SCALING OF INTERNAL WAVE MODEL PREDICTIONS FOR ε

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ABSTRACT

Both induced diffusion and the Henyey, Wright, and Flatté model predict the Gregg scaling law

$$\varepsilon \sim \left(\frac{S_{10}}{N}\right)^4 N^2 f \tag{1}$$

when the Garrett-Munk spectrum is used. This note discusses the scaling in a more general context. The more general scaling is needed:

1. near the equator, where f=0.

2. if the shear spectrum is not white in vertical wavenumber.

INTRODUCTION

Various model predictions of the energy flux through the internal wave field have been made. These model predictions involve not only the intrinsic physics put into the model, but also the assumed spectral form for the internal waves. Invariably, the Garrett-Munk (GM) spectrum is used. According to the models discussed here, GM is appropriate in steady-state conditions far from the equator. The question arises, what if these conditions are not met?

Gregg (1989) showed that a particular form describes data well. His form is a version of that predicted by both induced diffusion (ID) (McComas and Müller, 1981) and by the model of Henyey, Wright, and Flatté (1986) (HWF). These two models differ in their overall level, but have the same scaling form when applied to a Garrett-Munk spectral shape.

GENERALIZED SCALING

In order to obtain more general scaling expressions for these two models, we will assume that small-scale IWs obey a power law spectrum in vertical wavenumber, but not necessarily that of GM. We will denote the RMS shear on any vertical scale by S; the shear spectral level scales as S^2/k_v, where k_v is the vertical wavenumber describing that scale. GM has $S \sim k_v^{1/2}$.

Induced diffusion is described by a diffusivity in vertical wavenumber space. This diffusivity is given by (Müller et al, 1986, eq. 81).

$$D = \frac{k_h^2}{2} \int d(S^2)\, \delta(\Omega - V_g \cdot K) \tag{2}$$

Ω and K are the frequency and wavenumber of the large scale waves contributing dS^2 to the shear variance. V_g is the group velocity. For scaling purposes, the delta function behaves as the reciprocal of its argument, and we use the term $V_{gv}K_v$ as being indicative of the scaling. Thus, we obtain

$$D \sim \frac{k_h^2}{V_{gv}} \frac{S^2}{K_v}\bigg|_{max} . \tag{3}$$

The maximum wavenumber of the shearing waves is of order the wavenumber of the sheared wave. (For GM, S^2/K_v is constant, making this observation not necessary.) The group velocity is

$$V_{gv} = \frac{-N^2}{\omega} \frac{k_h^2}{k_v^3} \tag{4}$$

where we have assumed here and in what follows $\omega << N$; $k_h << k_v$.

Putting it all together, we have

$$\epsilon = D \partial_{k_v} E \tag{5}$$

where E is the energy spectral level in vertical wavenumber, and scales as

$$E \sim \frac{(S/k_v)^2}{k_v} = \frac{S^2}{k_v^3} \tag{6}$$

Therefore,

$$\epsilon \sim \frac{\omega S^4}{k_v^2 N^2} \tag{7}$$

Under steady state conditions, this form must be independent of k_v , leading to the GM S ~ $k_v^{1/2}$. More generally, for an evolving spectrum, the flux is only ε at the smallest scales. These scales are presumably determined by a Richardson number criteria S~N, so

$$\epsilon \sim \omega N^2/k_c^2 \tag{8}$$

where k_c is the wavenumber at some fixed Richardson number.

The HWF model is based on deterministic evolution at small scales. The steady-state spectral form is not obtained analytically. Numerical results (Flatté, Henyey, and Wright, 1985) demonstrate S ~ $k_v^{1/2}$ in steady state for this model. Energy dissipation scales as

$$\epsilon \sim \left|\frac{dk_v}{dt}\right| E \tag{9}$$

at some fixed Richardson number.

The change of vertical wavenumber is $|\ dk_v/dt\ | \sim Sk_h$, so

$$\epsilon \sim Sk_h\left(\frac{S^2}{k_v^3}\right) \tag{10}$$

$$\sim\left(\frac{k_h}{k_v}\right)\frac{N^3}{k_c^2} \tag{11}$$

As long as ω-f is not typically much smaller than f, k_h/k_v scales as ω/N. Thus,

$$\epsilon \sim \omega N^2/k_c^2, \tag{12}$$

exactly the same scaling as ID has. Except near the equator, typical values of ω are of order f. This is the origin of the f factor in the previously published scaling expressions. The models do not assume any process happens on time scales of 1/f. If there were no separation between horizontal and vertical scales, ω would scale as N, giving $\varepsilon \sim N^3/k_c^2$, which could be obtained by dimensional arguments. The reason the models predict otherwise is that the aspect ratio is a dimensionless parameter.

We now assume

$$\frac{S^2}{k_v} \sim A k_v^p \tag{13}$$

so p=0 for GM. Evaluating this at fixed Richardson number we get

$$N^2 \sim A k_c^{p+1} \tag{14}$$

Eliminating the unknown coefficient A, we get

$$\left(\frac{S}{N}\right)^2 \sim \left(\frac{k_v}{k_c}\right)^{p+1} \tag{15}$$

we solve for k_c and put it into ε, obtaining

$$\epsilon \sim \frac{\omega\ N^2}{k_v^2}\left(\frac{S}{N}\right)^{\frac{4}{P+1}} \tag{16}$$

Following Gregg, we fix k_v at 2π/10m, so we get

$$\epsilon \sim \omega N^2 \left(\frac{S_{10}}{N}\right)^{\frac{4}{p+1}} \tag{17}$$

This agrees with Gregg for the GM values

$$\omega \sim f \tag{18}$$

$$p=0. \tag{19}$$

and is the appropriate prediction in more general cases from both models.

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