PARADIGM LOST?

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ABSTRACT

It appeared some years ago that a number of theoretical and observed features of the internal wave field can be reconciled with a diapycnal diffusivity of no more than about 10^{-5} m^2s^{-1}. This "paradigm" is re–examined here, with particular attention to the arguments concerning the alleged universality of internal waves in space and time. A key question concerns the influence of nonlinear interactions on horizontal spreading, but the simple picture still seems reasonable for the main thermocline. However, there are clearly significant departures from it in various regions, such as the upper ocean or close to the sea floor. The influence of a sloping bottom on both diapycnal fluxes and the velocity boundary condition for low frequency flows is emphasized; some general theoretical results are presented but there is a need for further observations. Finally, a simple model for the mixing by, and decay of, an interfacial solitary wave is summarised.

1. INTRODUCTION

In January 1984 I had the pleasure of attending the Third 'Aha Huliko'a Hawaiian Winter Workshop on "Internal Gravity Waves and Small-Scale Turbulence". Figure 1 shows my attempt then to present a "zeroth order view" of the effect of internal waves on the ocean interior, summarising and perhaps oversimplifying the theoretical and observational progress made by many people. Seven years later it seems worthwhile to ask whether this paradigm should be abandoned or merely qualified.

Section 2 thus briefly reviews and extends the arguments leading to Figure 1, pointing out their weaknesses, but suggesting that the synthesis may still be appropriate for the main thermocline. It probably is not relevant, however, near the sea surface or sea floor. The role of the bottom boundary in determining the spectral shape of the internal wave spectrum is discussed briefly in Section 3. Diapycnal mixing by shear instability of bottom reflected internal waves, and the way in which this may control the velocity boundary condition for low frequency flows, is also reviewed; the need for more observational work near sloping boundaries is emphasized. Returning to the upper ocean, Section 4 briefly presents a simple model for the shear-induced decay of an internal solitary wave. Section 5

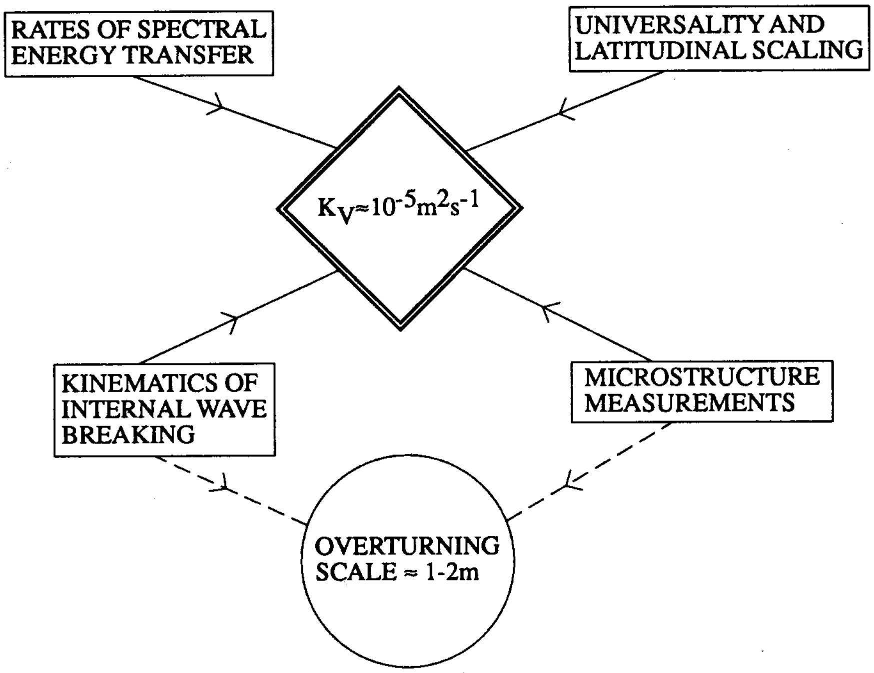

Figure 1: Schematic of a zeroth order view of various features of internal waves and turbulence that seem to be consistent with a diapycnal diffusivity of about 10^{-5} m^2s^{-1}. (From Garrett, 1984).

is a personal impression of topics that need to be emphasized in future research.

2. THE PARADIGM

The purpose of Figure 1 was to demonstrate that a number of theoretical and observational properties of internal gravity waves in the main thermocline are compatible with a vertical eddy diffusivity there of no more than about 10^{-5} m^2s^{-1}. Specifically:

(*i*) The rate of energy transfer through a typical internal wave spectrum has been estimated by McComas and Müller (1981b) using weak interaction theory and by Henyey *et al.* (1986) with a theory based on ray tracing. As pointed out by Gregg (1989), the two theories give essentially the same form E^2N^2 for the functional dependence of the energy flux on buoyancy frequency N and total internal wave E, but differ by a factor of nearly 7. Both theories depend on assumptions of limited validity; it remains to be established whether they are complementary, by applying in different regions of wavenumber-frequency space, and whether the factor of 7 discrepancy is an artifact of different, but hidden, assumptions.

The calculations of McComas and Müller (1981b) have been criticised for violating the assumption of weak nonlinearity (Holloway, 1980), but the assumption does seem to be valid for the regions of the spectrum at low frequency and low vertical wavenumber which, after all, contain most of the internal wave energy. Thus, provided that these regions are not affected by back fluxes from the much less energetic regions where the assumption breaks down, it does seem reasonable to accept the McComas and Müller (1981b) estimates of the energy transfer to high wavenumber and ultimately to dissipation ϵ. For typical spectral levels, and with the increase in potential energy of the basic state being 20 to 25% of ϵ (Osborn, 1980; Oakey, 1982), the McComas and Müller (1981b) results then lead to a depth-independent K_V of less than $2 \times 10^{-5}\ \mathrm{m^2s^{-1}}$ (following Gregg (1989) in allowing for the energy transfer from low modes by both induced diffusion and parametric subharmonic instability).

(*ii*) Gregg's (1989) comparison of direct measurements of ϵ at various locations and depths led him to adopt a parameterisation for ϵ similar in form to those of McComas and Müller (1981b) and Henyey *et al.* (1986), but a factor of about 3 smaller than obtained by the former and a factor of 2 more than obtained by the latter. This in turn led to K_V less than about $5 \times 10^{-6}\ \mathrm{m^2s^{-1}}$ for typical internal wave energy levels.

(*iii*) For $K_V \simeq 10^{-5}\ \mathrm{m^2s^{-1}}$, with $\epsilon = 4$ to $5K_VN^2$ per unit mass and with an e-folding depth of 1,300 m for N, the typical vertically integrated internal wave energy level of about 4,000 $\mathrm{Jm^{-2}}$ is dissipated or converted to mean potential energy in about 40 to 50 days. A decay time of this order or even longer is compatible with the rather small variability in internal wave energy levels in space or time. If, for example, we think of the internal wave energy level E in an area as being governed by the equation

$$dE/dt + E/\tau = F \qquad (1)$$

where τ is the decay time and F some exterior forcing, then the response to $F = F_0 + F_1 \cos\omega t$ is

$$E = \tau[F_0 + F_1(1+\omega^2\tau^2)^{-1/2}\cos\omega(t-t_0)] \qquad (2)$$

where the time lag $t_0 = \omega^{-1}\tan^{-1}\omega\tau$. This may have some relevance to the finding by Briscoe and Weller (1984) and Briscoe (1984) of a lag of 2 to 3 months between internal wave energy levels and seasonal variations in wind stress. Taking $t_0 = 75$ days and $\omega = 2\pi/365$ days implies $\tau = 200$ days, somewhat longer than the 50 days or so (requiring $t_0 = 40$ days) corresponding to $K_V = 10^{-5}\ \mathrm{m^2s^{-1}}$ but not much more than the 100 days (requiring $t_0 = 60$ days) or more associated with Gregg's (1989) value of less than 5×10^{-6} m^2s^{-1} for K_V.

Equation (2) also implies that the variation of E about its mean should be less than the variation of the forcing by a factor $(1+\omega^2\tau^2)^{-1/2}$. For $\tau = 50, 100, 200$ days this factor is 0.76, 0.52, 0.28. Briscoe's (1984) North Atlantic data (his Figures 6b and 8) do suggest that the internal wave energy has less variation than the monthly average wind stress which is assumed to be the driving mechanism; a factor of about 0.4 seems appropriate, corresponding to a decay time of a bit more than 100 days.

Any further discussion of this should be regarded as over-interpretation, but it does seem possible that the time lag and variability of the internal wave energy level is associated with a decay time of about 100 days. Briscoe (1984) suggested that the lag represents the time for wind-forced mesoscale eddies to pass on their energy to the internal wave field, but has later also recognised the connection with the decay time of the internal wave spectrum (Mel Briscoe, personal communication) .

It has also been suggested in the past that the tendency for internal wave energy levels to be similar from place to place, in spite of spatially variable forcing, is a consequence of the rather long decay time which permits significant horizontal spreading of a patch of high energy. An important unpublished paper by Cox and Johnson (1978) discussed the way in which the spread of a patch of high internal wave energy is a diffusive process, with a horizontal diffusivity K_H proportional to the time for horizontal anisotropy to be removed by wave-wave interactions. An appropriate model in this situation, for forcing that varies in strength with wavenumber l in the north-south direction y, is

$$K_H d^2E/dy^2 - E/\tau + F_0 + F_1 \cos ly = 0, \tag{3}$$

with τ the internal wave decay time as before. The response is

$$E = \tau[F_0 + (1 + K_H\tau l^2)^{-1} F_1 \cos ly] \tag{4}$$

so that variations in forcing are smeared out if $K_H\tau l^2 \gg 1$.

For Cox and Johnson's (1978) estimate of 1.4×10^4 m^2s^{-1} for K_H, taking $\tau = 100$ days and with $l = 1.9 \times 10^{-6}$ m^{-1}, as for the 30° north-south wavelength of wind stress in the Atlantic (Briscoe, 1984), the factor $(1 + K_H\tau l^2)^{-1} = 0.70$. Thus significant variations in E, as reported by Briscoe (1984), should be observed, but this conclusion is highly sensitive to K_H and hence to the time for horizontal anisotropy to be removed. Cox and Johnson (1978) took this to be 72 hours for the frequency of $1.3f$ contributing the most to their integral expression for K_H, but further analysis is long overdue.

It is also quite likely that it is dangerous to lump together all the frequencies in the inertio-internal wave spectrum. The seasonal variations reported by Briscoe and Weller (1984) and Briscoe (1984) were for high frequency internal waves (either $\omega > 1.15f$ or $\omega > 0.1$ cph in different analyses); the inertial band energy at the LOTUS site discussed by Briscoe and Weller (1984) showed a more complicated time dependence, with occasional bursts of high energy.

Spatially, Fu (1981) found that the inertial peak in the abyssal North Atlantic over smooth topography scaled as if it had originated as super–inertial energy at a lower latitude (though elsewhere it had a component due to local generation). This seems to imply that the wave does not lose much of its energy as it travels to its turning latitude, is internally reflected and travels back equatorward again. Taking y northward, with $y = 0$ at the latitude where $f = \omega$, then $f = \omega + \beta y$ with $\beta = \omega R^{-1} \cot\phi$, R being the earth's radius and ϕ the latitude. A wave of frequency ω and horizontal wave number then proceeds towards its turning latitude at a speed $\partial\omega/\partial l$, where the dispersion relation near the inertial frequency f is approximately

$$(\omega^2 - f^2)^{1/2} = (k^2 + l^2)^{1/2} c_j, \quad c_j = N_0 b (j\pi)^{-1} \tag{5}$$

for modenumber j in an ocean with a buoyancy frequency that is N_0 near the surface and decreases exponentially downwards with a scale depth b (e.g. Garrett and Munk, 1972). Hence, within the limits of ray tracing,

$$dy/dt = \partial\omega/\partial l \approx (2\beta/\omega)^{1/2}\, c_j(-y + y_0)^{1/2} \tag{6}$$

where $y_0 = -c_j^2 k^2 (2\omega\beta)^{-1}$ defines the turning latitude for east-west wavenumber k. Assuming that this WKB approximation is adequate to estimate the travel time T from some position $-Y$ to $-y_0$, then

$$T = [2R(Y - y_0)\tan\phi]^{1/2} c_j^{-1}. \tag{7}$$

If we take $Y - y_0 = 1{,}000$ km, $\phi = 45°$, $N_0 = 5.2 \times 10^{-3}\ \mathrm{s}^{-1}$ and $b = 1{,}300$ m, then $T = 20j$ days. This suggests that any processes that would tend to remove the inertial peak in the spectrum cannot do so in less than several tens of days.

On the other hand, as also recognised by Eric Kunze (personal communication), the latitudinal scaling of spectra is a two-way process; perhaps some of the energy at frequencies above f originates as inertial energy at a more poleward location. In fact, the higher-than-WKB inertial peak found by Fu (1981)in some locations suggests that the extra energy must be lost rather quickly.

To summarise this discussion:

(a) The temporal variability of the typical high frequency internal wave field suggests a decay time of order 100 days, with K_V no more than 10^{-5} m^{2}s^{-1}.

(b) Spatial variability of annual average high frequency energy levels can occur even if the decay time of the energy is 100 days or so, though this conclusion is strongly dependent on the poorly-known rate at which horizontal anisotropy is removed from a spectrum.

(c) To the extent to which the inertial peak is a latitudinal turning point effect, it also seems to imply a lifetime of several tens of days. However, regions with more elevated peaks must lose their energy in a time less than this.

(*iv*) The last main feature of Figure 1 concerns the kinematics of breaking waves. Garrett (1984) argued that typically observed shear spectra in the ocean would lead to shear instability and overturning on a vertical scale of order 1 m (as typically observed; e.g. Gregg, 1989) and that, if mixing events occur no more frequently than shear maxima, then again K_V is no more than about 10^{-5} m^{2}s^{-1}. These arguments seem as valid now as they did in 1984, and it does seem appropriate to point out that the estimate of the vertical scale was based on integral properties of the whole shear spectrum rather than on the vertical wavenumber of some particular kink in the spectrum!

In conclusion, to the extent to which Figure 1 represents a paradigm, it still seems reasonably valid for the ocean interior away from the surface or bottom topography where bursts of inertial energy may lead to a local increase in K_V. As before, though, it largely remains as a reminder of some of the different conceptual elements which need to be considered in developing any unified picture of the mixing produced by internal waves. It must also be remembered that processes other than internal wave breaking can lead to diapycnal mixing.

3. THE ROLE OF THE BOTTOM BOUNDARY

3.1 Effect on the Spectral Shape.

Most discussions of the internal wave wavenumber-frequency spectrum assume that it is determined by some combination of generation (most likely at the sea surface by wind) and nonlinear interaction in the ocean interior. Calculations by Eriksen (1982), Rubenstein (1988), Garrett and Gilbert (1988) and Xu (1990), however, show that reflection off bottom topography can scatter energy to higher wavenumber (though the frequency of each wave is unchanged). Thus waves which can reach the sea floor in a time short compared with their interaction time may have their energy scattered to higher wavenumbers, at the same frequency, sooner than they lose energy to other frequencies and wavenumbers by nonlinear interactions.

The vertical component of the group velocity of a wave of frequency ω and vertical wavenumber m is

$$c_{gz} = \left(\frac{N^2-\omega^2}{N^2-f^2}\right)\left(\frac{\omega^2-f^2}{\omega m}\right) \quad (8)$$

$$\simeq \left(\frac{\omega^2-f^2}{\omega m}\right) \quad \text{for } \omega \ll N \quad (9)$$

with $m \simeq j\pi N(bN_0)^{-1}$ in an ocean with an exponential N profile of scale height b (Munk, 1981), where j is the mode number. If b is significantly less than the ocean depth, the vertical travel time of waves with frequency less than N at the bottom is then $\pi\omega j(\omega^2-f^2)^{-1}$, or $\pi j\omega^{-1}$ away from f.

Now McComas and Müller (1981a, b) showed that, for a typical internal wave spectrum, the time scale for parametric subharmonic instability to drain energy from low modes with $\omega > 2f$ is given by $(0.27Em_*mN^{-2})^{-1}\omega^{-1}$ where m_* is the vertical wavenumber bandwidth. Taking $m = j\pi N(bN_0)^{-1}$, $m_* = j_*\pi N(bN_0)^{-1}$, $j_* = 3$, $N_0 = 5\times10^{-3}\ \mathrm{s}^{-1}$, $b = 1300$ m and $E = 2\pi\times10^{-5}b^2N_0N$, this time scale becomes $2\times10^3(N_0/N)j^{-1}\omega^{-1}$. Ignoring the depth dependence here by taking $N = N_0$, we see that this interaction time is greater than the vertical travel time $\pi j\omega^{-1}$ if $j < 25$.

Thus for the energetic low modes at frequencies greater than $2f$, interactions with bottom topography might be as significant as interactions with other waves in the water column. Conceivably the tendency for non-inertial waves to have a less red vertical wavenumber spectrum than inertial waves (Tom Sanford, this volume) may reflect this, although such a

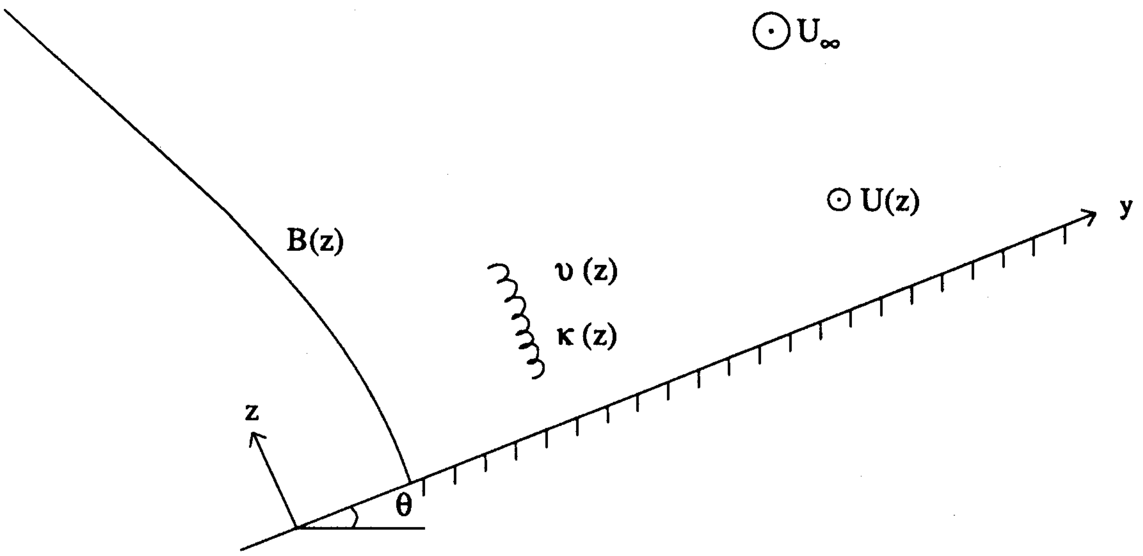

Figure 2: Definition sketch for mixing near a sloping boundary.

tendency may also be a consequence of nonlinear wave interactions (McComas and Müller, 1981a).

3.2 Mixing by Internal Waves Breaking Near a Slope

Figure 2 presents a schematic of a plane bottom slope at which internal waves are reflected, mainly into shorter vertical wavelengths with higher energy (e.g. Eriksen, 1982). This may well lead to shear instability and mixing close to the boundary (Eriksen, 1985; Garrett and Gilbert, 1988), though less so in concave regions (Gilbert and Garrett, 1989). It is assumed that the subsequent eddy fluxes of momentum and buoyancy can be represented by eddy viscosity and diffusivity profiles $\nu(z)$, $\kappa(z)$ as functions of the bottom-normal coordinate z. (This does not assume that a mixing length theory is valid; ν and κ are the eddy fluxes divided by the mean gradients). The mean buoyancy $B(z)$ is reduced near the boundary by the mixing, but $\partial B/\partial z \to N^2 \cos\theta$ as $z \to \infty$ with N^2 the constant interior stratification. In the upslope direction $\partial B/\partial y = N^2 \sin\theta$ for all z as the problem is assumed independent of y to lowest order.

The physics and mathematics of this boundary mixing problem are discussed by Phillips *et al.* (1986), Thorpe (1987) and Garrett (1990) amongst others. Here I wish merely to draw attention to some key factors that suggest the need and opportunities for observational work in this region.

Garrett (1990) showed that the vertical buoyancy flux is the value $\int_0^\infty \kappa(z)N^2 dz/\sin\theta$ that it would take if the stratification was unchanged by the mixing, multiplied by an "effectiveness"

$$I = \int_0^\infty \kappa(z) \left[\sin^2\theta + \left(\frac{\partial B/\partial z}{N^2 \cos\theta} \right)^2 \cos^2\theta \right] dz \Big/ \int_0^\infty \kappa dz \tag{10}$$

This is 1 if $\partial B/\partial z = N^2 \cos\theta$ throughout the boundary layer but is reduced if $\partial B/\partial z < N^2 \cos\theta$. Due to secondary circulation the reduction factor in the second term in (10) is the square of what it would be due to reduced stratification alone. Now if ν_0, κ_0 are the eddy coefficients close to the boundary, then $\partial B/\partial z$ is very small over a thickness of order h_{mixed} where

$$h_{\text{mixed}} = \left(\frac{f^2}{\nu_0^2} + \frac{N^2 \sin^2\theta}{\nu_0 \kappa_0} \right)^{-1/4} \tag{11}$$

$$= h_{\text{Ekman}}(1 + S\ Pr)^{-1/4}. \tag{12}$$

Here $h_{\text{Ekman}} = (\nu_0/f)^{1/2}$ is an Ekman depth, $S = N^2 \sin^2\theta/f^2$ is the Burger number based on the ocean interior stratification and the bottom slope and $Pr = \nu_0/\kappa_0$ is the eddy Prandtl number close to the boundary.

If significant mixing dies away outside a boundary layer of thickness approximately h_{mixed}, then the boundary mixing effectiveness from (10) is small since $\partial B/\partial z$ is small where κ is large and vice versa. If, however, vigorous mixing extends well into the region above h_{mixed}, where buoyancy driven flows can restore the stratification, then I can be significant. This may be the case for internal wave breaking. The stratification is, of course, tied to the values of $\nu(z)$, $\kappa(z)$. Garrett (1991) shows that if $\nu(z)$, $\kappa(z)$ fall off to very small values over a distance much greater than h_{mixed}, but assuming $\nu/\kappa = Pr$ =constant for simplicity, then, for $z \gtrsim h_{\text{mixed}}$,

$$\partial B/\partial z \simeq N^2 \cos\theta \left(\frac{S\ Pr}{1 + S\ Pr} \right). \tag{13}$$

The nature of this region is that both the thermal wind equation for the along-slope flow and an advective (upslope)/diffusive (bottom–normal) balance for buoyancy are satisfied. For this solution, (10) gives

$$I \simeq \left(\frac{S\ Pr}{1 + S\ Pr} \right)^2 \tag{14}$$

so that the mixing is particularly effective if $S\ Pr$ is not small.

3.3 The Alongslope Flow

Steady state boundary mixing theories also lead to formulae for the alongslope mean flow $U(z)$. For arbitrary profiles of $\nu(z)$, $\kappa(z)$ it can be shown (Garrett, 1990) that

$$U(z) = \int_0^z f \cot\theta [\kappa_\infty - \kappa(\partial B/\partial z)/N^2 \cos\theta] \nu^{-1} dz \tag{15}$$

where κ_∞ is the (small) value of κ as $z \to \infty$. The bottom-normal buoyancy gradient $\partial B/\partial z$, which tends to $N^2 \cos\theta$ as $z \to \infty$, is, of course, part of the solution. For $Pr = \nu(z)/\kappa(z) =$ constant and with mixing, as before, extending over a distance H from the boundary much greater than h_{mixed}, (15) shows that as $z \to \infty$, $U(z) \to U_\infty$ given by

$$U_\infty \simeq -fH \cot\theta \left(\frac{S}{1 + S\ Pr}\right). \tag{16}$$

As argued by MacCready and Rhines (1991) and Garrett (1991), it is this downwelling-favourable alongslope flow that becomes the boundary condition for the ocean interior velocity and very slowly diffuses into the interior.

These steady state solutions for the density stratification, mixing effectiveness and alongslope flow will undoubtedly turn out to be a gross oversimplification of events occurring in the real ocean. They do, however, draw attention to the need for measurements of eddy momentum fluxes as well as eddy buoyancy fluxes near the sloping sea floor. An observational program on a sloping bottom, using acoustic doppler current profilers to measure Reynold's stresses with the techniques described by Plueddemann (1987) and Lohrmann *et al.* (1990), could be very rewarding.

4. A SIMPLE MODEL FOR THE DECAY OF AN INTERFACIAL SOLITARY WAVE

Much of the study of internal waves has been concerned with their generation, interaction and dissipation in the open ocean. They also exist, however, in the shallower water of continental shelves, where they may have very different wavenumber/frequency spectra and have significantly different behaviour and effects.

In particular, near-surface solitary waves, usually originating from nonlinear internal tides at the shelf break, are a rather common feature of stratified continental shelf waters and also occur in the deep ocean (e.g. Ostrovsky and Stepanyants, 1989). In any location they provide a possible mechanism for vertical mixing.

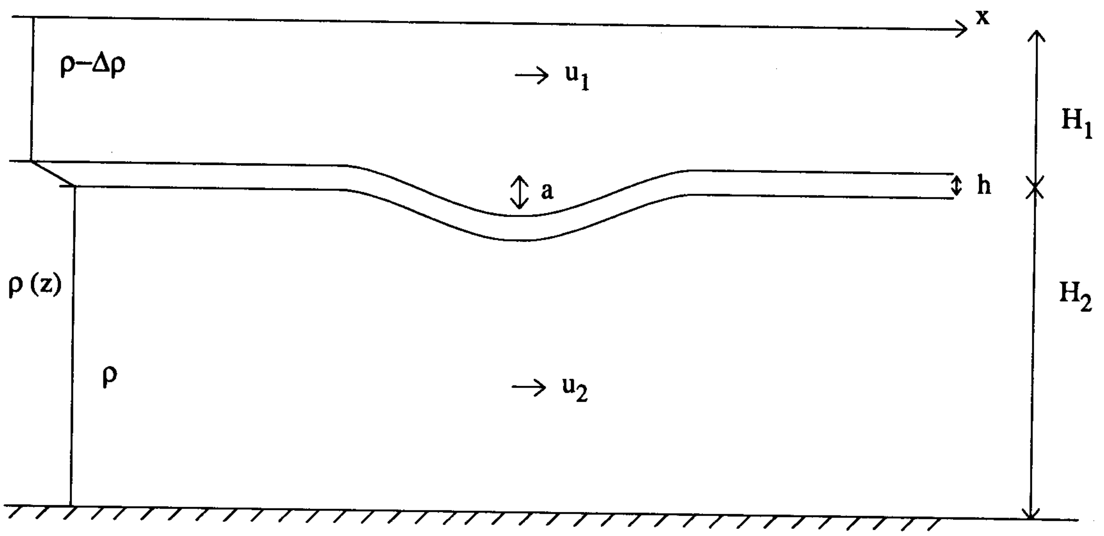

Figure 3: Schematic of an internal solitary wave in a two layer ocean with a thin interface.

In this section I summarise the simple criteria for mixing, and associated solitary wave decay, derived by Bogucki and Garrett (1991) for a fluid with two homogeneous layers separated by a thin interface. The situation envisaged is sketched in Figure 3; layers with thicknesses H_1, H_2 and densities $\rho - \Delta\rho$ and ρ are separated by a thin interface of thickness h ($\ll H_1$, H_2). A solitary wave of amplitude a generates currents u_1, u_2 in the two layers.

In a frame of reference moving with the wave speed c, continuity in the two layers requires

$$(c - u_1)(H_1 + a) = cH_1, \quad (c - u_2)(H_2 - a) = cH_2 \tag{17}$$

$$u_1 = ca(H_1 + a)^{-1}, \qquad u_2 = -ca(H_2 - a)^{-1} \tag{18}$$

if the horizontal length scale L of the solitary wave is much greater than H_1 and H_2. The Richardson number in the interface is then

$$Ri = g'h(u_1 - u_2)^{-2}, \quad g' = g\Delta\rho/\rho \tag{19}$$

$$= g'h(H_1 + a)^2(H_2 - a)^2 c^{-2}(H_1 + H_2)^{-2} a^{-2}. \tag{20}$$

In the simplest situation, where $H_2 \gg H_1$ and $a \ll H_1$, we have $c^2 \simeq g'H_1$ and

$$Ri = hH_1/a^2. \tag{21}$$

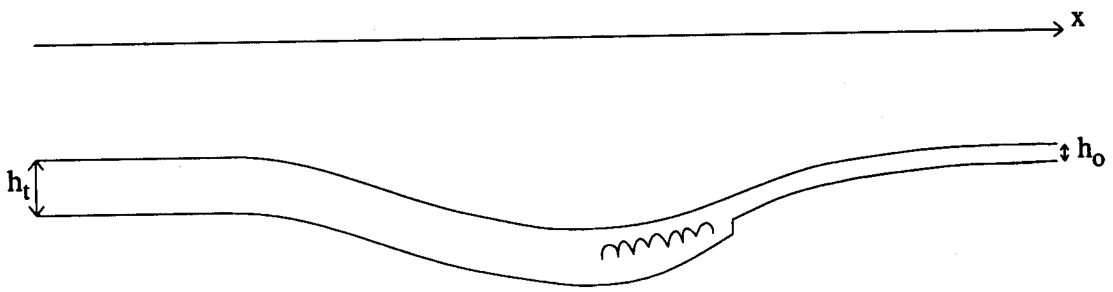

Figure 4: The internal solitary wave is progressing in the x direction and triggers shear instability at some point on the forward face. The interface quickly thickens and keeps thickening until the wave's maximum displacement is reached.

Hence $Ri < \frac{1}{4}$ for instability requires

$$a > 2(hH_1)^{1/2}. \tag{22}$$

This simple criterion for instability, that the solitary wave amplitude should be greater than twice the geometric mean of upper layer and interface thicknesses, also applies if we relax the assumption that $L \gg H_2$. In that case, as for "Benjamin-Ono" solitons (e.g. Ono, 1975) which require $L \ll H_2$, we still have $u_2 \ll u_1$ and (22) still applies subject to $a \ll H_1$ as before.

For solitary waves of amplitude greater than $2(hH_1)^{1/2}$ it seems plausible that shear instability would occur on the forward face as soon as the interfacial Richardson number drops below some critical value Ri_c which is presumably $\frac{1}{4}$. The growth and collapse of Kelvin-Helmholtz billows, and restratification of the fluid into a thicker interface layer, may all occur in a time scale given by some multiple of N^{-1}, where N is the buoyancy frequency in the interface and is given by $N^2 = g'/h$. Hence $N^{-1} = (hH_1)^{1/2}(g'H_1)^{-1/2}$. This is the time taken by the wave to travel a distance $(hH_1)^{1/2}$ which is very much less than L, indicating that the mixing proceeds rapidly compared with the passage time of the wave. Hence one might expect that, after the initial instability, the interface thickens rapidly to achieve some interfacial Richardson number Ri_t ($\simeq 0.4$ according to Thorpe (1972)). As sketched in Figure 4, the interface might then continue to grow until the wave crest passes, after which it remains at the same thickness given by

$$h_t = Ri_t \, a^2/H_1. \tag{23}$$

This thickening of the interface, from an initial thickness of h_0, has an associated increase in potential energy of $g'\rho(h_t^2 - h_0^2)/24$. If we assume that this energy is a fraction α of the total energy lost by the propagating soliton, which is assumed to hold together in spite of the dissipation, we have

$$\alpha\, dE/dx \;=\; -g'\rho(h_t^2 - h_0^2)/24 \qquad (24)$$

$$\;=\; -g'\rho Ri_t^2 H_1^{-2}(a^4 - a_0^4)/24 \qquad (25)$$

where $a_0 = (H_1 h_0/Ri_t)^{1/2}$ is less than the critical amplitude $(H_1 h_0/Ri_c)^{1/2}$ at which decay of the soliton would cease.

Further development of (25) requires a formula for the total soliton energy E. For $L \gg H_2$ the soliton interface displacement ζ as a function of x is given by $\zeta = a\,\mathrm{sech}^2(x/L)$ where, for $a \ll H_1$, $aL^2 = \frac{4}{3}H_1^2 H_2$ (e.g. Benney, 1966). Hence

$$E = \frac{4}{3}g'\rho a^2 L = \left(\frac{4}{3}\right)^{3/2} g'\rho H_1 H_2^{1/2} a^{3/2} \qquad (26)$$

and, from (25) with $Ri_t = 0.4$ and $\alpha = 0.2$ (Oakey, 1982),

$$da/dx = -0.014 H_1^{-3} H_2^{-1/2} a^{-1/2}(a^4 - a_0^4). \qquad (27)$$

If, on the other hand, $L \ll H_2$, the "Benjamin-Ono" solitons (Ono, 1975) have $\zeta = a[(x/L)^2 + 1]^{-1}$ with $aL = \frac{4}{3}H_1^2$ so that

$$E = (\pi/2)g'\rho a^2 L = (2\pi/3)g'\rho H_1^2 a. \qquad (28)$$

Hence, with $\alpha = 0.2$ and $Ri_t = 0.4$ as before,

$$da/dx = -0.016 H_1^{-4}(a^4 - a_0^4). \qquad (29)$$

In both situations, therefore, waves which meet the instability criterion (22) decay rather rapidly at first before eventually stabilising at $a_c = (H_1 h_0/Ri_c)^{1/2}$. As $a_c^4 = (Ri_t/Ri_c)^2 a_0^4 = 2.56 a_0^4$ for $Ri_t = 0.4$ and $Ri_c = 0.25$, the decay rate is rather insensitive to h_0; typical decay distances in either situation are several tens of kilometres (Bogucki and Garrett, 1991).

Application of these simple ideas to the real ocean may require allowance for other effects such as geometrical spreading and, more importantly, extension of the model to allow for a continuously stratified upper layer.

5. DISCUSSION

A picture of a rather persistent internal wave field, slowly losing energy to dissipation and vertical mixing with $K_V \simeq 10^{-5}\ \mathrm{m^2 s^{-1}}$, still seems appropriate for the main thermocline. As remarked by Gregg (1987), diapycnal mixing may then be a rather unimportant process compared with other processes such as ventilation from the surface mixed layer. One part of this picture is the prediction of ϵ from nonlinear internal wave interaction theories. The validity of the model of Henyey *et al.* (1986) is still a matter of debate (e.g. Gargett, 1990), but it is, in a sense, a local model, giving the local flux of energy to small scales and hence mixing for a typical spectrum. The calculations of McComas and Müller (1981a) do seem to be valid for the energetic low–frequency, low–mode, parts of the spectrum, but it is just these spectral regions which are sufficiently long-lived to propagate into a region with a different inertial frequency or to interact with bottom topography. Thus the energy losses from the energetic part of the spectrum in the McComas and Müller (1981b) theory do not immediately lead to turbulence. They need to be integrated with consideration of other processes affecting the internal wave spectrum before we can be confident of the implications for local mixing.

In particular, the pioneering study by Cox and Johnson (1978) of the consequences of horizontal anisotropy in the internal wave spectrum need to be re-examined; the problem is central to any consideration of the evolution of a spectrum that is distorted by bottom reflection, surface generation or just the lateral spreading of a patch of high energy.

The simple scenario shown in Fig. 1 is probably not valid near the sea surface. In particular, if an elevated inertial peak is generated there which is not connected in a WKB fashion to spectra at lower latitudes, then some extra local dissipation must be occurring.

The region near the seafloor is also likely to be anomalous. It has been emphasized here that the mixing produced by internal waves near a sloping bottom may be significant not only for basin-average diapycnal fluxes, but also for the velocity boundary condition for low frequency flows. There is a need to mount more observational programs, capable of measuring eddy momentum and buoyancy fluxes, near sloping bottoms, particularly those that are convex (to avoid destructive interference of reflected internal waves) and have a Burger number greater than 1 (to emphasize slope effects).

Our developing understanding of the deep–sea internal wave field will probably be of limited applicability on continental shelves, though there are the same concerns with surface generation and the role of a sloping bottom. A reasonably common phenomenon of shelf seas is the internal soliton; this paper has presented a simple preliminary model of the mixing produced at a thin interface by an internal soliton which is itself damped in the process.

In summary, therefore, the paradigm of Fig. 1 may be appropriate for some parts of the ocean, though some of its foundations are still rather shaky. Many interesting and important questions remain, however, about parts of the ocean where this picture is not appropriate but where internal waves have significant effects. In other words, our paradigm may hold in regions where internal waves do not matter and be inadequate in places where they do!

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