TESTING THE CRITICAL REFLECTION HYPOTHESIS

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ABSTRACT

According to linear internal wave theory, the reflection of internal waves off a bottom of uniform slope, in a uniformly rotating, uniformly stratified fluid, should lead to energy enhancement and a cross-isobath alignment of motions at the frequency for which the wave ray slope equals the bottom slope. Current meter data from the near-equatorial Atlantic and from the continental rise and slope off Nova Scotia are used to test this hypothesis.

1 INTRODUCTION

The reflection of internal waves off a plane rigid surface differs markedly from the reflection of electromagnetic or acoustic waves. In Optics or Acoustics, the incident and reflected wave rays make the same angle with respect to the normal to the reflecting surface, whereas for internal waves, the incident and reflected wave rays make the same angle with respect to the vertical. The unusual nature of the law of reflection for internal waves is a direct consequence of their dispersion relation which states that, for waves of a given frequency, energy must propagate at a given angle with respect to the vertical. As linear theory requires frequency to be conserved upon reflection, the angles which the incident and reflected wave rays make with respect to the vertical must be the same.

Close to the critical frequency ω_c for which the wave ray slope equals the bottom slope, simple arguments (Phillips 1977, p.227) show that, upon reflection, the wavenumber, energy density, and shear associated with the incident waves are greatly amplified, so that shear instability and energy dissipation are more likely. Phillips (1963) first pointed this out for inertial waves incident on a bottom of constant slope in a rotating, homogeneous, inviscid fluid, and later generalised the theory to internal gravity waves in Phillips (1966). In both cases he assumed that the direction of energy propagation of the incident waves was normal to the isobaths.

For arbitrary angle of incidence with respect to the isobaths, Sandstrom (1966) pointed out that whereas the component of the wavenumber vector normal to the isobaths is generally amplified upon reflection, the component of the wavenumber vector parallel to the isobaths remains unchanged. Consequently, the major axis of the current ellipses should become more closely aligned with the cross-isobath direction upon reflection. Sandstrom (1966, p.78) was the first to report observational evidence of energy enhancement at ω_c, using thermistor data from the Bermuda slope (Haurwitz, Stommel and Munk, 1959) [1]. He also performed laboratory experiments which clearly

[1] Wunsch (1972) also used that data and drew attention to the energy enhancement at ω_c.

demonstrated the amplification of parcel motions that results from internal wave reflection off a sloping bottom (figure 21, p.71), and published that work in Sandstrom (1969).

The most recent surge of interest for the problem of internal wave reflection off sloping bottoms is largely due to the work of Eriksen (1982), who presented observational evidence of energy and shear enhancement near the critical frequency at a few mooring sites, and also provided evidence for a cross-isobath alignment of motions near ω_c in a few of these cases. In a later paper, Eriksen (1985) explored the implications for ocean mixing of internal wave reflection off sloping bottoms. He argued that internal wave breaking at sloping boundaries may cause diapycnal mixing of global oceanic significance, possibly accounting for Munk's (1966) canonical value of $K_v \approx 10^{-4}\mathrm{m}^2\mathrm{s}^{-1}$ in the abyssal ocean. This possibility was further examined by Garrett and Gilbert (1988).

In section 2 of this paper, we give a brief summary of the linear theory of internal wave reflection off a bottom of constant slope, in a uniformly rotating, uniformly stratified fluid. In section 3, we examine current meter data from the Western Boundary Sill Experiment to try to verify whether a near-bottom enhancement of motions with 3–4 day periods can be attributed to internal wave reflection, as Eriksen (1982) suggested. In section 4, current meter data from the continental rise and slope off Nova Scotia are used in order to look for evidence of energy enhancement and/or cross-isobath alignment of motions near ω_c. A summary and general discussion follow in section 5.

2 THEORY

The dispersion relation for internal waves is given by (Gill, 1982)

$$\omega^2 = N^2 \cos^2 \theta + f^2 \sin^2 \theta \tag{1}$$

$$\text{or} \qquad \tan \theta = \left(\frac{\mathrm{N}^2 - \omega^2}{\omega^2 - \mathrm{f}^2} \right)^{1/2}, \tag{2}$$

where N is the buoyancy frequency, f is the inertial frequency, and θ is the angle which the wavenumber vector $\mathbf{k} = (k, \ell, m)$ makes with respect to the horizontal. Since the direction of energy propagation is perpendicular to the direction of phase propagation ($\mathbf{c_g} \perp \mathbf{k}$) for internal waves, the frequency at which the wave ray slope matches a bottom slope of $\tan \alpha$ can be obtained by substituting $\theta = \pi/2 - \alpha$ into (1), yielding

$$\omega_c^2 = N^2 \sin^2 \alpha + f^2 \cos^2 \alpha. \tag{3}$$

It can be shown (Eriksen, 1982, Gilbert, 1990) that for constant values of f, N and $\tan \alpha$, the wavenumber amplification is given by

$$\frac{m_r}{m_i} = \frac{a^2 + 2a \cos \phi_i + 1}{a^2 - 1}, \qquad a = \tan \alpha \tan \theta, \tag{4}$$

where a is a frequency-dependent parameter (2), ϕ_i is the angle which the horizontal component of $\mathbf{k_i}$ makes with the onslope direction, and m_i, m_r denote the vertical components of the incident and reflected wavenumber vectors respectively. By requiring that $\mathbf{u} \cdot \hat{\mathbf{n}} = 0$ at the bottom, where $\hat{\mathbf{n}}$ is a unit vector normal to the bottom, it can also be shown that (Eriksen, 1982)

$$\frac{E_r}{E_i} = \left(\frac{m_r}{m_i}\right)^2, \tag{5}$$

and $$\phi_r = \sin^{-1}\left(\left|\frac{m_i}{m_r}\right| \sin\phi_i\right), \tag{6}$$

where E_i, E_r are the energy density of the incident and reflected waves respectively, and ϕ_i, ϕ_r are the angles which the horizontal components of $\mathbf{k_i}$, $\mathbf{k_r}$ make with the onslope direction. There is a singularity in (4) when $a \to 1$ ($\omega \to \omega_c$), so that linear inviscid theory (5) predicts we should observe enhanced energy density near ω_c above sloping bottoms. It also predicts (6) a cross-isobath alignment of motions near ω_c ($|m_i/m_r| \to 0$ and hence $\phi_r \to 0$ or π when $a \to 1$).

3 DATA FROM THE WESTERN BOUNDARY SILL EXPERIMENT

To measure the flow of Antarctic Bottom Water entering the Northwest Atlantic Basin, the buoy group of the Woods Hole Oceanographic Institution deployed two moorings between the Ceara Rise and the mid-Atlantic ridge from December 9, 1977 to December 5, 1978 (Whitehead and Worthington, 1982, hereafter WW82). Mooring 636 was deployed at 4°2.5′N, 39°40.6′W, and mooring 637 was deployed at 4°1.3′N, 39°19.0′W. Both moorings had current meters at 10m, 50m, 100m and 200m above the bottom. The local depth was 4456m at mooring 636, and 4304m at mooring 637 (see Fig. 1).

Eriksen's (1982) most convincing evidence of energy enhancement near ω_c came from mooring 636 of this experiment. This is shown here on Fig. 2, where we see that the near-bottom energy enhancement ranges roughly from 0.005 cph to 0.05 cph (periods between 20 and 200 hours), with a peak in energy density centered at 0.0117 cph (85.5 hour or 3.56 day period). The inertial frequency at that location is 0.00587 cph (170.3 hour period), so that the energy peak occurs well within the internal waveband at $\omega \sim 2f$.

Following Eriksen (1982) in taking $N \approx 0.8$ cph, and assuming that the bottom slope at mooring 636 is 0.015, as estimated from the East-West transect shown on Fig. 1, we obtain $\omega_c = (N^2 \sin^2\alpha + f^2 \cos^2\alpha)^{1/2} \approx 0.0133$ cph. This estimate of the critical frequency compares well with the energy enhancement shown on Fig. 2.

However, Eriksen (1982, p. 533) pointed out that while the energetic motions with $\omega \approx 0.012$ cph at mooring 636 could be due to critical internal wave reflection, they violate one of the basic predictions of specular reflection theory (6), namely that the major axis of the current ellipses should be oriented normal to the isobaths. Instead the current ellipses are roughly parallel to the inferred North-South orientation of the isobaths

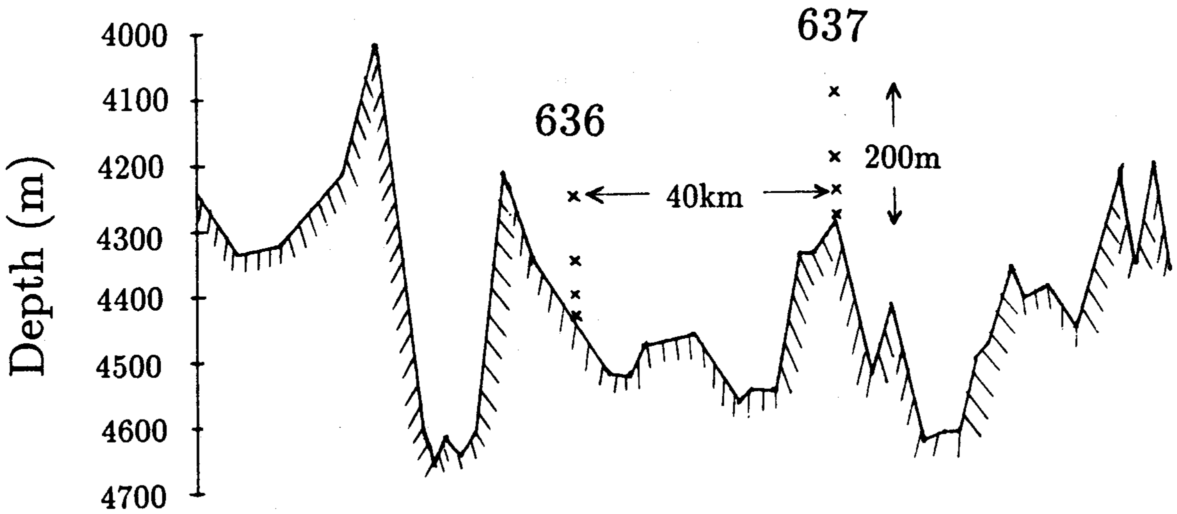

Figure 1: Topography in the vicinity of moorings 636 and 637 of the Western Boundary Sill Experiment, based upon the East-West transect at 4°N shown on figure 4 of Whitehead and Worthington (1982).

(see Eriksen's figure 9). This is not a trivial discrepancy, as the mechanism leading to energy enhancement near the critical frequency involves a 'squeezing' of the reflected wavebeam (relative to the width of the incident wavebeam) in the vertical plane normal to the isobaths (Phillips, 1977, figure 5.13). It is hard to envisage how internal wave reflection could lead to enhanced motions in a vertical plane roughly parallel to the isobaths. Further examination of the current meter data from the Western Boundary Sill Experiment therefore seems warranted.

Comparison with mooring 637

Figure 3 shows the kinetic energy spectra 50m[2], 100m and 200m above the bottom at mooring 637. A first obvious difference in the kinetic energy spectra is that an inertial peak ($f = 5.87 \times 10^{-3}$ cph) appears to be present at mooring 637, whereas it could not be discerned at mooring 636 (Fig. 2). A second difference is that while mooring 637 shows some near-bottom kinetic energy enhancement at $\omega = 0.0117$ cph, the energy levels are not nearly as elevated as at mooring 636; comparing the 50m records from each mooring, we find that the spectral level at mooring 637 is about 5 times lower than at mooring 636. The range of frequencies over which we observe near-bottom kinetic energy enhancement is also narrower at mooring 637.

Nonetheless, since moorings 636 and 637 both display near-bottom energy enhancement at $\omega = 0.0117$ cph, it seems worthwhile to find out whether motions at the two moorings are

[2]Data from the current meter 10m above the bottom were judged to be of lesser quality by NODC (National Oceanographic Data Center) and so were not sent to us.

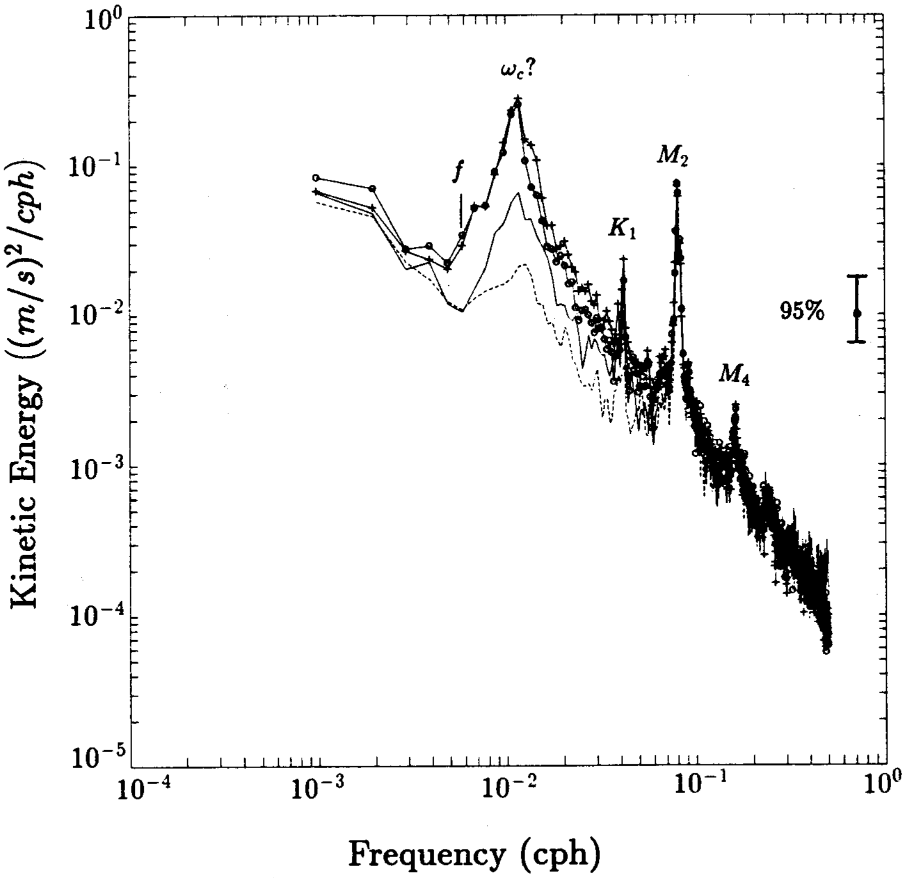

Figure 2: Kinetic energy spectra at 10m (+), 50m (o), 100m (—) and 200m (- - -) above the bottom at mooring 636 ($\nu = 30$). The inertial frequency is $f = 5.87 \times 10^{-3}$ cph.

correlated at that frequency. Figure 4 shows cross-spectra between the current meter 200m above the bottom at mooring 636, and the current meter 50m above the bottom at mooring 637 (4256m and 4254m deep respectively).

For the clockwise velocity signals, there is a strong coherence peak at the semidiurnal tide (not shown here), a few weak peaks near the diurnal tide, and another peak spreading across four adjacent frequency bins from 0.0117 cph to 0.0146 cph, where the signals are not significantly out of phase. The latter coherence peak coincides perfectly with the range of frequencies with the most kinetic energy within the internal waveband 50m above the bottom at mooring 637 (Fig. 3). The temperature signals are also correlated in the 0.0137–0.0146 cph band, where they are not significantly out of phase.

Thus Fig. 4 shows that for $\omega \approx 0.012$ cph, the motions at mooring 636 are coherent and not significantly out of phase with those at mooring 637, some 40 km away. Significant coherences within the internal waveband over horizontal distances of the order of a few tens of kilometers are rare but not unheard of in the deep ocean. For example, Fu (1981)

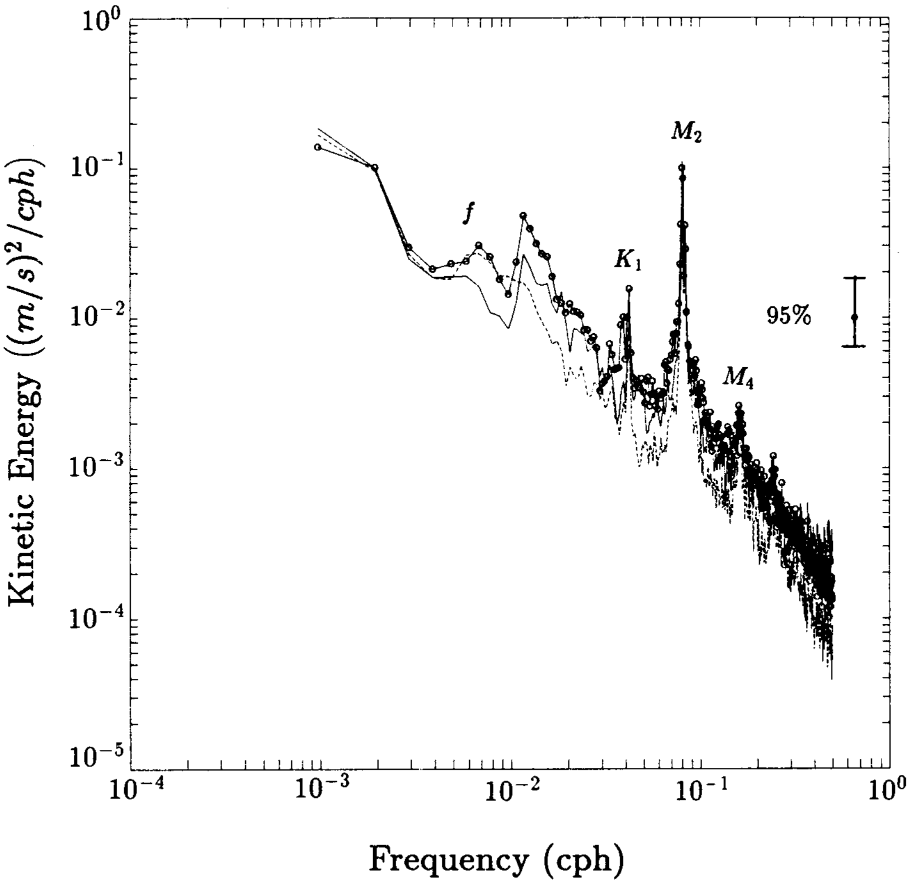

Figure 3: Kinetic energy spectra 50m (o), 100m (—) and 200m (- - -) above the bottom at mooring 637 ($\nu = 30$).

obtained significant coherences over horizontal separations up to 70 km at mid-latitudes for the inertial band. For the mid-latitude internal wave model proposed by Garrett and Munk (1972), the horizontal distance at which the coherence drops to 0.5 is given by

$$\Delta X_{0.5} = \frac{58 \text{ m} \cdot 3 \text{ cph}}{(\omega^2 - f^2)^{1/2}}, \tag{7}$$

where ω and f are expressed in cph. For $\omega = 1.17 \times 10^{-2}$ cph and $f = 5.87 \times 10^{-3}$ cph, this yields $\Delta X_{0.5} = 17$ km, so that the coherence should be less than 0.5 (or the squared coherence should be less than 0.25) at a separation of 40 km. Wunsch and Webb (1979) showed that at low latitudes, currents are generally coherent over smaller spatial scales than would be predicted by (7), so that the observed coherence is rather surprising. It becomes even more puzzling when we consider the fact that based on the critical reflection hypothesis, the observed field of motions, if dominated by the reflected wavefield, should be unusually rich in motions with small horizontal and vertical scales [3] near the critical frequency (4). Cross-spectra between various pairs of current meters at mooring 636 suggest that the dominant vertical scales of motion at $\omega \approx 0.12$ cph are also large (Gilbert, 1990).

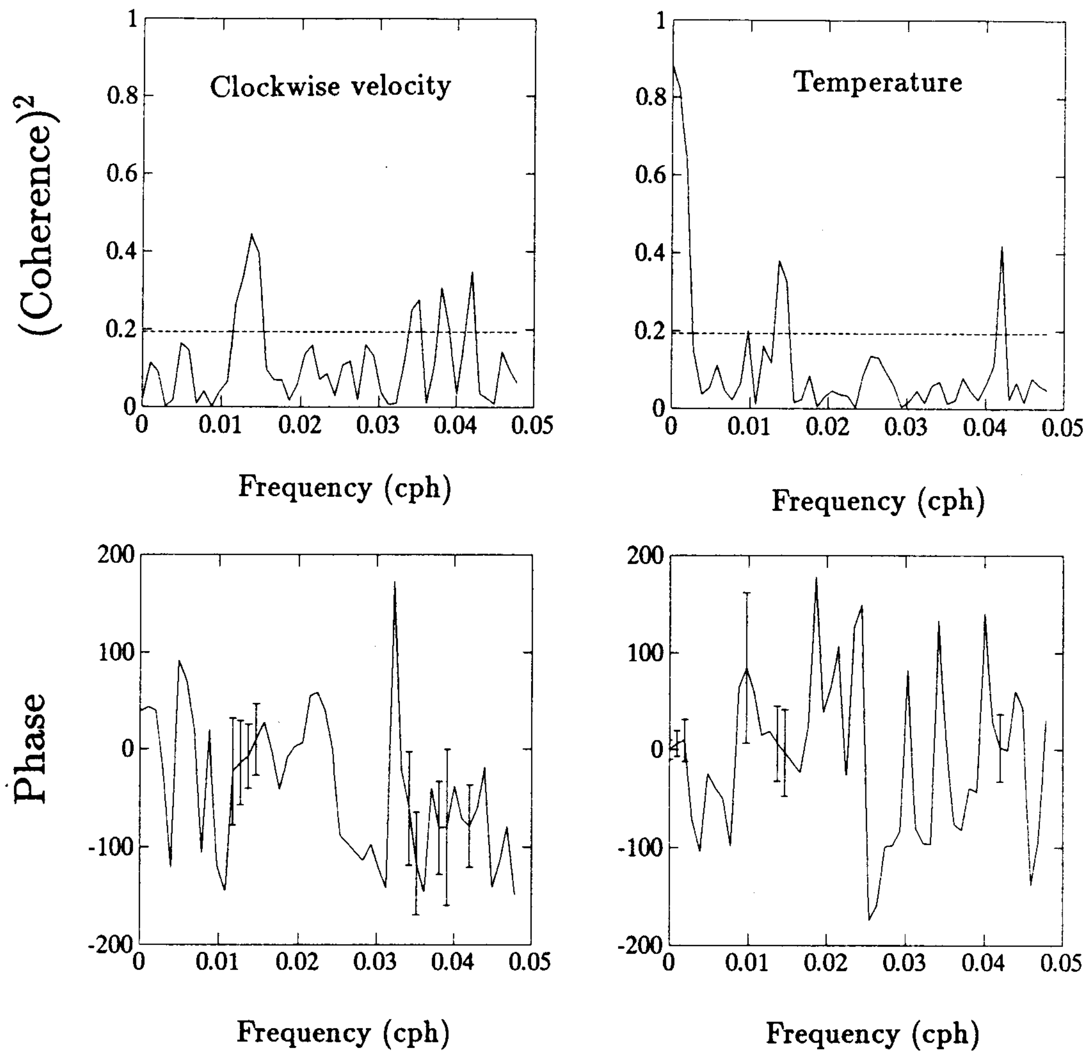

Figure 4: Cross-spectra between the current meter 200m above the bottom at mooring 636, and the current meter 50m above the bottom at mooring 637, both of which were at the same depth. The cross-spectrum for clockwise velocity is shown on the left, and that for temperature is shown on the right. A positive phase means that the signal at mooring 637 leads the signal at mooring 636. The dashed line represents the 95% significance level for zero true coherence on the upper plots, and the 95% confidence intervals for phase are shown on the lower plots ($\nu = 30$).

[3]The horizontal and vertical scales of motion should be reduced by the same factor upon reflection, because conservation of frequency implies that $\tan\theta_r = \tan\theta_i$, which in turn implies that $m_r/m_i = \kappa_r/\kappa_i$, where m and κ are the vertical and horizontal components of the wavenumber vector respectively.

Rotary spectra and the ratio P_{++}/P_{--}

Using the clockwise (u_-) and anticlockwise (u_+) rotary velocity components

$$u_- = (u - iv)/\sqrt{2}, \tag{8}$$
$$u_+ = (u + iv)/\sqrt{2} \tag{9}$$

as in Müller et al. (1978), we computed the clockwise and anticlockwise autospectra P_{--} and P_{++} at each current meter. For linear, freely-propagating internal waves, the ratio of anticlockwise to clockwise kinetic energy should be given by (Fofonoff, 1969)

$$\frac{P_{++}}{P_{--}} = \left(\frac{\omega - f}{\omega + f}\right)^2. \tag{10}$$

On Fig. 5, we plotted P_{++}/P_{--} as a function of frequency at 10m, 50m, 100m and 200m above the bottom at mooring 636. Interestingly enough, we find large deviations from (10) in the range of frequencies for which near-bottom kinetic energy enhancement is observed. More interesting still, we find that in the neighbourhood of the energy peak ($\omega \approx 0.012$ cph), the largest departure from (10) occurs at 10m above the bottom, followed by the departures at 50m, 100m and 200m in that order. There thus seems to be a one to one correspondance between kinetic energy enhancement and the degree of departure from (10) near $\omega \approx 0.012$ cph. Eriksen (1982, p.533) hinted at that when he mentioned that "the current ellipses at $\omega \sim 2f$ are more narrow than would be expected from linear internal waves, even if waves were unidirectional."

Trapped waves with $\omega > f$?

Thompson and Luyten (1976) have provided evidence for the existence of Rhines' (1970) bottom-trapped waves at frequencies lower than the inertial frequency f. In the present context however, what is interesting about those waves is that the maximum allowed frequency, $N \sin\alpha \approx 0.8$ cph $\times$ $0.015 \approx 0.012$ cph, is larger than f, so that bottom-trapped motions could in principle occur in the frequency band normally reserved to freely-propagating internal waves ($f < \omega < N$). The possibility of bottom-trapped waves with $\omega > f$ has not received much attention in the literature, but Huthnance (1989) points out that "continuity of mode forms near $\omega = f$ (Huthnance, 1978) suggests that trapped waves approaching $\omega = f$ from lower frequencies should continue as nearly-trapped waves for ω exceeding f. The bottom trapped waves of Rhines (1970) in realistic contexts are obvious candidates." Huthnance and Baines (1982) called for a more detailed investigation of the phenomenon, pointing out that we do not know much about the possible radiational energy losses of those waves when $N \sin\alpha > f$.

The bottom-trapped motions described by Rhines (1970) are rectilinear, implying the ratio of anticlockwise to clockwise kinetic energy

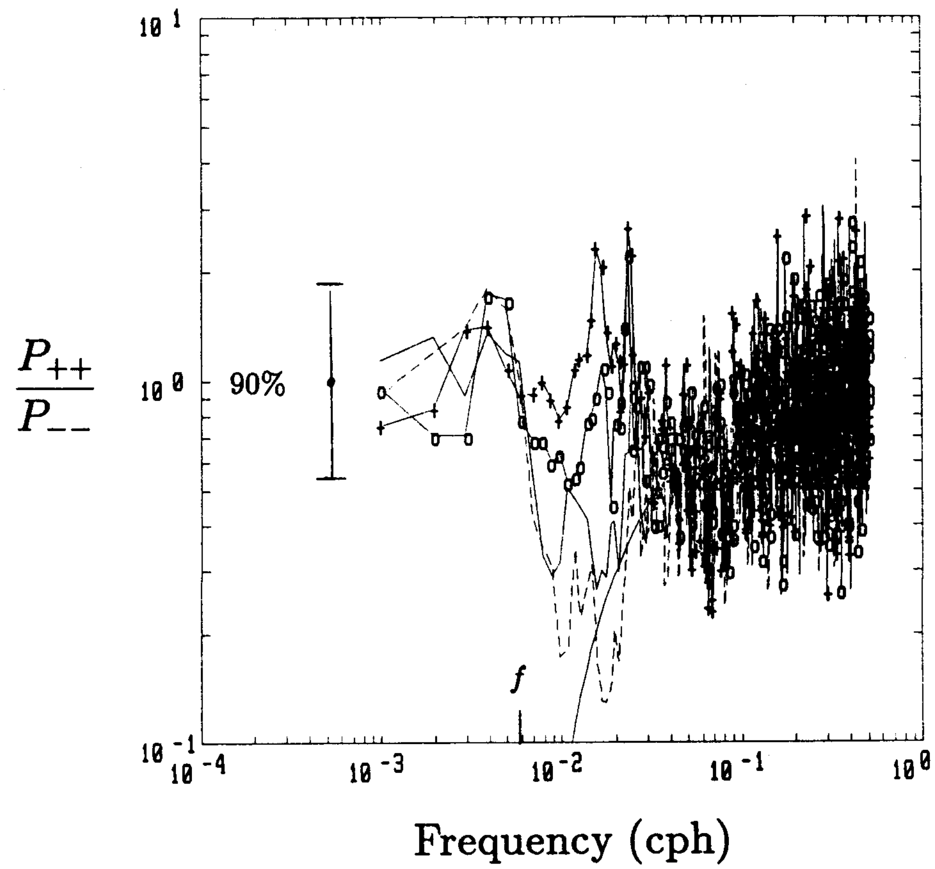

Figure 5: Ratio P_{++}/P_{--} at 10m (+), 50m (o), 100m (—) and 200m (- - -) above the bottom at mooring 636 ($\nu = 30$). The theoretical ratio (10) for linear, freely propagating internal waves, is also shown for the purpose of comparison. The 90% confidence intervals assume a Fisher's F distribution for the ratio of two independent chi-squared distributions (Jenkins and Watts, 1968, p.85), and so may not be appropriate.

$$\frac{P_{++}}{P_{--}} = 1. \tag{11}$$

Looking at Fig. 5, we find that at 10m and 50m above the bottom, P_{++}/P_{--} is closer to 1 than to the internal wave line for $\omega \leq 0.025$ cph. This result prompted Gilbert (1990) to try to verify whether the orientation ϕ of these motions changes with frequency according to

$$\phi = \cos^{-1}\left(\frac{\omega}{N \sin \alpha}\right), \tag{12}$$

as in Rhines' theory [4], but he found no significant changes of ellipse orientation within that range of frequencies. Thus whereas Eriksen's (1982) interpretation of the 3–4 day motions at mooring 636 as critically reflected internal waves fails to explain why the

[4]The notation used here differs from that of Rhines; ϕ is measured counterclockwise from the onslope direction to ensure consistency with the earlier definition of ϕ given in section 2.

current ellipses near ω_c are not perpendicular to the inferred North–South orientation of the isobaths, an interpretation of those motions in terms of Rhines' internal edge waves (Gilbert, 1990) fails to explain the lack of turning of the current ellipses with frequency.

4 DATA FROM THE CONTINENTAL RISE AND SLOPE OFF NOVA SCOTIA

The Bedford Institute of Oceanography deployed several moorings on the Scotian Rise and Slope during the 1970's and 1980's. Those moorings were deployed for the purpose of studying sub-inertial motions (e.g. Louis et al., 1982, Smith and Petrie, 1982), not for the purpose of studying internal wave reflection off sloping bottoms. Consequently, the vertical spacing between current meters is large at most moorings; there is usually only one current meter in the first 100m above the bottom, which does not allow us to study the vertical structure of some of the small-scale processes known to occur above sloping topography (e.g. Thorpe et al., 1990). Nevertheless, it should be possible to look for evidence of near-bottom energy enhancement at ω_c in a manner analogous to Eriksen (1982).

Tables 1 and 2 provide a convenient summary of the current meter data used here, and Fig. 6 shows the location of the moorings. More detailed information about the data can be found in Lively (1979a, 1979b and 1984).

A list of criteria

Linear internal wave reflection theory predicts we should observe energy enhancement (5) at the critical frequency ω_c above sloping topography. For the data set under consideration here, we could try to verify this by looking for

1a) a spectral peak above the background energy level at ω_c

1b) near-boundary energy enhancement at ω_c for instruments at different heights above the bottom, but on the same mooring

1c) near-boundary energy enhancement at ω_c for instruments at the same depth, but on different moorings

1d) a spectral level at ω_c larger than that given by the GM79 model spectrum.

The other major prediction of linear internal wave reflection theory is that current ellipses should tend to orient themselves normal to the isobaths upon reflection (6). Hence we should also look for

2) a pronounced anisotropy at ω_c, the major axis of current ellipses being oriented in the cross-isobath direction.

Criterion 1a was used by Sandstrom (1966) and Eriksen (1982) at mooring sites where the critical frequency was well separated from the energetic tidal and inertial frequencies.

Such a separation is necessary if we wish to unambiguously interpret a spectral peak at ω_c as the signature of internal wave reflection. Unfortunately, ω_c is close to f or M_2 at nearly all the mooring sites considered here (see table 3), so that criterion 1a was only used at mooring S2.

Table 1: Mooring locations on the Scotian Rise and Slope. The letter S stands for the Shelf Break experiment, and R stands for the Risex experiment.

Mooring	Latitude	Longitude	Bottom depth	Current meter depths
S1	42°48.6′N	63°30.0′W	250m	20,50,100,150,230m
S2	42°46.8′N	64°00.0′W	250m	20,50,230m
S3	42°45.0′N	63°30.0′W	710m	230,500,690m
S4	42°40.4′N	63°30.0′W	1010m	50,100,150,500,690,990m
S5	42°30.5′N	63°30.0′W	1550m	50,150,1530m
S6	43°00.5′N	63°30.0′W	170m	20,50,100,150m
S7	42°41.7′N	64°00.0′W	710m	230,690m
S8	42°01.0′N	63°30.0′W	2550m	70,1500,2530m
R1	41°20.0′N	63°58.0′W	3600m	200,500,1000,2900,3500m
R2	41°27.0′N	63°30.0′W	3600m	3500m
R3	41°00.0′N	65°00.8′W	3600m	3500m
R4	41°38.6′N	64°17.7′W	3000m	200,1000,2900m
R5	41°53.2′N	64°31.7′W	2500m	200,500,1000,2400m

Table 2: Mooring deployment periods on the Scotian Rise and Slope. Periods A to G are from the Shelf Break Experiment, and periods H to J are from the Risex Experiment.

Mooring deployment period	Deployment date (dd/mm/yy)	Recovery date (dd/mm/yy)	Length of deployment (days)	Comments
A	13/12/75	06/04/76	115	Mooring S1 only.
B	06/04/76	05/07/76	90	S1 and S4 only.
C	05/07/76	17/10/76	104	
D	17/10/76	16/12/76	60	
E	16/12/76	02/04/77	107	
F	02/04/77	09/07/77	98	
G	09/07/77	04/01/78	179	Data from S1, S2,S4,S6 only.
H	04/11/80	03/03/81	119	Poor data return, except for R5.
I	03/03/81	06/05/81	64	Poor data return.
J	06/05/81	24/10/81	171	Good data return, except for R5.

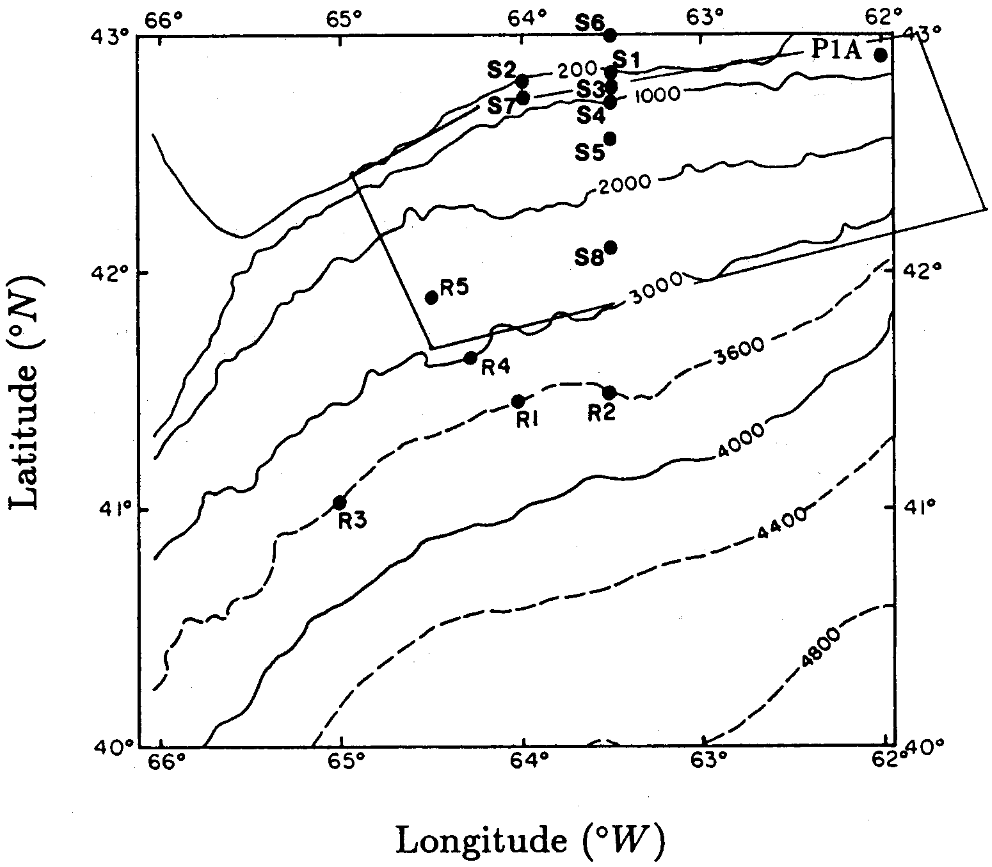

Figure 6: Map of the Scotian Rise and Slope showing the location of moorings used in this paper. The letter S refers to moorings from the 1975–78 Shelf Break experiment, R refers to moorings from the 1980–81 Risex Experiment, and mooring P1A is Petrie's (1975) mooring 1A. The hydrographic data used to compute the Brunt-Vaisala frequency $N(z)$ comes from subarea 33 of Drinkwater and Trites (1987), whose perimeter is shown here as a thin line.

Criterion 1b was also used by Eriksen (1982) at a few moorings. It can be used at most of the moorings considered here, as the majority of them have two or more current meters in the vertical. However, we should bear in mind that this criterion fails to take into account the slantwise propagation of internal wave energy, as pointed out by Gilbert and Garrett (1989), and so may be ambiguous. In all cases where we have used criterion 1b, the kinetic energy spectra from instruments at different depths were normalised with respect to a common value of N, taking into account WKB scaling (Briscoe, 1975, Gill 1982, p.300).

Criterion 1c was used by Thorpe (1987a, figure 11) at a site where the critical frequency was close to the inertial frequency ($\omega_c \approx f$), a situation where this criterion should be most useful due to nearly horizontal wave rays. It was used in a few cases for the present data set.

Criterion 1d does not on its own constitute evidence of critical internal wave reflection, but it could be used to support successful tests based on other criteria, especially when $\omega_c > M_2$, where there is some degree of universality in the spectral levels (Wunsch, 1976). Fu (1981) has shown that such universality does not exist near f in the deep ocean, so that criterion 1d was not used at moorings where $\omega_c \sim f$. It was not used at moorings where $\omega_c \sim M_2$ either, since internal tides were excluded from the internal wave model of Garrett and Munk (1972). Application of criterion 1d was thus limited to mooring S2 for the present data set.

Criterion 2 was successfully used by Eriksen (1982) for mooring sites on Muir Seamount and the New England continental slope, but failed at mooring 636 of the Western Boundary Sill Experiment. Application of criterion 2 was limited here to those moorings where ω_c was appreciably greater than f (near-inertial motions are quasi-circular and so the orientation of their 'major' axis tends to be less stable).

Hundreds of autospectra were computed from the data set described in tables 1 and 2, some of which are shown in Gilbert (1990). A summary of his results is given in table 3.

Table 3: Results of tests aimed at determining whether critical internal wave reflection was observed at moorings from the Shelf Break and Risex Experiments. In column 1 the critical frequency (in cph) at each of the mooring sites is written in parentheses. For the purpose of comparison, note that $f \sim 0.056$ cph, and $M_2 = 0.081$ cph. In columns 2 and 3, the capital letters refer to the mooring periods listed in table 2, and the numbers in parentheses refer to the criteria described in this section.

Mooring	Successful tests (95% significant)	Unsuccessful tests (not significant)	Comments
S1(.086)		D(2),E(2),G(2)	
S2(.132)	D(1d,2),E(1d,2), G(1d,2)	D(1a),E(1a), G(1a)	
S3(.076)	E(2),F(2)	D(1b,2),E(1b,1c), F(1b,1c)	
S4(.060)	C(1b),E(1b)	F(1b)	Large bump at $\approx 1000m$
S5(.058)	C(1b,1c),F(1b)	D(1b,1c),F(1c)	
S6(.059)		F(1c)	
S7(.076)	C(1b,2),E(2), F(1b,2)	D(1b,2),E(1b)	
S8(.057)		F(1b)	sig. at 80% level
R1(.055)		J(1b)	K.E.(3500m) > K.E.(2900m)
R2(.056)			K.E.($\omega = f$) $\sim$ R1 (3500m)
R3(.055)			K.E.($\omega = f$) $\sim$ R1 (3500m)
R4(.056)	J(1c)	J(1b)	
R5(.057)		H(1b)	

5 SUMMARY AND DISCUSSION

The WBSE data

In section 3 of this paper, we reexamined data from mooring 636 of the Western Boundary Sill Experiment to try to verify whether the energetic 3–4 day motions seen at that mooring are due to critical internal wave reflection, as postulated by Eriksen (1982). We pointed out, as Eriksen did, that those motions are parallel to the inferred orientation of the isobaths, not normal to them, thus violating one of the predictions of specular internal wave reflection theory.

We also pointed out that the 3–4 day motions are coherent over a horizontal scale of at least 40 km (Fig. 4). Such a large coherence scale appears to be inconsistent with the transfer of energy to smaller scales that should occur due to internal wave reflection. Gilbert (1990) also showed that the 3–4 day motions at mooring 636 are coherent over a vertical scale of at least 200m, and the measured phase leads or lags are generally not significantly different from zero over that distance, implying large vertical scales of motion.

Another puzzling feature of the 3–4 day motions at mooring 636 is that they do not satisfy the consistency relation (10) for freely-propagating internal waves (Fig. 5). At 10m and 50m above the bottom, those motions are essentially rectilinear, apparently more consistent with the prediction of Rhines (1970) for internal edge waves (11). Agreement with Rhines' theory is quite limited however, as the orientation of those quasi-rectilinear motions does not change with frequency according to (12) (Gilbert, 1990).

Perhaps this illustrates the need for a careful investigation of the manner in which the consistency relations for internal waves should change as we approach a sloping bottom. Let us consider here the inviscid problem of a wave which reflects off a vertical wall with $\phi_i \neq 0$. The onslope velocity component of the incident and reflected waves must cancel each other at the wall in order to satisfy the boundary condition of no normal flow, but the alongslope velocity component of the incident and reflected waves are phase-locked at the wall and add up constructively. Consequently, in the immediate vicinity of a vertical wall, motions should be rectilinear, and we should have $P_{++}/P_{--} = 1$ instead of $P_{++}/P_{--} = [(\omega - f)/(\omega + f)]^2$ (Barry Ruddick, personal communication). We need to verify whether this result would still hold above sloping topography, as it may help explain the variation of P_{++}/P_{--} with height above the bottom [5] observed at mooring 636 for example (Fig. 5). Preliminary results suggest that a slope 2 to 3 times steeper than 0.015 might be consistent with Fig. 5, but that would leave the energy peak at $\omega \approx 0.012$ cph shown on Fig. 2 unexplained.

Our lack of success in trying to interpret the 3–4 day motions at mooring 636 is largely due to our poor knowledge of the topography. Figure 1 is only based on an East–West

[5] Mooers (1973) suggests that departure from the ratio (10) may be a good indicator of the validity of the low spatial coherence hypothesis of Fofonoff (1969).

transect, and so cannot tell us what is the orientation and magnitude of ∇h. The most detailed bathymetric map available for the area, produced by Moody et al. (1979) and shown on figure 2 of WW82, does not help us solve that problem either due to its lack of resolution.

Proximity to the equator ($f \sim 10^{-5}s^{-1}$ at 4°N) and the possibility of equatorially trapped waves (Moore and Philander, 1977, Eriksen, 1980) may also complicate the analysis, a fact that was overlooked by Gilbert (1990) and probably deserves closer attention. Among other things, the traditional approximation of neglecting the horizontal component of the Earth's rotation vector $\mathbf{\Omega}$ in the equations of motion may have to be reexamined near the equator.

The Scotian Slope data

In section 4, historical current meter data from the Scotian Rise and Slope were used in order to look for evidence of critical internal wave reflection. To that end, a set of criteria was proposed and discussed, and then applied to the data. Energy enhancement and/or cross-isobath alignment of motions near ω_c was found to be significant at the 95% level in some cases (see table 3), but was generally not very pronounced. This could be due to the overall concavity of the Scotian Rise and Slope (Gilbert and Garrett, 1989), but other factors may also affect the likelihood of observing energy enhancement at the critical frequency.

Gilbert (1990) suggests that when $N \sin \alpha \ll f$ at a given mooring site, so that $\omega_c \sim f$ (3), the orientation of the isobaths could be one such factor. The linear reflection laws of Eriksen (1982), valid on an f-plane, predict that maximum wavenumber amplification upon reflection should occur for onslope incident energy propagation (i.e. $\phi_i = 0$). However on a β-plane, due to the turning latitude effect, near-inertial motions are more likely to have $\phi_i \approx \pm\pi/2$ when the onslope direction is poleward (e.g. Kroll, 1975). This is roughly the case on the Scotian Slope, possibly explaining the overall absence of pronounced energy enhancement at $\omega_c \sim f$ [6] (note that $\omega_c \approx f$ at 9 of the 13 moorings in table 3). A more quantitative investigation of this phenomenon could probably be carried out using the wave functions of Munk and Phillips (1968).

Finally we would like to draw the attention of the reader to the non-linear, specular reflection theory of Thorpe (1987b), which raises the possibility of singularities for wave ray slopes different from the bottom slope. When a train of finite amplitude internal waves travelling in a vertical plane normal to the slope gets reflected, resonance between the incident and reflected waves is possible at second order for a bottom slope less than 0.15 and a wave ray slope less than 0.58. This condition is met almost everywhere on the Scotian Rise and Slope for example. Gilbert (1990) offered an explanation for the unusual

[6] Fu (1981) found that the largest energy peaks at $\omega = f$ in the abyssal ocean occured in the vicinity of the Mid-Atlantic Ridge, which runs roughly North-South, so that the onslope direction is zonal and near-inertial waves are more likely to have $\phi_i \approx 0$.

size of the M_4 peak seen at Petrie's (1975) mooring 1A (see Fig. 6) in terms of that theory, and also explained some aspects of the observations at moorings S3 and S7 using Thorpe's theory. Unlike the first order resonance, higher order resonances should be unaffected by the boundary concavity criterion of Gilbert and Garrett (1989).

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REFERENCES

Briscoe, M.G. 1975. Preliminary results from the trimoored Internal Wave Experiment (IWEX). *J.Geophys.Res.*, **80**: 3872–3884.

Drinkwater, K.F. and R.W. Trites, 1987. Monthly means of temperature and salinity in the Scotian Shelf region., *Canadian technical report of fisheries and aquatic sciences No.1539*, iv + 101p.

Eriksen, C.C. 1980. Evidence for a continuous spectrum of equatorial waves in the Indian Ocean. *J. Geophys. Res.*, **85**: 3285–3303.

Eriksen, C.C. 1982. Observations of internal wave reflection off sloping bottoms. *J. Geophys. Res.*, **87**: 525–538.

Eriksen, C.C. 1985. Implications of ocean bottom reflection for internal wave spectra and mixing. *J. Phys. Oceanogr.*, **15**: 1145–1156.

Fofonoff, N.P. 1969. Spectral characteristics of internal waves in the ocean. *Deep-Sea Res.*, **16**: 58–71.

Fu, L.L. 1981. Observations and models of inertial waves in the deep ocean. *Rev. Geophys. Space Phys.*, **19**: 141–170.

Garrett, C.J.R. and W.H. Munk 1972. Space-time scales of internal waves. *Geophys. Fluid Dyn.*, **2**: 225–264.

Garrett, C.J.R. and D. Gilbert, 1988. Estimates of vertical mixing by internal waves reflected off a sloping bottom. In *Small-scale turbulence and mixing in the ocean.*, J.C.J. Nihoul and B.M. Jamart (editors), Elsevier, Amsterdam, pp. 405–423.

Gilbert, D. 1990. Theory and observations of internal wave reflection off sloping topography. *Ph.D. Thesis,* Dalhousie University, xix + 183 pp.

Gilbert, D. and C.J.R. Garrett 1989. Implications for ocean mixing of internal wave scattering off irregular topography. *J. Phys. Oceanogr.*, **19**: 1716–1729.

Gill, A.E., 1982. *Atmosphere-Ocean Dynamics.* Academic Press, New York, 662 pp.

Haurwitz, B., H. Stommel and W.H. Munk, 1959. *On the thermal unrest in the ocean. In* The atmosphere and sea in motion, Rossby Memorial Volume, B. Bolin (editor), Rockefeller Univ. Press, pp. 74–94.

Huthnance, J.M. 1978. On coastal trapped waves: analysis and numerical calculation by inverse iteration. *J.Phys.Oceanogr.*, **8**: 74–92.

Huthnance, J.M. 1989. Internal tides and waves near the continental shelf edge. *Geophys. Astrophys. Fluid Dynamics*, **48**: 81–106.

Huthnance, J.M. and P.G. Baines 1982. Tidal currents in the Northwest African upwelling region. *Deep-Sea Res.*, **29**: 285–306.

Jenkins, G.M. and D.G. Watts, 1968. *Spectral analysis and its applications.*, Holden-Day, Oakland, California, 525 pp.

Kroll, J. 1975. The propagation of wind-generated inertial oscillations from the surface into the deep ocean. *J.Mar.Res.*, **33**: 15–51.

Lively, R.R. 1979a. Current meter and meteorological observations on the Scotian Shelf: December 1975 to January 1978. Volume 1: December 1975 to December 1976. Bedford Institute of Oceanography data series report BI–D–79–1, 280 pp.

Lively, R.R. 1979b. Current meter and meteorological observations on the Scotian Shelf: December 1975 to January 1978. Volume 2: December 1976 to January 1978. Bedford Institute of Oceanography data series report BI–D–79–1, 368 pp.

Lively, R.R. 1984. Current meter observations on the Scotian Rise for November 1980 to October 1981. *Canadian technical report of hydrography and ocean sciences no. 50,* 156 pp.

Louis, J.P., B.D. Petrie and P.C. Smith 1982. Observations of topographic Rossby waves on the continental margin off Nova Scotia. *J.Phys.Oceanogr.*, **12**: 47–55.

Moody, R., D.E. Hayes and S. Connary 1979. Bathymetry of the continental margin of Brazil, *Map number 832,* Am. Assoc. Petrol. Geol., Tulsa, Oklahoma.

Mooers, C.N.K. 1973. A technique for the cross spectrum analysis of pairs of complex-valued time series, with emphasis on properties of polarized components and rotational invariants. *Deep-Sea Res.*, **20**: 1129–1141.

Moore, D.W. and S.G.H. Philander 1977. Modeling of the tropical oceanic circulation, in *The Sea,* vol. 6, pp. 319–361, Interscience, New York.

Müller, P., D.J. Olbers and J. Willebrand 1978. The Iwex spectrum *J.Geophys.Res.*, **83**: 479–500.

Munk, W.H. 1966. Abyssal recipes. *Deep-Sea Res.*, **13**: 207–230.

Munk, W.H. and N. Phillips 1968. Coherence and band structure of inertial motion in the sea. *Rev.Geophys.Space Phys.*, **6**: 447–472.

Petrie, B. 1975. M2 surface and internal tides on the Scotian shelf and slope. *J. Mar. Res.*, **33**: 303–323.

Phillips, O.M. 1963. Energy transfer in rotating fluids by reflection of inertial waves. *Phys. Fluids*, **6**: 513–520.

Phillips, O.M., 1966. *The Dynamics of the Upper Ocean.* Cambridge University Press, New York, 261pp.

Phillips, O.M., 1977. *The Dynamics of the Upper Ocean. 2nd ed.,* Cambridge University Press, New York, 336 pp.

Rhines, P.B. 1970. Edge-, Bottom-, and Rossby waves in a rotating stratified fluid. *Geophys. Fluid Dyn.*, **1**: 273–302.

Sandstrom, H., 1966. The importance of topography in generation and propagation of internal waves. *Ph.D. thesis,* University of California at San Diego, xi + 105 pp.

Sandstrom, H. 1969. Effect of topography on propagation of waves in stratified fluids. *Deep-Sea Res.*, **16**: 405–410.

Smith, P.C. and B.D. Petrie 1982. Low-frequency circulation at the edge of the Scotian Shelf. *J. Phys. Oceanogr.*, **12**: 28–46.

Thompson, R.O.R.Y. and J.R. Luyten 1976. Evidence for bottom-trapped topographic Rossby waves from single moorings. *Deep-Sea Res.*, **23**: 629–635.

Thorpe, S.A. 1987a. Current and temperature variability on the continental slope. *Phil. Trans. R. Soc. London, Series A*, **323**: 471–517.

Thorpe, S.A. 1987b. On the reflection of a train of finite amplitude internal gravity waves from a uniform slope. *J. Fluid Mech.*, **178**: 279–302.

Thorpe, S.A., P. Hall and M. White 1990. The variability of mixing at the continental slope. *Phil.Trans.R.Soc.Lond.A*, **331**: 183–194.

Whitehead, J.A. and L.V. Worthington 1982. The flux and mixing rates of Antarctic Bottom Water within the North Atlantic. *J. Geophys. Res.*, **87**: 7903–7924.

Wunsch. C. 1972. The spectrum from two years to two minutes of temperature fluctuations in the main thermocline at Bermuda. *Deep-Sea Res.*, **19**: 577–593..

Wunsch, C. 1976. Geographical variability of the internal wavefield: A search for sources and sinks. *J. Phys. Oceanogr.*, **6**: 471–485.

Wunsch, C. and S. Webb 1979. The climatology of deep ocean internal waves. *J. Phys. Oceanogr.*, **9**: 235–243.