OBSERVING "INTEGRATING" VARIABLES IN THE OCEAN

Douglas S. Luther
Department of Oceanography, School of Ocean and Earth Science and Technology
University of Hawaii at Manoa, Honolulu, Hawaii 96822

Alan D. Chave
Department of Geology and Geophysics, Woods Hole Oceanographic Institution
Woods Hole, Massachusetts 02543

ABSTRACT

Some physical variables are natural spatial integrals of oceanic water motion or state properties. Observation of these variables permits isolation of physical processes that might otherwise be difficult to examine because of the superposition of many phenomena at one place. Independent of a particular physical model, observations of such integrating quantities frequently enable direct determination of relatedness between variables at different locations, and direct determination of causality, while more traditional point observations may fail to find such relationships. Furthermore, integral quantities such as volume and heat transport, which are now being studied with great fervor because of their climatic importance, are likely more accurately estimated using observations of "integrating" variables than using a set of point measurements. Examples of integrating types of variables, such as horizontal electric fields, vertical acoustic travel time and bottom pressure, are used to demonstrate the ideas above with examples drawn from the study of (a) atmospherically forced, mesoscale motions, and (b) the volume and heat transports of the Gulf Stream.

INTRODUCTION

At any particular location in the oceans, the sub-inertial water motions and fluctuations of state properties are likely to be due to a superposition (and, possibly, interaction) of a variety of phenomena that each have specific and different balances between acceleration, advection, Coriolis forces, pressure, dissipation, external forcing, and so on. Time-dependent boundary layers exist as a result of property fluxes to and from the atmosphere and earth. Semi-permanent meso- and gyre-scale currents (O(100 km) and O(1000 km), respectively) of the "general circulation" are forced by the winds and property fluxes, and, through instabilities, produce meso- and gyre-scale variability in the form of meandering currents, coherent vortices, radiated waves, and so on. Meso- and gyre-scale variability can also be directly driven by the atmosphere. Each of these, and many other unlisted phenomena, exist at a variety of space scales for each time scale, so that they overlap each other not only in physical space and time, but in frequency and wavenumber space as well.

To decipher the ocean's physics, it is often preferable to examine a single phenomenon at a time. Then one has to consider "contamination" from the other phenomena that would inhibit, for instance, direct detection of relatedness among oceanic variables and between oceanic and atmospheric variables.

There are of course a number of strategies for isolating particular phenomena in order to study their kinematics and dynamics. Sometimes, time series of variables are all that is needed to separate phenomena according to their characteristic frequencies. Other times, spatial information is needed, which raises the cost and difficulty of a field experiment, but which allows discrimination of wavenumbers or principal components and thereby possible discrimination of different processes. Frequently, experiments are designed so that there is a reasonable certainty that the phenomenon to be studied dominates all other processes. However, there are many instances when this cannot be accomplished. In these cases, observations are usually compared with model output visually, graphically, statistically, or through dynamical parameter estimation. Such comparisons can lead to the identification of the quality of the dynamical hypotheses as a function of frequency and/or wavenumber. It is not unusual for experiments to be designed to take advantage of most if not all of the strategies above.

The purpose of this note is to point out that there now exists an additional observational strategy, most components of which are rather new to oceanography, for isolating phenomena that are large scale in the vertical and/or horizontal. This strategy is based on the measurement of integrating variables. The spatial filtering inherent in these variables frequently enables statistical confirmation of important large scale kinematic and dynamic relationships which might otherwise go undetected except with a formidably large array of point measurements. Yet, in deference to the theme of this workshop, it must be acknowledged that isolating large scale phenomena does not imply that the phenomena observed, or statistics of these phenomena estimated from integrating variables, are homogeneous over large scales as well. This inhomogeneity complicates, if not invalidates, the application of many statistical procedures that assume homogeneity.

We define integrating variables as those that are natural spatial integrals of oceanic water motion or state properties. Table 1 lists a few of the more important integrating variables being observed today. These integrating variables are ones that by their very nature tend to filter out the shorter spatial scale variability. The techniques we'll discuss in this note are those whose usefulness is well-established and which offer the advantage of cost-effectiveness. In addition, these techniques may have greater accuracy in comparison to using a suite of point measurements when the ultimate goal of an investigation is the measurement of an integral quantity such as volume transport.

Table 1. Examples of Integrating Variables.

Variable	Principal component of integrand	References
Horizontal electric fields at a point	Conductivity-weighted horizontal water velocity, from seafloor to sea surface	Sanford (1971) Chave & Luther (1990) Luther et al. (1991)
Voltages across fixed horizontal distances (typically, using abandoned undersea telephone cables)	Conductivity-weighted horizontal water velocity (one component only), from seafloor to sea surface over a fixed horizontal distance	Larsen & Sanford (1985) Larsen (1992) Chave et al. (1992b)
Vertical acoustic travel time	Inverse sound speed from seafloor to sea surface	Watts & Rossby (1977) Pickart & Watts (1990)
Bottom pressure	Horizontal water velocity near the seafloor, over a fixed horizontal distance	Brown et al. (1975) Whitworth & Peterson (1985)
Horizontal acoustic travel time (acoustic thermometry)	Inverse sound speed along horizontal, depth-varying ray paths	Munk & Forbes (1989)
Reciprocal acoustic travel time	Water velocity along horizontal, depth-varying ray paths	Worcester (1977) Worcester et al. (1991)
Orientation of the earth's axis of rotation	Global mass distribution (especially in hydrologic reservoirs)	Chao (1988) Eubanks (1993)
Rotation rate of the earth	Global atmospheric angular momentum (principally, fluctuations in zonal winds)	Hide & Dickey (1991) Eubanks (1993)

Measurement of integrating variables allows the investigator to focus immediately on a specific region of wavenumber space, without the "contamination" of shorter scale variability that may depend on processes other than the one being sought. Furthermore, such restriction of the wavenumber space may enable the detection of properties (like spatial coherence or air-sea coherence) that tend to zero as the wavenumber bandwidth increases and may provide more useful constraints for numerical model simulations than do point measurements.

In the sections that follow, we will describe applications of three of the more underutilized, yet most cost-effective, integrating variables listed in Table 1, including point measurements of horizontal electric fields (HEFs), vertical acoustic travel time (VATT), and bottom pressure (P_b). We will show how observations of HEFs and P_b in the Barotropic, Electromagnetic and Pressure Experiment (BEMPEX) provided definitive evidence of the existence of gyre-scale motions that are directly forced by sea surface winds. Horizontal electric field data from the Synoptic Ocean Prediction (SYNOP) experiment will be shown that suggest the greater accuracy of these integrating variables in estimates of volume transport. And, we will outline the potential utility of combining HEF and VATT observations to obtain nearly direct estimates from the seafloor of heat transport and the gravest vertical structures of horizontal currents and temperature fluctuations.

HORIZONTAL ELECTRIC FIELDS

Motional electromagnetic induction is now theoretically well understood in certain idealized settings (e.g., Sanford, 1971; Chave and Luther, 1990). Assuming distant continental boundaries and a flat seafloor with laterally homogeneous conductivity, then for the low-frequency limit where the aspect ratio of ocean currents is small, where the effect of self induction is weak, and where the vertical velocity can be neglected in comparison with the horizontal velocity, it can be shown that the point HEFs are related to horizontal water velocity by

$$\vec{E}_h(x,y,t) = CF_z k \times < \vec{v}_h(x,y,t) >^* + \frac{\vec{j}^*}{\sigma} + \vec{N}(x,y,t), \tag{1}$$

where

$$< \vec{v}_h(x,y,t) >^* = \frac{\int_{-H}^{0} dz\, \sigma(x,y,z,t)\, \vec{v}_h(x,y,z,t)}{\int_{-H}^{0} dz\, \sigma(x,y,z,t)} \tag{2}$$

and is called the conductivity-weighted, vertically averaged (CWVA) water velocity; $\vec{v}_h(x,y,z,t)$ is horizontal water velocity; $\sigma(x,y,z,t)$ is seawater electrical conductivity; F_z

is the vertical component of the geomagnetic field; and $H(x,y)$ is ocean depth. The scale factor C depends on $\sigma(z \le -H)$; C can be estimated by intercomparisons, but extensive geophysical evidence suggests that $C = 0.95 \pm 0.05$ almost everywhere in the deep oceans (e.g., Chave and Luther, 1990). A noise term $\vec{N}(x,y,t)$ is composed of externally produced (i.e., in the ionosphere and magnetosphere) electromagnetic fields that dominate for periods shorter than a few days but are negligible at longer periods (e.g., Chave et al., 1989).

Locally and non-locally produced electric currents are represented by $\vec{J}^*$. Given usual oceanic scales of motion at sub-inertial periods (greater than half a pendulum day), locally produced electric currents are theoretically negligible if the bottom is flat (Chave and Luther, 1990) or the flow is aligned along isobaths (Stephenson and Bryan, 1992). Local generation of $\vec{J}^*$ may be sufficient to inhibit accurate estimation of ocean water currents with electric fields only where the currents cut across isobaths and then only if the underlying sediments are relatively non-conductive (Larsen, 1992; Stephenson and Bryan, 1992). Meandering of a narrow current like the Gulf Stream can theoretically produce non-zero $\vec{J}^*$ outside of the stream boundaries (the principal example of non-local generation of $\vec{J}^*$), which theoretically could be a large noise relative to the electric field signal induced by the smaller water currents there. However, Sanford (1986) has pointed out that observations have shown $\vec{J}^*/\sigma$ to be small [yielding errors of O(1 cm/s)] and generally negligible. And our own work with the SYNOP data has shown that the best agreement between the moored current meter data and the horizontal electrometer data occurs where the currents are moderate to weak, resulting in no detectable $\vec{J}^*$. Therefore, in the following, $\vec{J}^*$ is ignored.

Dropping the horizontal dependences and letting $\sigma(z,t)$ equal a vertical average part plus a residual, i.e.,

$$\sigma(z,t) = <\sigma(t)> + \hat{\sigma}(z,t), \text{ where } <\sigma(t)> = \frac{1}{H}\int_{-H}^{0} dz\, \sigma(z,t), \tag{3}$$

then Eq. 2 becomes

$$<\vec{v}_h(t)>^* = <\vec{v}_h(t)> + \frac{\int_{-H}^{0} dz\, \hat{\sigma}(x,t)\, \vec{v}_h(x,t)}{H <\sigma(t)>}. \tag{4}$$

The first term on the right hand side (RHS) of Eq. (4) is just the vertical average of horizontal water velocity (or depth-normalized transport per unit width). The second term on the RHS of Eq. (4) is a small contribution because seawater conductivity is a weak function of depth. Note that $|\hat{\sigma}(z,t)| \ll <\sigma(t)>$; $<\sigma(t)>$ is always greater than 3 Siemens m^{-1}; to a good approximation, $\sigma(z,t) = 3.0 + 0.09T(z,t)$ Siemens m^{-1}, where $T(z,t)$ is unless the horizontal currents are strong and baroclinic (i.e., have large vertical shear). Assuming low-frequency motions so that $\vec{N}(t)$ can be ignored, and using Eq. (4), the components of Eq. (1) in the northern hemisphere become

$$\frac{E^{u}_{-y}(t)}{C|F_z|} = <u(t)> + \frac{1}{H}\int_{-H}^{0} \frac{\hat{\sigma}(z,t)}{<\sigma(t)>} u(z,t)\,\mathrm{d}z, \tag{5a}$$

$$\frac{E^{v}_{x}(t)}{C|F_z|} = <v(t)> + \frac{1}{H}\int_{-H}^{0} \frac{\hat{\sigma}(z,t)}{<\sigma(t)>} v(z,t)\,\mathrm{d}z, \tag{5b}$$

where the subscript on E denotes the direction in which that term is positive and the superscript indicates the horizontal water velocity component to which it is proportional. Clearly, the HEFs are integrating variables in the sense defined in the introduction, being proportional to the vertical average of horizontal water velocity plus a small "contamination" due to conductivity. The conductivity contribution is negligible if conductivity is independent of depth in the ocean (as it is at very high latitudes) or if the horizontal water velocity has little vertical shear (as frequently occurs at mid- to high-latitudes).

Normal Modes

To elucidate further the relationships in Eq. (5), it is useful to expand the various quantities in terms of the dynamical normal modes. While any complete basis set could be used, the dynamical normal modes have a vertical dependence that permits rapid convergence of the expansions of horizontal velocity and seawater conductivity and temperature. Despite the fact that the dynamical normal modes are obtained from the equations of motion by various simplifying assumptions such as no mean flow and linearity, in using these modes here we are not making any assumptions about the underlying dynamics of the quantities being observed. The modes are simply the most convenient basis set for our purposes.

The dynamical modes are obtained from the equations of motion by assuming no mean flow, mean stratification in hydrostatic balance, and a flat bottom. With horizontal velocity and pressure proportional to $\phi_i(z)$, and vertical velocity and density perturbations proportional to $\theta_i(z)$, the equations for Boussinesq linear waves (small perturbations) then separate into sets of equations for the horizontal/time dependence and vertical dependence. Specifically, with

$$\begin{Bmatrix} \vec{v}_h(z,t) \\ p(z,t)/\rho_* \end{Bmatrix} = \sum_{i=0}^{\infty} \begin{Bmatrix} \vec{\mathrm{a}}_{h,i}(x,y,t) \\ \mathrm{d}_i(x,y,t) \end{Bmatrix} \phi_i(z) \tag{6a}$$

and

$$\begin{Bmatrix} w(z,t) \\ \rho'(z,t)g/N^2(z) \end{Bmatrix} = \sum_{i=0}^{\infty} \begin{Bmatrix} f_i(x,y,t) \\ g_i(x,y,t) \end{Bmatrix} \theta_i(z) \tag{6b}$$

where $\rho = \rho_* + \overline{\rho}(z) + \rho'(z,t)$ and $N^2(z) = -\frac{g}{\rho_*}\frac{d\overline{\rho}}{dz}$, then ϕ_i and θ_i satisfy

$$\phi_i = \frac{d\theta_i}{dz} \text{ and } \frac{d\phi_i}{dz} = -\frac{N^2}{\gamma_i^2}\theta_i \tag{6c}$$

with the boundary conditions

$$\theta_i = 0 \text{ or } \frac{d\phi_i}{dz} = 0 \text{ at } z = -H, \tag{6d}$$

$$\frac{d\theta_i}{dz} - \frac{g}{\gamma_i^2}\theta_i = 0 \text{ or } \frac{d\phi_i}{dz} + \frac{N^2}{g}\phi_i = 0 \text{ at } z = 0. \tag{6e}$$

Equations (6c) are solved numerically with the appropriate boundary conditions to produce the vertical structure functions and eigenvalues, γ_i^2. The structure functions are orthogonal and are normalized so that

$$\frac{1}{H}\int_{-H}^{0} \phi_i \, \phi_j \, dz = \delta_{ij}. \tag{6f}$$

This normalization means that the ϕ_i are non-dimensional, while the θ_i have dimensions of length. We now have a complete orthonormal basis set for describing any quantity that varies with depth. The fact that these modes are "tuned" to the depth-dependences of real oceanographic variables makes them very useful, since it means expansions in terms of these modes should converge rapidly. For our purposes in this section, it is not important what assumptions were used to obtain the vertical structure equations in Eq. (6c).

Let's now expand conductivity in terms of the dynamical modes, viz.,

$$\frac{\hat{\sigma}(z,t)}{<\sigma(t)>} = \sum_{i=1}^{\infty} s_i(x,y,t)\,\phi_i(z), \tag{7}$$

where the i=0 (barotropic) mode has been dropped since the vertical average of $\hat{\sigma}$ is zero. Substituting this expression and those in Eq. (6a) into Eq. (5), and truncating after mode number 1, yields

$$\frac{E_{-y}^{u}(t)}{C|F_z|} \approx a_{x,0} + s_1 a_{x,1}, \tag{8a}$$

$$\frac{E_{x}^{v}(t)}{C|F_z|} \approx a_{y,0} + s_1 a_{y,1}, \tag{8b}$$

The truncation in Eq. (8) is quite reasonable given the expected decrease in modal current amplitudes with increasing mode number, and given the examples in Table 2 of mean s_i, calculated using Levitus (1982) data to compute structure functions and conductivity profiles. Table 2 suggests that in polar oceans and in the mid-latitude Pacific the conductivity-weighting contribution to the electric field is completely trivial. Using a year of electric field and current meter mooring data, Luther et al. (1991) have shown the validity of Eq. (5) in a region of the mid-latitude North Pacific with weak mean currents and weak baroclinic eddy activity. In that location, the conductivity-weighting contribution to the electric field was completely trivial.

Table 2. Expansion coefficients for conductivity from Eq. (7).

	s_i		
Mode (i)	32.5°N Atlantic	42.5°N Pacific	57.5°S Atlantic
1	0.119	0.017	-0.004
2	-0.014	0.021	-0.009
3	-0.012	-0.002	-0.004
4	0.008	0.008	-0.001

On the other hand, in the mid-latitude North Atlantic, equal amplitudes of the barotropic (i=0) and first baroclinic (i=1) modes of current imply a ~12% relative contribution to the electric field from the last term on the RHS of Eq. (5). Rossby (1987) found first baroclinic mode amplitudes up to 70% greater than barotropic mode amplitudes in the Gulf Stream at 73°W, with very small second and third baroclinic mode amplitudes. Consequently, in the Gulf Stream we can expect up to 20% conductivity-weighting contributions to the electric field due to the first baroclinic mode of current. In fact, our work with SYNOP data has shown occasional conductivity-weighting contributions up to 30%, although the mean contribution is <15%.

The variation of conductivity with depth in the oceans (e.g., Luther et al., 1991) suggests dominance of the first baroclinic mode in the conductivity contribution to Eq. (5), which allows the use of another integrating measurement, vertical acoustic travel time, to estimate first baroclinic mode current and temperature amplitudes in order to remove the conductivity contribution from the HEF. This procedure is outlined later.

Horizontal Electrometer (HEM) Versus Mooring Estimates of Transport

In deployments of seafloor HEMs to date, where comparison of HEM estimates of the vertically averaged horizontal water velocity, $< \vec{v}_h(t) >$, with current meter mooring estimates of the same quantity have been possible, the HEM estimates have proven to be more accurate. These results provide an example of how measurement of an integrating variable provides a more accurate estimation of oceanic behavior than can be accomplished with a suite of conventional point measurements. In this specific case such accuracy has significant importance to climate studies that rely on estimates of transport (which is directly proportional to horizontal integrals of $H< \vec{v}_h(t) >$ for determining the world ocean's role in climate fluctuations.

The first HEM vs. mooring comparison of $< \vec{v}_h(t) >$ estimates was produced by Luther et al. (1991) from data collected during BEMPEX, an experiment that deployed a large number of HEMs and pressure gauges in the North Pacific to study direct atmospheric forcing of gyre-scale eddies (the results of which are discussed further below). The accuracy of the HEM estimates of $< \vec{v}_h(t) >$ was corroborated by current estimates made by reciprocal tomography, which is based on measuring reciprocal acoustic travel time differences (Table 1). The inaccuracy of the current meter mooring estimates was attributed primarily to stalling of the current meter rotors in the weak flows below 1000 meters depth. Another electrometer-mooring comparison presented below comes from the opposite extreme for oceanic flows, i.e., from the Gulf Stream which has strong currents at all depths so that rotor stalling is not expected to be a problem.

The Office of Naval Research provided funds for us (with Jean Filloux) to deploy four of Filloux' seafloor HEMs (Filloux, 1987) next to current meter moorings during the last year ('89-'90) of the SYNOP experiment in the Gulf Stream. The HEMs were deployed in an array centered around 37.5°N, 68.5°W, at depths near 4700 m. Near each HEM were sub-surface moorings deployed by J. Bane, T. Shay, R. Watts, and W. Johns, carrying current meters at nominal depths of 400 m, 700 m, 1000 m and 3500 m.

The LHSs of Eqs. (5a) and (5b) were obtained from the HEM data using C=0.95, as per estimates of C made by Sanford et al. (1985) in the western North Atlantic, and using an appropriate estimate of F_z for the time and location of the experiment, while the RHSs were estimated from the mooring data. The latter estimation included extrapolation of $\vec{v}_h$, temperature and pressure to a fictitious nominal 100-m depth, conversion of pressure to 'depth,' and estimation of conductivity using temperatures and a climatological temperature/salinity relation in an empirical formula. The currents and conductivities at the four real and one "fictitious" instruments were trapezoidally integrated, taking into account the time dependence of the depths of the instruments. The time series thus obtained, representing opposite sides of Eq. (5), are highly coherent, as shown in Figure 1.

While the coherence in Figure 1 is very encouraging, and the rms differences between the estimates of the LHS and RHS of Eq. (5) are no larger for instance than what has been considered very good agreement for testing schemes to remove the effects of mooring motion from current meter data (e.g., Hogg, 1991; Cronin, 1991), examination of the