applying these and other statistical methods, physical oceanographers must modify existing methods and must invent new ones to deal with the peculiarities of oceanographic problems in a sensible way. The workshop witnessed the considerable progress that is being made at this task.

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