UNSTEADY SHALLOWING MIXED LAYER

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ABSTRACT

An analytical analysis of the mechanical energy budget for an unsteady shallowing oceanic surface mixed layer with a downward surface buoyancy flux B_o and a water surface friction velocity u_* provides insight into the relative roles of vertical mixing by turbulence generated within the Obukhov depth zone, $L = 2u_*^3/B_o$, and by turbulence generated by a mean flow dynamic instability below L.

An increase in the downward surface buoyancy flux decreases the Obukhov length and causes an initially well-mixed turbulent boundary layer to be separated into a surface layer, which decreases in thickness to L, and a region below L which is isolated. The turbulence in the new surface layer achieves an equilibrium state on an adjustment time scale L/u_*, and the turbulence in the isolated region is dissipated. The thickness δ of the interface between these two mixed layers grows due to a mean flow dynamic instability, achieving its maximum value of about 0.18 u_*/f after about 0.37 of an inertial period.

INTRODUCTION

The purpose here is to examine the unsteady ocean surface mixed layer which shallows in response to a change in atmospheric forcing. This situation may arise if either the downward surface buoyancy flux, B_o, is increased or if the surface wind stress, u_*^2, is decreased, causing the Obukhov length scale,

$$L = 2\, u_*^3/B_o \tag{1}$$

to be reduced to a value less than the previously established depth of turbulent mixing, z_i.

Many applications of the vertically integrated turbulent kinetic energy (TKE) budget (see reviews by Zilitinkevich et al., 1979; Garwood, 1979; and Price et al., 1987) have shown the length scale attributed to Obukhov (1946, 1971) to be the steady state solution for a shallowing, buoyancy-flux dominated ocean surface mixed layer. Recently, Price et al. (1986) have focused interest on the development of transient thermoclines due to the diurnal increase in the downward buoyancy flux. From such observations as these it has become evident that it is insufficient to consider entrainment alone in developing and verifying mixed layer models. In fact, some of the theoretical differences between alternative mixed layer parameterizations may be better examined and understood by simulation of the shallowing mixed layer than by simulation of the mixed layer which is being deepened by strong entrainment. Although much attention has been paid to the unsteady response of a deepening mixed layer, little attention has been given to the dynamics and thermodynamics of the shallowing surface mixed layers in the ocean.

DeSzoeke and Rhines (1976) first demonstrated the asymptotic regimes of an unsteady TKE budget for a mixed layer during the initial deepening into a linearly stratified pycnocline, $\partial b/\partial z = N^2$, where N is the buoyancy frequency and b is the mean buoyancy. They found that unsteadiness in the TKE was significant in determining mixed layer depth only during the first stage of deepening, or for $t \lesssim 1/N$. However, DeSzoeke and Rhines considered only the wind-driven case, and they did not include a surface buoyancy flux. Therefore a principal difference here is the inclusion of a downward surface buoyancy flux, resulting in the restratification of a previously well mixed layer. Following DeSzoeke and Rhines, however, this study shall include the unsteadiness in the TKE (first suggested by Zilitinkevich, 1975, in an application to the atmospheric turbulent boundary layer) in order to reveal the time scale for the TKE budget to approach equilibrium and for the surface mixed layer depth to achieve an approximate steady state, $h \sim L$.

The Obukhov scale L is not the only possible length scale for turbulent boundary layers in equilibrium. In the case of the neutral atmospheric planetary boundary layer (zero surface buoyancy flux), the limiting depth of the turbulence has traditionally (Rossby and Montgomery, 1935) and in observational studies (Clarke, 1970) and in model applications of the atmosphere (Deardorff, 1972; Wyngaard et al., 1974) been assumed to be proportional to the neutral planetary boundary layer scale,

$$L_o = u_*/f. \qquad (2)$$

For the oceanic planetary boundary layer (OPBL), and in particular for the stable ($B_o > 0$) OPBL, the relative importance of L_o and L has not been clear. The effort here may help resolve this question.

The method will be to examine the energetics of the turbulence, via the TKE equations and of the mean flow with regard to the possibility of dynamic instability below the surface-controlled turbulent boundary layer. A sudden change in the surface forcing will result in the transient decay of turbulence in the surface layer. A new shallower turbulent boundary layer will be established as the turbulent kinetic energy budget approaches a new equilibrium. As the surface buoyancy flux and wind stress cause the mean buoyancy and momentum to increase in the new shallower layer, both the mean shear and the mean buoyancy gradient at the base of the new layer will be expected to increase. Apart from the surface-generated TKE, a mean flow instability may occur below the new layer, causing momentum and buoyancy to be mixed to a greater depth (below z = -L).

THE UNSTEADY TKE BUDGET

The turbulent kinetic energy equations for the three constituents, $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{w'^2}$, are integrated vertically across a well-mixed and turbulent layer of depth h:

STORAGE	WIND-SHEAR PRODUCTION	PRESSURE REDISTRIBUTION	VISCOUS DISSIPATION	BUOYANT DAMPING

$$\langle \overline{u'^2} h/2\rangle_t = \cos^2(\theta) m_3 u_*^3 + m_2 \; E^{1/2}(E-3\langle \overline{u'^2}\rangle) - M_1/3\; E^{3/2} \qquad (3)$$

$$\langle \overline{v'^2} h/2\rangle_t = \sin^2(\theta) m_3 u_*^3 + m_2 \; E^{1/2}(E-3\langle \overline{v'^2}\rangle) - M_1/3\; E^{3/2} \qquad (4)$$

$$\langle \overline{w'^2} h/2\rangle_t = m_2 \; E^{1/2}(E-3\langle \overline{w'^2}\rangle) - M_1/3\; E^{3/2} - B_o h/2 \qquad (5)$$

The functional parameterization of the pressure redistribution and dissipation terms above follow the methods of Garwood (1977). The brackets, ⟨ ⟩, denote averages over the vertical extent (h) of the surface mixed layer. The wind direction is θ, and $E = \langle \overline{u'^2 + v'^2 + w'^2} \rangle$ is the total TKE. In eq. (5) there is no entrainment buoyancy flux because it is required that there be no entrainment at the base of the mixed layer. Here the entrainment is not occurring because $\overline{w'^2}$ is assumed to vanish at the base of the mixed layer, $z = -h$. In this special state of no entrainment, the ratio of net vertical TKE to total TKE is required to follow the similarity constraint,

$$r = 3 \langle \overline{w'^2} \rangle / E, \tag{6}$$

where r is expected to be a small dimensionless constant. For isotropic turbulence, r would have a value of unity, and in general r would vary with changes in the entrainment buoyancy flux. However, in the limiting case here of strong buoyant damping and vanishingly small $\overline{w'^2}$ at $z = -h$ and no entrainment, it is expected that r will achieve a limiting value much less than unity. Although there is no direct experimental evidence for the size of r in the OPBL, a reasonable value for r may be determined from laboratory observations of stable grid-stirred turbulence. In any case, with r prescribed, eqs. (3-6) provide a closed system of equations for mixed layer depth, h, and the three constituent components of the vertically averaged TKE, $\langle \overline{u'^2} \rangle$, $\langle \overline{v'^2} \rangle$, $\langle \overline{w'^2} \rangle$. For the purposes here, an approximate and very direct solution for a shallowing h is achieved by assuming $r \sim 0$ and solving eq. (5) simultaneously with eq. (7), the total TKE equation:

$$\langle Eh/2 \rangle_t = \overset{\text{"G"}}{m_3 u_*^3} - \overset{\text{"D"}}{m_1 E^{3/2}} - B_o h/2 \tag{7}$$

<u>Steady state solution</u>

The steady state solutions for mixed layer depth and dissipation are given by

$$h_\infty = \left[\frac{m_2/m_1 - 1/3\ (1 - r)}{m_2/m_1 + 2/3\ (1 - r)} \right] \left[\frac{2\ m_3\ u_*^3}{B_o} \right] \tag{8}$$

$$D_\infty = m_1\ E^{3/2} = 6 m_3\ u_*^3 /\ (7 - 3r) \tag{9}$$

where D is the vertical integral of TKE dissipation over the mixed layer. The values $r = 0$, $m_2/m_1 = 0.5$, and $m_3 = 7$ as taken as representative values for the model constants, m_i. These values are based upon laboratory observations and model vertification (Garwood, Gallacher and Müller, 1985 a, b; Martin, 1985; Gallacher, 1987; Gaspar, 1987). Then eqs. (8) and (9) give

$$h = 2u_*^3/B_o = L, \text{ and}$$

$$D = (6/7)\ G$$

where $G = m_3 u_*^3$ is the total wind-shear production. Hence, for the steady balance, 6/7 of the wind stress-generated turbulence (G) is dissipated, and only 1/7 of this energy is used to increase the system potential energy via the buoyancy flux.

Unsteady adjustment of shallowing mixed layer

The total TKE, the mixed layer depth, and time are both nondimensionalized on the boundary condition parameters, the downward surface buoyancy flux, B_o, and the water surface friction velocity, u_*:

$$E^* = (D/D_\infty)^{2/3} = 0.3028E/u_*^2 \tag{10}$$

$$h^* = h/L = B_o h/(2u_*^3) \tag{11}$$

$$t^* = B_o t/u_*^2 \tag{12}$$

The dimensionless coefficients are included in eqs. (10-11) so as to normalize both the TKE and the mixed layer depth on their respective steady state values. Thus eq. (7) for the total TKE budget becomes

$$(6)^{2/3} \langle E^* h/2 \rangle_{t^*} = 7 - 6(E^*)^{3/2} - h^* \tag{7a}$$

and the vertical TKE budget, eq. (5), reduces to

$$0 = (E^*)^{3/2} - h^* \tag{5a}$$

Figure 1 shows the solutions for $E^*(t^*)$ and $h^*(t^*)$, assuming that the mixed layer was much deeper than L at the time the downward buoyancy flux was initiated (and the wind stress was held steady). Notice that the depth of mixing is immediately reduced to 7L/6, regardless of how deep z_i was prior to the inception of the buoyancy flux (as long as $z_i < -7L/6$. The remnant mixed region which is below the shallowing surface layer and above z_i, $-h > z > z_i$, will initially still be turbulent. However, this

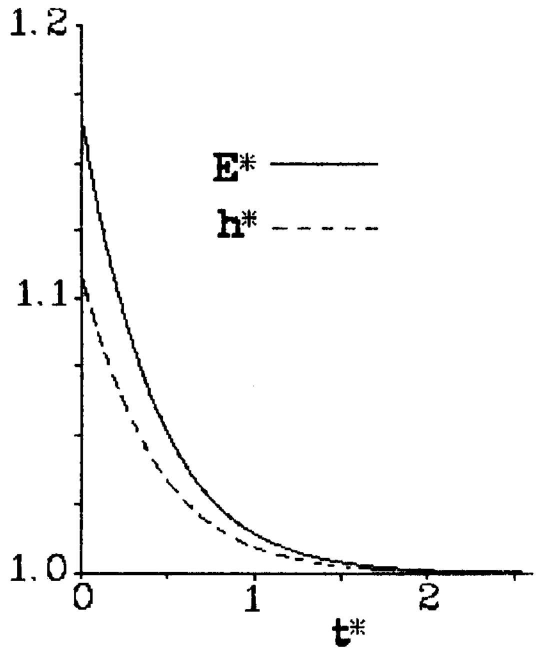

Fig. 1. Unsteady response of the dimensionless TKE, $E^* = E/u_*^2$, and the dimensionless mixed layer depth, h = h/L, versus the dimensionless time, $t^* = B_o t/u_*^2$, for a wind-forced (friction velocity = u_*) turbulent boundary layer which has a downward buoyancy flux (B_o) imposed at time t = 0.

region will now be isolated from surface fluxes, and the turbulence will necessarily decay by viscous dissipation unless there is an alternate source of turbulence (other than the wind stress). The schematic diagram in Figure 2 depicts the new surface layer which remains fully turbulent and the underlying remnant of the mixed layer with decaying turbulence.

The dimensionless solutions in Figure 1 show the time scale for adjustment by the new surface layer to a new equilibrium to correspond to $t^* = 1$, or $t = u_*^2/B_o$. For example, with a net downward heat flux of 400 W/m^2 and a wind speed of 8 m/s, the decay time period is about 1000 s for h to decrease to a new value of L = 20 m and for the total TKE to be about E = 0.0003 m^2s^{-2}. If the prior depth of mixing had been $z_i = -100$ m, for example, the remnant region between z = -20 m and z = -100 m would contain decaying turbulence. Because this region is isolated from the surface buoyancy flux, the decay time of the turbulence is not expected to be dependent upon the surface boundary conditions, and therefore will differ from the adjustment time for the surface mixed layer turbulence. Scale analysis of the TKE budget for this region suggests a decay time on the order of $(|z_i| - L)/u_*$, which could easily be much longer than the adjustment time scale for the turbulence in the surface layer.

Interfacial dynamic instability

In the hypothetical case here of a sudden step increase in the downward buoyancy flux, a turbulent boundary layer which is initially well mixed is separated into two distinctly different mixing layers. Typically, for a diurnally shallowing oceanic mixed layer, the turbulence in the lower remnant layer might be expected to decay on time scales of 10^3-10^4 s, provided there were no other sources of energy such as local breaking internal waves or turbulent transport from another shear production source region. Hence, because the buoyancy flux ceases throughout its vertical extent, this remnant layer is expected to continue to be well mixed until the time when another source of turbulence can penetrate the region.

An anticipated principal source of energy is that produced by a local dynamic instability. Both momentum and buoyancy will be relatively well mixed down to $z = -L$ as they are fluxed into the surface layer. However, at the lower interface between the two mixing layers, a dynamic instability of thickness,

$$\delta = (\Delta U^2 + \Delta V^2)\ (4\ \Delta\ b)^{-1} \qquad (13)$$

is predicted (Garwood, 1977) to develop from the requirement that

$$Ri_\delta = \Delta b/\ (\Delta U^2 + \Delta V^2) = Ri_{cr} \sim 1/4$$

in the interfacial region between the surface mixed layer and an underlying dynamically stable and nonturbulent region, where Ri_δ is the gradient Richardson number and Δb, ΔU and ΔV are the buoyancy and mean velocity changes across the interface. See Figure 2.

Although the intensity of the turbulence and dissipation may be relatively large in the interfacial region, it will not be well mixed. Rather, the shear and mean buoyancy gradient will be governed by eq. (13), together with equations for the mean kinetic energy and the mean buoyancy:

$$\Delta U^2 + \Delta V^2 = 2u_*^4 f^{-2}\ (1 - \cos ft)\ (L + \delta/2)^{-2} \qquad (14)$$

$$\Delta b = B_o\ t(L + \delta/2)^{-1}. \qquad (15)$$

For the case of a constant wind stress and constant buoyancy flux, an analytical expression for $\delta(t)$ may be derived from the mean kinetic energy and mean buoyancy budgets, giving

$$\delta + \delta^2/(2L) = 0.25\ u_* f^{-1}\ (1 - \cos ft)\ (ft)^{-1}. \qquad (16)$$

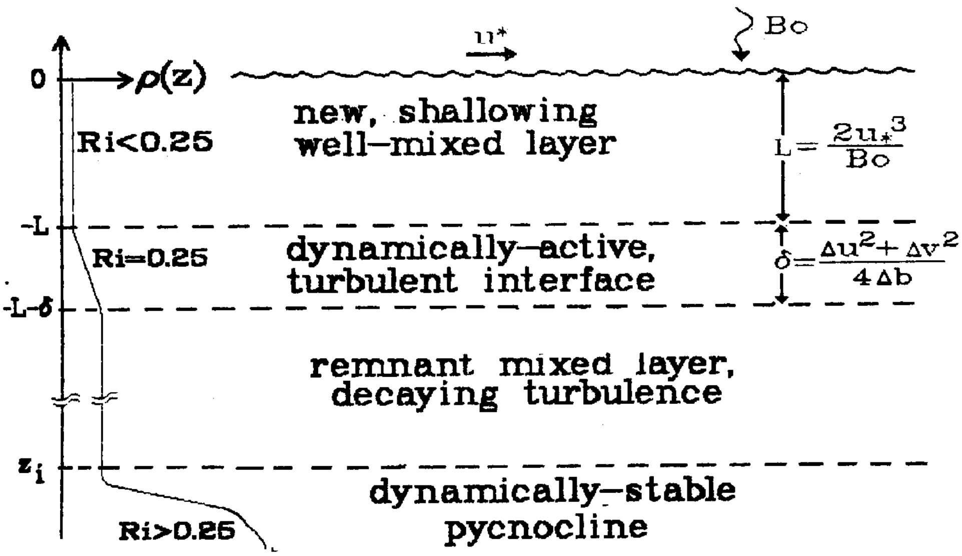

Fig. 2. Schematic diagram of the vertical domains of the ocean surface turbulent boundary layer as it responds to a downward buoyancy flux imposed on the surface. The density profile will evolve into two well- mixed regions separated by a stratified but dynamically active interface region of thickness δ. The surface layer turbulence will tend to maintain a well-mixed profile of thickness L after decaying from an initial depth of 7L/6.

Figure 3 shows the time dependence of $\delta(t)$ upon the dimensionless parameter,

$$B^* = u_*/ (fL) = B_o/ (2fu_*^2). \tag{17}$$

The parameter B^* is a measure of the relative importance of the neutral planetary boundary layer scale to the Obukhov scale, L_o/L.

For the small time, $ft \ll 1$, eq. (16) reduces to a linear dependence in time,

$$\delta \sim u_* t/4,$$

but the Coriolis effect limits this initial tendency. Although the maximum mean kinetic energy occurs at $ft = \pi$, deepest penetration of the dynamic instability occurs at $ft = 2.33$, or about 0.37 of an inertial period. For large L (small B^*), the limiting size of δ is

$$\delta_{max} = 0.18\, u_*/f \tag{18}$$

which is independent of B_o. For increasingly large B_o, the thickness δ

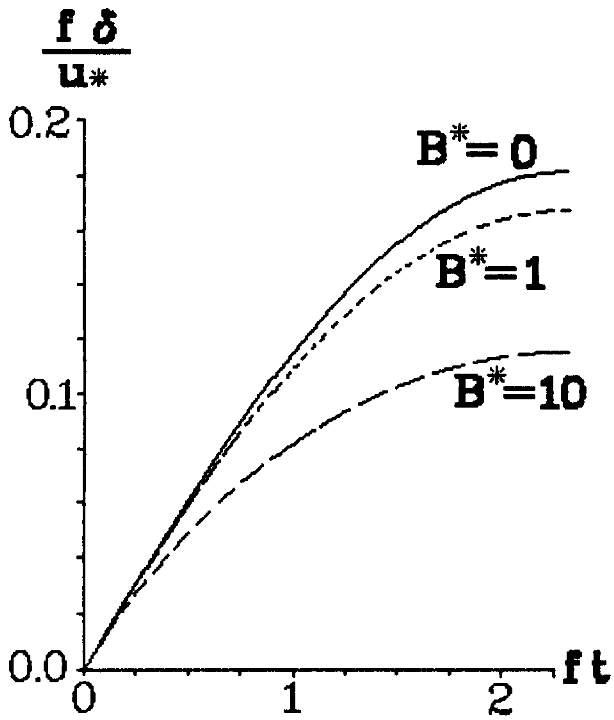

Fig. 3. Time-dependent growth of the interface thickness, δ, for three different cases of the buoyancy-to-rotation parameter, $B^* = u_*/(fL)$. Values of B^* less than unity correspond to transient thermocline formation for temperate oceanic regions. Values of B^* greater than 10 may occur regularly in tropical oceans.

becomes more dependent upon L. However, for $B^* \leq 10$ as for the summertime mixed layer in temperate oceans, δ_{max} depends mostly upon the Rossby and Montgomery neutral planetary boundary layer scale, $L_o = u_*/f$.

A fundamentally different scaling of the below-L interface will occur at low latitudes. With f vanishingly small, and $B^* \gg 1$, the mean kinetic energy budget given by eq. (14) becomes incorrect. Without the Coriolis limitation on the surface layer buildup of wind-driven mean kinetic energy, lateral friction and advection must have increasingly greater importance in the momentum budget. Also, the TKE itself may be influenced by planetary rotation in tropical regimes (Garwood, Müller and Gallacher, 1985). Nevertheless, as f is made smaller, δ will tend to increase proportionately, by eq. (18). Hence tropical and equatorial turbulent boundary layers might be expected to have relatively thick δ-regions created by intermittent dynamic instability and possibly strong turbulence below relatively shallow well-mixed layers of thickness L.

SUMMARY

A mechanical energy budget analysis of the unsteady shallowing oceanic surface mixed layer provides insight into the relative roles of vertical mixing by turbulence generated within the Obukhov depth zone, $0 < z < -L$, and by turbulence generated by mean flow dynamic instability below the Obukhov depth.

The special case of interest here is when the downward surface buoyancy flux (B_o) increases or the friction velocity (u_*) decreases sufficiently to cause the Obukhov depth, $L - u_*^3/B_o$, to decrease to a value less than the previously established turbulent boundary layer depth. Then equilibrium theory requires that there be no entrainment caused by the vertical transport of turbulence generated by wind-shear production. However, unsteadiness in the turbulent kinetic energy budget has an important effect upon the depth of the well-mixed part of the turbulent boundary layer for a period of time of approximately u_*^2/B_o. After this adjustment period the surface flux-controlled turbulent boundary layer depth approaches L asymptotically.

Mixing in the interfacial region below $z = -L$ must depend upon a source of energy other than turbulent transport. As both momentum and buoyancy which are fluxed through the surface are confined initially to the layer of depth L, with increasing time a local mean flow dynamic instability is expected at the base of the well-mixed layer. This instability causes mixing to penetrate below $z = -L$ to a maximum thickness of $\delta = 0.18u_*/f$. Hence the "interface" thickness, δ, is independent of the surface buoyancy flux and is a function of the history of the surface wind forcing and rotation. Thus both Obukhov scale and the neutral planetary boundary layer scale are shown to have important but different roles in the evolution of the shallowing transient oceanic turbulent boundary layer.

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