DIFFUSION PARAMETRIZATIONS FOR THE CLIMATOLOGICAL CIRCULATION OF THE NORTH ATLANTIC AND THE SOUTHERN OCEAN

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ABSTRACT

This paper is a review of the basic physics and some technical and conceptual problems of estimation procedures which attempt to determine mixing parametrizations for heat and salt from hydrographic data. The review is guided by applications of various different forms of the β-spiral method to the hydrography of the North Atlantic and the Southern Ocean. The results appear reasonable in areas where intense mixing is expected to occur. In regions of low mixing, however, as in the central North Atlantic, the coefficients often are only marginally significant. A fairly distinct pattern of mixing and fairly large diffusivity values are obtained in the Southern Ocean. The data, however, are not able to distinguish between different orientations of the mixing tensor: the question of horizontal versus isopycnal mixing is still open for dispute.

PROBLEMS

The inference of mixing properties from hydrographic data meets with many difficulties. Some of these are fairly obvious (though not easy to handle), as, e.g., the effects of errors in the data. Other difficulties are of a more subtle nature. Of the various assumptions which have to be made in the course of any estimation procedure, some are hidden so deeply in the approximation of physical principles or the mathematical estimation techniques that they may elude even careful, experienced people. In this section we discuss various causes of possible failure.

Searching for a small signal

In a diagnostic approach the density and tracer data are given from the hydrographic observations. Any determination of mixing parameters has to make use of the tracer balance

$$u\,\sigma_x + v\,\sigma_y + w\,\sigma_z = D[\sigma] \tag{1}$$

governed by the advection of the tracer σ by the current (u,v,w) and by the mechanism of mixing, which will be assumed here to be of a diffusive nature. In the applications the tracer σ will be any in adiabatic conditions conserved functional $\sigma(\Theta,S)$ of potential temperature Θ and salinity S, such as potential density or veronicity (for a definition of this tracer see Veronis, 1972 or Olbers et al., 1985). Obviously the estimation of a parametrization of $D[\sigma]$ by Eq. (1) is inherently coupled to the estimation of the velocities u, v and w as well, since these are generally not known. The relative roles of advection and diffusion can be found by scaling the above set of equations (see, e.g., Pedlosky, 1978). Taking here vertical diffusion for the purpose of demonstration

$$D[\sigma] = K \sigma_{zz}$$

we obtain for motions of planetary scale the well-known ratio of diffusive to advective terms

$$\text{diffusion / advection} \approx \delta_d/\delta_a$$

where $\delta_a = (fa^2W/g\delta\rho)^{1/2}$ is the advective vertical scale (a = earth radius, $\delta\rho$ = density scale) and $\delta_d = K/W$ is the diffusive vertical scale. With a K of 10^{-4} m^2/s—as proposed by Munk (1966)—and a vertical velocity scale W of 10^{-6} m^2/s the ratio of diffusive to advective terms becomes 0.1 (or less for smaller values of K, as presumably appropriate for local conditions). It appears that mixing is only a small contribution to the tracer balance. We must expect that determination of mixing parameters from such a balance might be a delicate problem unless applied in region of strong mixing.

This problem can readily be seen in the diffusivity estimates obtained from the North Atlantic application of the β-spiral method (Olbers et al., 1985). As an example, Fig. 1 displays the diapycnal diffusivity corresponding to the depth range 100 to 800 m determined from the balance of veronicity (the reference velocities used in the tracer balance were determined before from data between 800 and 2000 m, see below). The most obvious feature in this figure is the marked correspondence of large diffusivities with the regions of strong currents where mixing is more likely to occur. In the calm region between the Gulf Stream and North Atlantic Current on the one side and the equatorial current system on the other the diffusivities are low, in fact they are set to zero in many places by the inversion scheme which constrains the diffusivities to be positive. The determination of these low values of mixing (isopycnal values below 10^2 m^2/s and diapycnal values below 10^{-5} m^2/s) is entirely spoiled by noise in the data. It may as well point toward an inadequate parametrization of the mixing mechanism.

The level-of-no-motion problem

The velocities which are necessary to utilize Eq. (1) for the estimation of the diffusion processes are generally determined from the geostrophic and hydrostatic approximations of the momentum balance. These are quite appropriate for the large-scale oceanic currents but hydrographic data only allow us to determine the baroclinic pressure. Without any

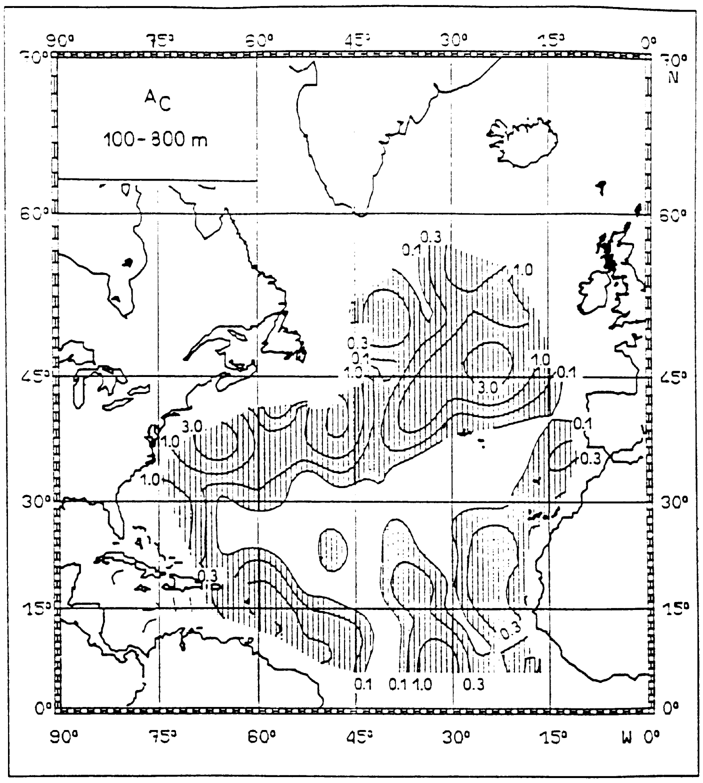

Fig 1. Map of the diapycnal diffusion coefficient in the North Atlantic for the depth range 100–800 m. Units are 10^{-4} m^2/s, contours are logarithmically spaced with interval 0.5.

knowledge about surface topography we are faced with a classical problem of large-scale oceanography––the determination of the absolute velocity profile given the vertical shear by hydrography in form of the thermal wind relations

$$u_z = (g/f)\ \rho_y$$
$$v_z = -(g/f)\ \rho_x \qquad (2)$$

and the local version of the vortex stretching equation

$$w_z = (\beta\ v - F)/f \qquad (3)$$

Here turbulent diffusion of vorticity is included by a term F, since all terms retained in Eq. (3) are of higher order than the geostrophic momentum terms in Eq. (2). Integration of these relations from a fixed reference level $z = z_o$

$$u = u_o + (g/f) \int_{z_o}^{z} dz' \; \rho_y = u_o + u'$$

$$v = v_o - (g/f) \int_{z_o}^{z} dz' \; \rho_x = v_o + v' \qquad (4)$$

$$w = w_o + (1/f) \int_{z_o}^{z} dz' \; (\beta_v - F)$$

$$= w_o + (\beta/f)(z - z_o)v_o - (1/f) \int_{z_o}^{z} d_z' \; F + w'$$

reveals the set of unknowns—the reference velocities u_o, v_o and w_o—which any estimation scheme must determine in conjunction with the mixing parameters. Inserting the integrated thermal wind and vorticity Eqs. (4) into the tracer balance Eq. (1) we find the β-spiral equation in the form

$$u_o \; \sigma_x + v_o \; [\sigma_y + (\beta/f) \; (z - z_o) \; \sigma z] + w_o \; \sigma z$$

$$- D - (1/f) \; \sigma_z \int_{z_o}^{z} d_z' \; F \qquad (5)$$

$$= - (u' \; \sigma_x + v' \; \sigma_y + w' \; \sigma_z)$$

Given a parametrization for the diffusion term D for the tracer and F for vorticity this equation may be applied at some levels to set up a formally overdetermined inverse problem for the unknown mixing parameters and the reference velocities. A powerful method for solving such a problem is the technique of singular value decomposition (SVD) described for oceanographic aspects, e.g., in Olbers (1989) and Wunsch (1989). The problem may be only formally overdetermined since the rank of the set of equations Eq. (5) may effectively (not mathematically) be less than the number of unknowns. As demonstrated below this occurs frequently in the determination of reference velocities.

Ill-posedness

One of the most frequent fallacies of an inverse solution arises from the attempt to extract from the data information that is not really contained there, i.e., parameters should be estimated which are not or not well enough constrained by the relations and data considered. At first, it seems that such a mistake can easily be avoided by simple inspection of the data and the relevant physical mechanisms responsible for the shape of the data. And indeed, in low-order problems a close inspection generally will sort out such failures. However, in case of problems with many data and unknowns the failure may only be found deep in the elements of the inversion tools.

An example of an ill-posed inverse problem is taken here from the North Atlantic application of the β-spiral method (Olbers et al., 1985). In that paper the attempt was made to determine the three unknown reference velocities u_o, v_o and w_o from about ten relations of the form Eq. (5) with vanishing D and F. The tracer was the potential density at levels between 100 and 800 m depth taken from Levitus' atlas (Levitus, 1982). Accepting the physical assumptions which lead to Eq. (5) and the data accuracy, this problem with three unknowns and ten relations should at first sight give a well-behaved inverse solution. However, the matrix condition (ratio of the smallest to the largest eigenvalue of the 3 x 3 matrix corresponding to Eq. (5)) points out severe problems. The model was applied to the entire North Atlantic south of 60°N. Over more than half of this area the condition index is less than 10^{-3} (Fig. 2). This indicates that we have tried to estimate some parameter combination which is not constrained by the hydrographic fields. The data structure leading to this singular behaviour in the low index areas of Fig. 2 is

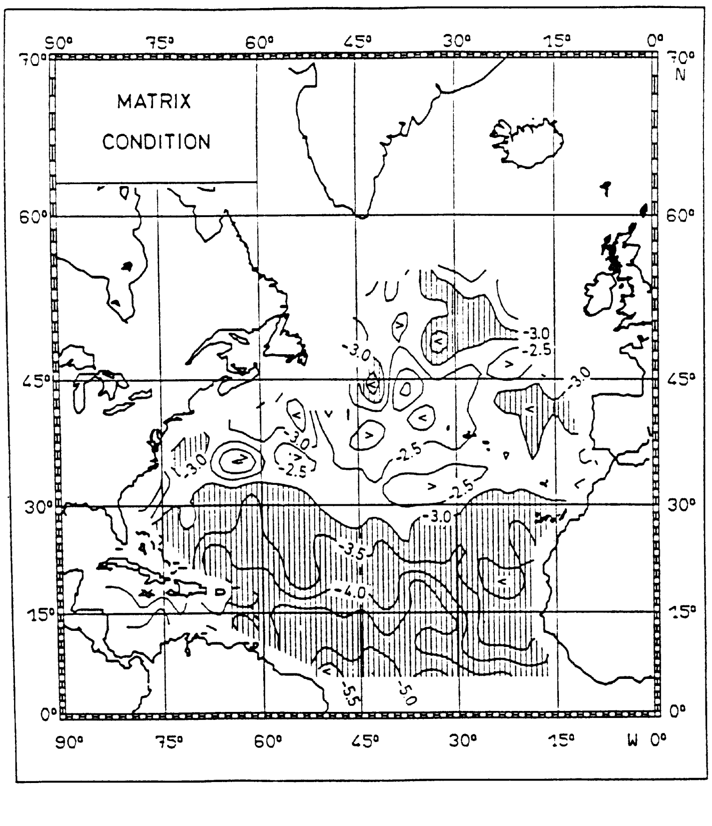

Fig 2. Condition index for the adiabatic form of the set of equations (5) at levels between 100 and 800 m in the North Atlantic. Contours are logarithmically spaced, the area with a condition index below 10^{-3} is shaded.

similar to the one considered below for the circulation in the Southern Ocean: the horizontal gradient of the tracer does not turn rapidly enough with depth, deteriorating the effective rank of the problem.

The next vivid example of singular behaviour occurred during the construction of an inverse model for the Antarctic Circumpolar Current (Olbers and Wenzel, 1989) based on a collection of hydrographic data from the Southern Ocean prepared by Gordon et al. (1982). For our work we used the gridded version of this atlas, for which the temperature and salinity fields are interpolated on a grid with dimensions of 1° of latitude and 2° of longitude on 42 standard levels in the depth range 0 to 7000 m.

As conserved tracer we used potential density referred to the surface. Figure 3 displays the absolute geostrophic circulation in 2000 m depth, obtained by fitting the model to the 100 to 2000 m part of the data. In view of the well-known eastward surface circulation of the Antarctic

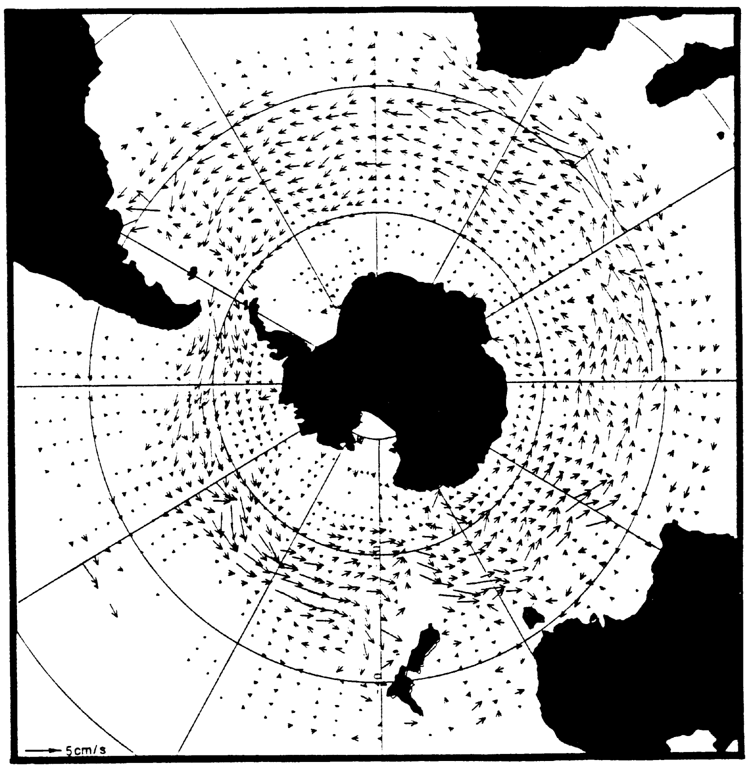

Fig 3. The absolute horizontal velocity at 2000 m depth in the Southern Ocean, obtained from data for 100–2000 m and an adiabatic β-spiral model. Notice the wrong direction of the current in this ill-conditioned solution.

Circumpolar Current (ACC) the level-of-no-motion in this solution must lie well above 2000 m depth, which entirely contradicts our expectation that the ACC is an almost barotropic eastward current. The matrix condition of the solution has very low values (below 10^{-3}) in most of the latitudes of the ACC. The separation of the total circulation of Fig. 3 into three components of the eigenvector basis associated with the eigenvalues of Eq. (5) revealed that the contributions from the two large eigenvalues are small and mainly meridional, whereas the contribution from the smallest eigenvalue is almost identical to the total solution.

The reason for this behaviour must be sought in the zonality of the potential density field. Over most of the upper part of the water column the zonal gradient is much smaller than the meridional one so that the reference velocity u_o is not sufficiently constrained by the β-spiral equation Eq. (5). Unreliable data structures may then be amplified as the consequence of near singular relations and spoil the total solution. Cutting off the contribution from the smallest eigenvalue, as suggested by the SVD approach, leaves us with the sum of the two contributions of the two large eigenvalues, which is still an unsatisfactory circulation. One way out of this dilemma appears to explicitly prescribe u_o from some a priori knowledge, but u_o of course is the most essential part of the signal we actually would like to extract from the data. We consider two other possibilities later: taking deeper data and expanding the model by the bottom boundary condition. Both extensions break the zonality and thus yield better condition indices.

Notice that the almost-singularity does not imply invalidity of the physical model Eq. (5); we are just asking too much if we want three independent parameters from this model. Notice further the potential fallacy in such a situation: the inversion always provides a set of parameters (unless the condition index is exactly zero) and only deeper insight into the inverse machinery and possibly a priori information about the expected solution prevents the acceptance of unreliable results.

Noise

We have mentioned the possible failure of parameter fitting because of noisy observations. Hydrographic data contain errors from different sources. Instrumental noise may be important in great depths and high latitudes where temperature and salinity vary only a few thousandths of a degree or per mille in the entire water column. Individual sections generally contain aliasing by internal waves and small-scale eddies because of undersampling. The presence of data noise arising from such observational limitations apparently may lead to a masking of the mixing signatures so that mixing parameters may become undeterminable. A more fundamental mistake may be introduced if the original data have been processed in some interpolation or objective analysis scheme which necessarily implies smoothing. This may yield a systematic bias of the mixing effects and estimated mixing coefficients will be oversized.