WIND FORCING AND OBSERVED OCEANIC WAVENUMBER SPECTRA

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ABSTRACT

If internal waves are forced by wind stress applied as a body force (uniform vertical gradient of stress) in a surface layer, their spectrum has the vertical wavenumber shape of the Garrett-Munk internal wave spectrum. Bandwidth and high wavenumber shape are functions of the layer thickness and shape, respectively. Conversely, if the Garrett-Munk wavenumber shape is determined by wind forced waves at near-inertial frequencies, then the observed shape implies that the vertical gradient of stress from the surface is nearly uniform and cuts off sharply at a depth comparable to observed seasonal thermocline depths.

INTRODUCTION

Models of wind driven ocean circulation commonly use one of two methods to prescribe how wind stress drives the ocean. The more common method is to prescribe surface stress and assume that small scale processes can be parameterized by a given (normally uniform) eddy viscosity. The stress then vanishes exponentially with depth with a scale small compared to the ocean depth. Vertical velocities at the base of this frictional layer are independent of the form of friction, arise simply from any divergence pattern of transports within the layer, and drive the deeper circulation. This is the familiar construct of Ekman theory. The other choice, more common for equatorial models where the Ekman decay scale becomes singularly large, is to assume that wind stress acts as a body force distributed over some upper layer. As in the Ekman theory, stress is imagined to vanish beneath some (normally shallow) depth. Rather than specify a vertical velocity at this depth, the (normally uniform) body force is projected onto the available free modes of ocean oscillation (one barotropic plus an infinite number of baroclinic modes). This is the construct of Lighthill (1969) applied to the problem of generating a low-latitude western boundary current (the Somali current). As pointed out by Knox and Anderson (1985), the Lighthill approach is merely an alternate parameterization of small scale frictional processes to that chosen six decades earlier by Ekman. The key aspect shared by both models is that wind stress is applied completely to a near-surface layer of the ocean.

The details of how wind stress is applied to the upper ocean have implications for the character of response to time dependent forcing. For wind systems which move relatively rapidly compared to wave speeds of free baroclinic modes, oceanic response is largely at near-inertial frequencies. The amplitude of each mode excited depends on how the mode projects onto the forcing. It turns out that the vertical wavenumber shape of forced internal waves depends not on the spectrum of forcing but on how the vertical distribution of stress projects onto the modes.

The vertical wavenumber shape of observed internal wave spectra tends to -2 at high wavenumbers, as observed by many profile measurements of horizontal current components and/or temperature. At low wavenumber, the spectrum has not been measured directly but has been inferred from coherence measurements at different separations. The kinematic model fit of Garrett and Munk (GM 81) chooses a wavenumber shape $(n^2 + n_*^2)^{-1}$ where n is vertical mode number and n_* is a measure of wavenumber bandwidth (Munk, 1981). The transition vertical mode (or equivalent vertical wavenumber) scale n_* has been inferred from coherences to be 3. The GM model assumes that wavenumber spectral shape is independent of frequency, a notion which is challenged by upper ocean measurements (Pinkel, 1985). The profiles upon which the GM wavenumber shape is based are dominated by contributions to the joint frequency-wavenumber spectrum near the inertial frequency. This fact has been exploited to infer near-inertial fluxes from sequences of profiles rather coarsely spaced in time (see D'Asaro and Perkins, 1984, and others cited therein).

Explanations of the observed internal wave spectrum are incomplete. Rather complex theories of wave interaction, scattering, and turbulence in stratified media have been invoked to explain the GM spectrum (Müller et al., 1986). The wave interaction models find the GM shape to be in equilibrium but do not explain the shape of the most energetic part of the spectrum. "The transfers within the energetic low-frequency, low-wavenumber region are weak and not dominated by any particular process (McComas and Müller, 1981a)". They do not explain, either, the total energy level or the wavenumber bandwidth n_* (McComas and Müller, 1981b; Müller et al., 1986). An alternative model will be used here to explain the wavenumber shape.

The internal wave wavenumber shape at near-inertial frequencies can be a consequence of forcing. We adopt the view that the near-inertial part of the internal wave spectrum is dominated by response to wind forcing, consisting of a number of overlapping wakes to traveling stress patterns. Near-inertial waves can be generated efficiently by sharp changes in wind stress, as demonstrated by D'Asaro (1985) and others. Atmospheric wind systems often take the form of fronts which travel at speeds greater than the free wave speeds of all but near-inertial internal modes. The internal wave wake behind a particular front may last for several days during which time other fronts may pass a given site. Energy is continually being added to the ocean by the wind stress jump across each front, but because the source translates, the energy density at a fixed site behind the front is invariant. Forcing by traveling wind systems predisposes the near-inertial part of the oceanic internal wave spectrum to have a particular vertical wavenumber shape.

INTERNAL WAVE GENERATION BY WIND STRESS

The linearized perturbation equations for a stratified f-plane resting ocean of uniform depth D can be written, using conventional notation, as:

$$u_t - fv = -p_x + \tau^x(y,y,t)Z(z) \tag{1a}$$

$$v_t + fu = -p_y + \tau^y(x,y,t)Z(z) \tag{1b}$$

$$N^2 w = -p_{zt} \tag{1c}$$

$$u_x + v_y + w_z = 0 \tag{1d}$$

following McCreary (1985) where wind stress τ is given at the surface and its vertical gradient is specified by $Z(z)$. The form of $Z(z)$ is crucial to the vertical wavenumber spectrum of forced wave response. For completeness, we specify that the vertical integral of $Z(z)$ over the total depth D be unity. If $G_n'(z)$ is the vertical structure function for horizontal current components u and v and reduced pressure p and $G_n(z)$ that for vertical velocity w, then the equations (1) reduce when projected on these modes to:

$$u_{nt} - fv_n + c_n p_{nx} = \tau_n^x \tag{2a}$$

$$v_{nt} + fu_n + c_n p_{ny} = \tau_n^y \tag{2b}$$

$$u_{nx} + v_{ny} + p_{nt}/c_n = 0 \tag{2c}$$

where n is the vertical mode number corresponding to eigenvalues c_n of the vertical structure equation

$$G_n''(z) + \frac{N^2(z)}{c_n^2} G_n(z) = 0 \tag{3}$$

These modes are the barotropic (n=0) and baroclinic (≥ 1) free modes of the flat-bottom ocean considered. The equations (2) express the components of velocity and pressure associated with each mode n. Forcing takes the form

$$\tau_n = \frac{\tau \int_{-D}^{0} Z(z)G_n'(z)dz}{\int_{-D}^{0} G_n'^2(z)dz} \tag{4}$$

where τ_n is called the coupling coefficient linking wind stress τ (= τ^x or τ^y) to the horizontal structure equations for a particular vertical mode. Without loss of generality, the vertical modes $G_n'(z)$ can be normalized so that their mean square value averaged vertically is unity (so that the denominator in (4) is D).

The common choice for distributing stress in a surface layer of thickness h is to prescribe $Z(z) = h^{-1}$. If this surface layer is identified with the mixed layer depth (or is even shallower than the mixed layer), the equation (3) becomes

$$G_n''(z) = 0 \quad \text{for} \quad -h < z \leq 0 \tag{5}$$

since $N(z) = 0$ for (at least) the layer in which stress is distributed. Typically h is identified with the mixed layer depth. Then $G_n'(z)$ = constant = $G_n'(0)$ over the interval $-h < z \leq 0$. This implies the coupling coefficients τ_n are:

$$\tau_n = \frac{\tau}{D} G_n'(0) \tag{6}$$

Notice that the depth of the layer depth h over which stress is distributed does not appear explicitly. This is because the vertical structure functions $G_n'(z)$ are constant over this depth range.

Some simple examples of different distributions Z(z) are illustrative. For convenience, consider an ocean of uniform stratifiction N without a mixed surface layer. The functions $G_n'(z) = \sqrt{2}\cos(n\pi z/D)$ satisfy the normalization chosen above. The coupling coefficients take the form of a discrete cosine transform given by

$$\tau_n = \frac{\tau\sqrt{2}}{D} \int_{-D}^{0} Z(z) \cos(n\pi z/D)dz \tag{7}$$

Now if stress is concentrated as a delta function at the surface $Z(z) = \delta(0)$, then τ_n is independent of n. That is, it has a white spectrum. If, however, stress is distributed uniformly over depth h as in the example of the previous paragraph, then τ_n is proportional to sinc($n\pi h/D$) where sinc(x) is the interpolating function $\sin(\pi x)/\pi x$. Finally, if stress is distributed as a ramp $Z(z) = 2(z/h+1)/h$ within the layer h and vanishes outside, then the coupling coefficients decay more steeply with vertical mode number n $\left(\tau_n \propto \mathrm{sinc}^2(n\pi h/D)\right)$. As these examples show, the distribution of stress gradient Z(z) determines the functional dependence of the coupling coefficients τ_n on mode number n. The layer thickness h relative to the total depth D determines the wavenumber bandwidth upon which forcing projects.

The form of Z(z) for real oceanic situations is uncertain and is even time and location dependent, since storms and heating modify mixed layer depth. Instead of a cosine transform, the projection of Z(z) onto the vertical modes of the vertical structure equation for non-uniform stratification determines the relative excitation of motions of different scales. The constant N examples do provide some intuition: sharp transitions in Z(z) imply energetic high wavenumber portions of the response spectrum.

To go from the coupling coefficients τ_n to the internal wave response spectrum requires solution of equations (2). These have been solved for various idealized geometries (open ocean, coastal equatorial) and forcings (stationary, traveling, irrotational, vortical, etc.). The general properties are that the most efficiently excited waves are those which are most closely matched by the wind in structure and scale; that is, nearly resonant. An illustrative case is the idealization of a moving atmospheric front to a moving line source of stress divergence. Kundu and Thomson (1985) demonstrate that the solution is composed of the available vertical modes, each with its own horizontal structure, traveling at the speed of the front. Waves whose minimum phase speed c_n exceeds the speed of the front (typically the barotropic mode) are excited as evanescent waves. Those with minimum phase speeds c_n slower than the frontal speed are excited as standing oscillations moving with the front. The most strongly excited waves are those modes for which c_n is near the translation speed U and the coupling coefficient is high. From their solutions, the average energy per unity horizontal area of each mode can be calculated as:

$$E_n = \frac{\rho_o}{2}\,\tau_n^{\,2}\,\frac{D}{f^2(1 - c_n^{\,2}/U^2)} = \frac{\rho}{2}\left(\frac{G_n{}'(0)\tau}{D}\right)^2 \frac{D}{f^2(1 - c_n^{\,2}/U^2)} \tag{8}$$

where τ is the stress jump across the front in the direction of its travel. (This calculation assumes Z(z) is a constant and non-zero only over a depth at most as deep as the surface mixed layer.) The two dependences of vertical mode number n are in the coupling coefficient (e.g., $G_n'(0)^2$) and resonance with the forcing $(1 - c_n^2/U^2)^{-1}$. For a fast moving front ($U \gg c_n$, $n \geq 1$, where $c_1 \sim 3$ m/s), the baroclinic modes are far from resonance so that the dependence on n of the internal wave spectrum enters only through the factors $G_n'(0)^2$. Examples of this behavior are given below.

A stratification appropriate to comparing forced vertical wavenumber spectra with the GM spectrum is the profile used by Garrett and Munk to calculate model vertical structure functions. It is a rough approximation to the stratification found at many oceanic locations: an exponential thermocline capped by a mixed layer. Garrett and Munk use:

$$N(z) = 0 \qquad -h < z < 0 \tag{9}$$

$$N_o e^{(z+h)/b} \quad -D \leq z < -h$$

where N_o = 3 cph and b = 1300 m. For illustration, we use D = 5 km and h = 50, 100, or 200 m. The vertical structure functions $G_n'(z)$ for this profile are analytic (Bessel functions) but can be calculated numerically for ease. Completeness implies that

$$\frac{h}{D} \sum_{n=0}^{\infty} G_n'(0)^2 = 1 \tag{10}$$

so that $hD^{-1}G_n'(0)^2$ is the relative contribution each mode makes to the total spectrum of response (neglecting any resonances, i.e., $U \gg c_1 > c_2 > c_3 \ldots$). That is, $hD^{-1}G_n'(0)^2$ gives the vertical wavenumber spectral shape of internal waves forced by fast-moving wind systems. Plots of this quantity are given in Figure 1 for three different choices of h/D, as labeled. Solid circles are drawn to highlight the values for n=1 to 10. As expected for the case of $Z(z) = h^{-1}$ for $-h < z < 0$ and zero otherwise, the forced spectra all asymptote to n^{-2} decay for high enough n. Also as expected, the transition mode number is inversely proportional to h/D.

The heavy curve labeled GM81 drawn in Figure 1 is $(n^2+n_*^2)^{-1}$ with $n_*=3$ normalized by its sum from n=1 to 100 (instead of infinity). It is, of course, the Garrett-Munk vertical wavenumber shape inferred from observed internal wave statistics. The curves $hD^{-1}G_n'(0)^2$ are remarkably similar to GM81. In particular, the curves labeled h/d = 0.02 and 0.04 are within

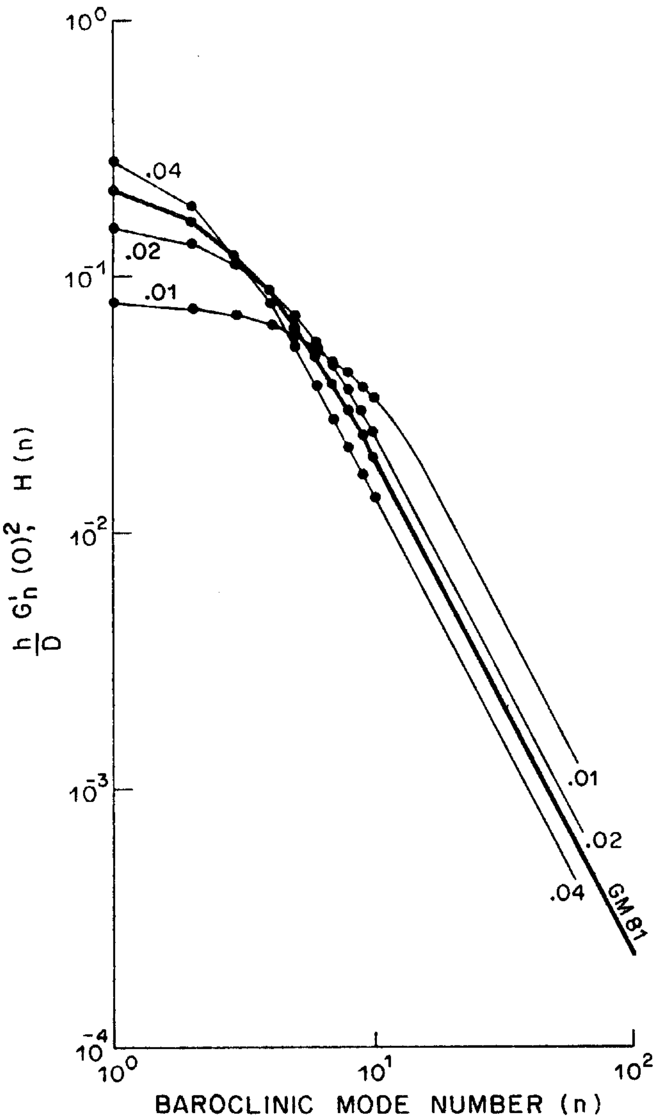

Fig. 1. Relative energy of baroclinic modes excited by wind stress applied as a uniform body force in a layer of depth h to an ocean of depth D with an exponential N(z) profile. Light curves marked 0.01, 0.02, and 0.04 correspond to h/D. Heavy curve is the Garrett-Munk vertical wavenumber shape (GM81). All curves are normalized to have the same (unit) energy. Evaluations for the first ten modes are marked with circles.

25% of GM81 for all but the lowest baroclinic mode (where the differences are less than 40%). These ratios imply depths of frictional influence of 100 and 200 m in a 5000 m deep ocean, respectively.

DISCUSSION

The close similarity of the model forced spectra and GM81 suggests that the oceanic internal wave spectrum at near-inertial frequencies is predisposed to its shape by wind-forced waves. The GM81 wavenumber shape is based on wavenumber spectra and coherence as a function of separation rather than on direct measurements of the joint frequency-wavenumber spectrum of internal waves. As such, it is dominated by contributions near the inertial frequency, since this part of the joint frequency-wavenumber spectrum is most energetic. The interaction theories which find GM81 to be in equilibrium do not explain the spectral shape in the neighborhood of this most energetic part of the spectrum (the lowest octave or two in frequency and wavenumbers not larger than the bandwidth). Forcing by traveling wind systems may provide an explanation for this part of the spectrum.

The spectral levels obtained from the wind-forced model are reasonable, given jumps in stress τ across model fronts of 1-5 dynes/cm^2 and frontal translation speeds U ~ 0(10 m/s) where deep ocean low baroclinic mode speeds are 0(1 m/s). Curiously, the energy in any particular mode is independent of the depth h over which stress is absorbed by the ocean (see equation (8)), but the total wave energy is proportional to $\tau^2 h^{-1} f^{-2}$ (which can be seen by summing (8) together with (10)). More complicated wind systems will yield different constants of proportionality, but the basic features of the model results must be preserved. These are that 1) energy density of a particular baroclininc mode should be independent of the stress layer depth, 2) total energy should be inversely proportional to stress layer depth, 3) bandwidth should be a monotonically decreasing function of stress layer depth, and 4) both energy density of a particular mode and the total energy should be proportional to stress squared (roughly wind speed to the fourth power!).

If the forced wave explanation of the near-inertial, low wavenumber portion of the internal wave spectrum is correct, the form of the spectrum has profound implications for the distribution of stress gradient from wind in the upper ocean. As the examples of the previous section demonstrate, concentrating the stress gradient as a delta function at the surface or as a linear ramp down from the surface gives vertical wavenumber spectral shapes which disagree strongly with those observed. A constant stress gradient confined to a shallow surface layer gives a spectral shape which agrees with the observed shape. Justification for this constant gradient stress layer model has been lacking in the past. If the forced wave model is correct, it implies that stress vertical divergence truly is nearly uniform throughout a shallow layer and cuts off sharply. As long as the transition zone between the region of nearly uniform stress divergence (of depth h) and the deep interior with vanishing stress divergence is small (compared to h), wind forcing will excite waves whose spectrum asymptote to a -2 slope in vertical wavenumber.

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