ON THE ZONAL MOMENTUM BALANCE AT THE EQUATOR

T. M. Dillon, J. N. Moum, T. K. Chereskin, and D. R. Caldwell

College of Oceanography, Oregon State University,
Corvallis, Oregon, 97331

ABSTRACT

The conventional view of equatorial dynamics requires that the zonal equatorial wind stress be balanced, in the mean, by the vertical integral of "large-scale" terms, such as the zonal pressure gradient, mesoscale eddy flux, and mean advection, over the upper few hundred meters. It is usually presumed that the surface wind stress is communicated to the interior by turbulent processes. Turbulent kinetic energy dissipation rates measured at 140°W during the TROPIC HEAT experiment and a production rate-dissipation rate balance argument have been used to calculate the zonal turbulent stress at 30 m to 90 m depth. The calculated turbulent stress amounts to only 20% of the wind stress at 30 m depth, and decreases exponentially with depth below 30 m. Typical large-scale estimates of the zonal pressure gradient, mesoscale eddy flux, and advection have a depth scale larger than the turbulent stress, and are inconsistent with the vertical divergence of the stress as estimated from the dissipation rate measurements. It is concluded that either

(1) the measured estimates of dissipation rate are too small,
(2) the actual large-scale zonal pressure gradient, mesoscale eddy flux, and advection during our observation period were highly atypical and had a very shallow depth scale,
(3) some process other than the simple diffusion of momentum through shear instabilities is transporting the momentum, or
(4) the assumption of a production-dissipation balance in the turbulent kinetic energy budget is incorrect.

The first two possibilities are unlikely.

1. INTRODUCTION

Two important features of the upper 100 m of the equatorial ocean are large vertical shears of zonal velocity and energetic turbulence. The Equatorial Undercurrent flows toward the east beneath the westward-flowing South Equatorial Current. Between these two currents, vertical shears as large as 0.02 s^{-1} are commonly observed. The most recent measurements of equatorial turbulence were done as part of the TROPIC HEAT experiment, during which one ship occupied an equatorial station from November 19 to December 1, 1984 (Moum and Caldwell, [1985], hereafter MC), and another ship occupied a station a few tens of km distant from November 25 to 30 (Gregg et al [1985], hereafter GPWOS). Both groups took vertical profiles 24 hours per day. New records were set for the number of profiles collected, with MC obtaining 1749 good casts, and GPWOS obtaining 385 good casts.

The large number of profiles collected in TROPIC HEAT make possible the calculation of some elementary balances of the momentum budget in the upper 100 m. The most important contributions to the zonal momentum budget at the equator are the zonal convergence of zonal momentum, upwelling of eastward momentum, the zonal pressure gradient, the divergence of the mesoscale eddy flux, and the turbulent stress (Bryden and Brady, 1985). Balancing these factors leads to the conservation equation

$$U\frac{\partial U}{\partial x} + W\frac{\partial U}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{\partial \overline{U'U'}}{\partial x} - \frac{\partial \overline{U'V'}}{\partial y} - \frac{\partial \overline{F}_x}{\partial z} , \qquad (1)$$

where U and V are mean zonal (positive eastward) and meridional (positive northward) velocities, P is mean pressure, ρ is density, x is the zonal direction (positive x eastward), and F_x is the vertical turbulent flux of zonal momentum. By "mean", we intend an average over several months to years, and thousands of kilometers; U′ and V′ are zonal and meridional velocity fluctuations in the mesoscale eddy band (i.e., fluctuations with time scales from a day to several weeks, length scales from tens of kilometers to hundreds of kilometers). Contributions of small-scale horizontal turbulent fluxes (i.e., turbulent fluxes with length scales smaller than tens of meters) are neglected on the grounds that horizontal velocity gradients are orders of magnitude smaller than vertical velocity gradients. The overbar indicates an average over several months to several years. It is assumed that the mean acceleration of fluid and the mean meridional velocity are small. The zonal balance of forces on a column of fluid of unit surface area extending from z=a to z=b is obtained by vertically integrating and rearranging eq.(1):

$$\int_a^b \{-\frac{1}{\rho}\frac{\partial P}{\partial x} - U\frac{\partial U}{\partial x} - W\frac{\partial U}{\partial z} - \frac{\partial \overline{U'U'}}{\partial x} - \frac{\partial \overline{U'V'}}{\partial y}\} \, dz - \overline{F}_x(b) + \overline{F}_x(a) = 0,$$

or

$$ZPG + ZD + UW + EF - \overline{F}_x(b) + \overline{F}_x(a) = 0 \qquad (2)$$

where -ZPG is the integral of the zonal pressure gradient divided by density, -ZD is the integral of the zonal divergence of mean zonal momentum, -UW is the integral of the upwelling of zonal momentum, and -EF is the integral of the mesoscale eddy flux.

If the momentum flux in the fluid interior is due to turbulent processes, $-F_x(z)$ equals $\tau(z)/\rho$, where τ is the zonal turbulent stress; at the sea surface, $-F_x(0)$ equals τ_0/ρ, where τ_0 is the zonal surface wind stress. An eddy viscosity K_m is defined by

$$- F_x(z) = K_m \langle \partial u/\partial z \rangle, \quad z < 0 , \qquad (3)$$

where the brackets < > denote some averaging process, for example, an ensemble average, an areal average, or perhaps a Monte-Carlo average. If a time average is used, it is an average over some arbitrary "short" time, longer than the time scale of turbulent fluctuations, but shorter than the overbar average. For the measurements described below, hourly averages of turbulence quantities are used, although it is more correct to think of these as ensemble averages rather than time averages.

Bryden and Brady (1985) (hereafter BB) made an order of magnitude estimate of K_m by assuming that the dominant term in eq.(2) is ZPG, the zonal pressure gradient term. Using historical measurements of $\partial P/\partial x$ and wind stress, BB estimated a mean equatorial eddy viscosity at 75 m depth of 1.7×10^{-3} m^2/s, an order of magnitude larger than has been estimated from dissipation measurements (GPWOS).

Our approach is to extend BB's method. We will use BB's estimates of the first four terms in eq.(2), and use MC's estimates of $K_m(\partial u/\partial z)$ and the surface wind stress to evaluate the last two terms of eq.(2) (see Fig. 1 for a schematic representation). We seek to determine whether eq.(2) can be balanced in this manner, and if not, which of the assumptions might be at fault.

In section 2, we review the dissipation method for estimating turbulent stresses. In section 3, we evaluate both sides of eq.(2), and are unable to find equality. At first sight, this approach seems peculiar. BB's estimates are for the mean circulation over a wide longitude range, while MC's turbulence data is for a single 12-day period at one location several years after BB's estimates were made. In principle, there is no reason to expect a balance to be achievable. BB's estimates can, however, be viewed as "typical" estimates, and the difference between BB's estimates and the actual realization can be viewed as an "anomaly". In section 4, we ask whether or not the zonal momentum budget residual can be explained by a <u>reasonable</u> anomaly in large-scale terms, and examine the consequences if reasonable anomalies cannot be found.

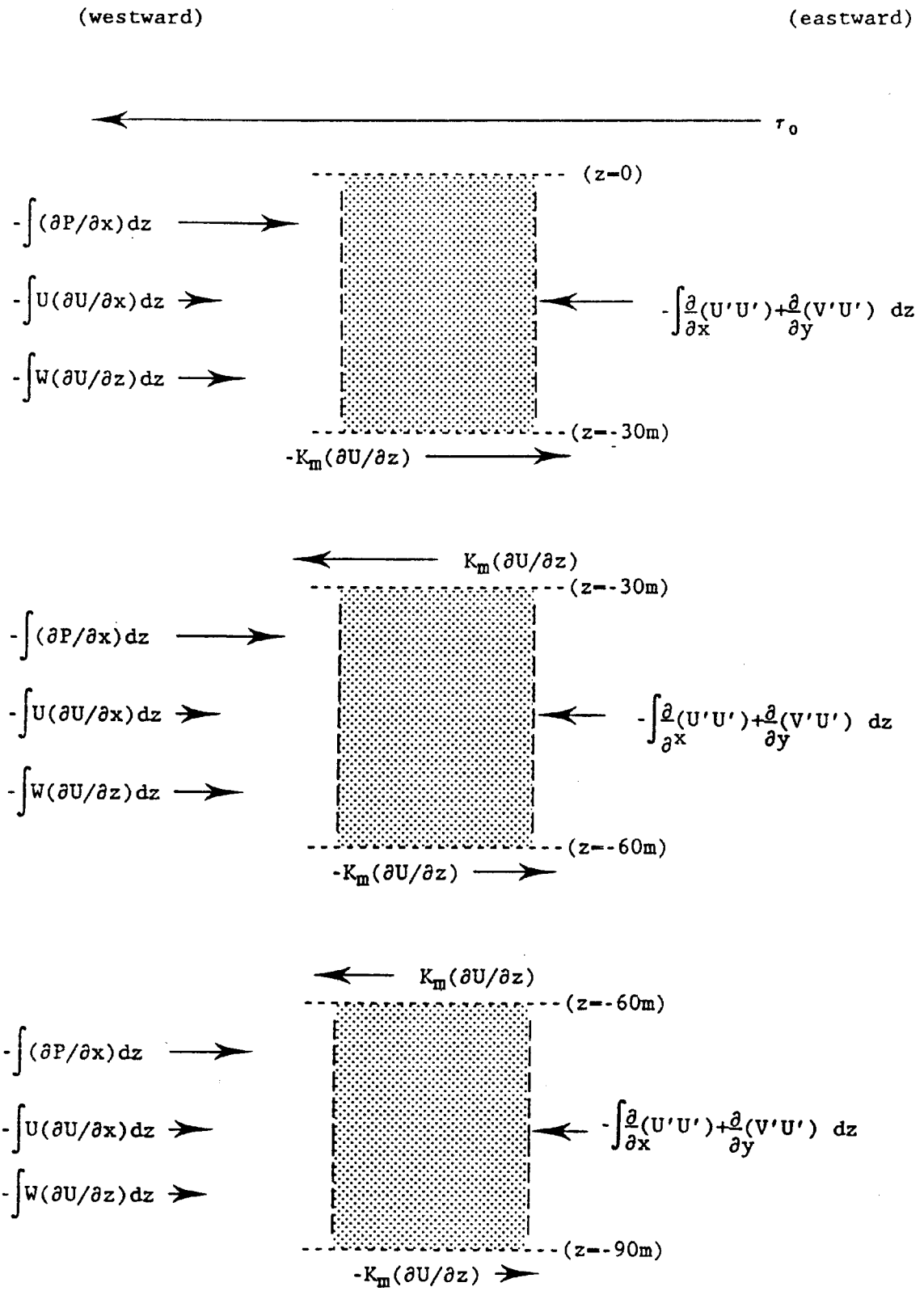

Fig. 1. Schematic view of balance of forces per unit surface area on control volumes in the upper three 30 m depth intervals. The terms represented are from eq.(2). The arrow length for each term is approximately scaled by the corresponding values in Table 1.

2. THE DISSIPATION METHOD

Estimates of eddy viscosities, diffusivities, and the turbulent fluxes of momentum and heat have been made using measurements of ϵ, the turbulent kinetic energy dissipation rate (e.g., Osborn, 1980; Crawford, 1982; GPWOS). The dissipation rate is calculated from measurements of large wavenumber vertical shears (vertical scale roughly between 0.01 and 1 m). Flux estimates are based on the turbulent kinetic energy equation (Wyngaard and Cote, 1971; Osborn, 1980; Dillon, 1984, or any standard text):

$$\frac{\partial \langle q^2\rangle}{\partial t} + W\frac{\partial \langle q^2\rangle}{\partial z} = \mathcal{P} - \epsilon + B - \frac{\partial}{\partial z}\{\langle w'p'/\rho\rangle + \langle w'q^2\rangle\} . \qquad (4)$$

Here, q^2 is the turbulent kinetic energy, W is the mean vertical velocity, and $\mathcal{P}$ is the rate of production of turbulent kinetic energy by shear instabilities,

$$\mathcal{P} = - \langle w'u'\rangle\langle\partial u/\partial z\rangle - \langle w'v'\rangle\langle\partial v/\partial z\rangle ; \qquad (5)$$

ϵ is the rate of kinetic energy dissipation, $B = (g/\rho)\langle w'\rho'\rangle$ is the buoyancy flux, u' v' and w' are zonal, meridional, and vertical velocity fluctuations, and p' is a pressure fluctuation. In obtaining eq.(4), it has been assumed that gradients of terms like q^2, $\langle p'u_i'\rangle$, and $\langle w'q^2\rangle$ are much larger in the vertical than in the horizontal because the forcing processes are, in an average sense, horizontally uniform over the region being considered.

The next step in using eq.(4) to obtain eddy viscosities is to assume the time scale for changes in the shear production is long compared to the time scale for adjustment of the fluctuation field, i.e., that $\partial q^2/\partial t$ is small compared to $\mathcal{P}$. This assumption makes no statement about the state of the turbulence in any single observation. <u>In any single realization of a particular overturning event</u>, $|\partial q^2/\partial t|$ may be much greater than $\mathcal{P}$ and the fluctuation field may be rapidly growing or rapidly decaying, yet the steady-state assumption is reasonable, provided only that enough samples are included in the average. An analogy may be useful here: the ensemble average of sea level changes slowly, even though any instantaneous measurement at a point may show rapid variations.

The second term on the left of eq.(4) is the advection of turbulent kinetic energy by a mean vertical velocity W. In many parts of the ocean this term may be safely neglected because W is small, but in equatorial waters, there is a net upwelling, with W in the range of 1 to 3 meters per day (BB). We can scale $W\langle\partial q^2/\partial z\rangle$ as Wq^2/h, where h is a typical equatorial mixing layer depth, say 50 m. Since ϵ scales as q^3/ℓ, where ℓ

is the typical size of an overturn (say, 1 to 5 m), we estimate that $W\langle\partial q^2/\partial z\rangle/\epsilon \approx (W\ell)/(hq) \approx 10^{-6}$. This term can be safely neglected.

Osborn (1980) argues that the last two terms on the right of eq.(4) are divergences, and hence can affect only the distribution of q^2 in space, not the total amount of turbulent energy, for stably stratified turbulent flows. This argument may be more nearly correct for the divergence of $\langle w'q^2\rangle$ than for $\langle w'p'\rangle/\rho$, because the former term is just the advection of turbulent kinetic energy by turbulent vertical velocity fluctuations, while the latter term is the internal wave energy flux. There is no a priori reason to believe that the amount of energy radiated away by internal waves is small, but we shall for the present assume that it is, and later will discuss the consequences of this assumption.
We are left with the common assumption that the rate of production of turbulent kinetic energy is balanced by its rate of dissipation plus the buoyancy flux. Experiments in the laboratory and in the atmospheric boundary layer (Osborn [1980] reviews some of these) indicate that the flux Richardson number, $R_f = B/\mathcal{P}$, is small, probably never exceeding 0.2. On these grounds, we neglect B in eq.(4), and arrive at a production-dissipation balance:

$$\mathcal{P} = \epsilon \quad . \tag{6}$$

If we define an eddy viscosity by

$$\langle u'w'\rangle = -K_m\langle\partial u/\partial z\rangle \ , \quad \langle v'w'\rangle = -K_m\langle\partial v/\partial z\rangle, \tag{7}$$

we can use eqs.(5), (6), and (7) to obtain the eddy viscosity and zonal shear stress in terms of measured dissipation rates:

$$K_m = \frac{\epsilon}{\langle\partial u/\partial z\rangle^2 + \langle\partial v/\partial z\rangle^2} \ , \tag{8}$$

$$-F_x = -\langle u'w'\rangle = K_m\langle\partial u/\partial z\rangle = \frac{\epsilon\langle\partial u/\partial z\rangle}{\langle\partial u/\partial z\rangle^2 + \langle\partial v/\partial z\rangle^2} \tag{9}$$

3. EQUATORIAL TURBULENCE ESTIMATES

Turbulent kinetic energy dissipation rates were determined from measurements made using the Rapid Sampling Vertical Profiler (RSVP; Caldwell, Dillon and Moum, 1985) and were averaged over one-hour time intervals and 12-m depth intervals. The zonal shear was measured using a hull-mounted Acoustic Doppler Current Profiler (ADCP) and averaged over the same time and depth intervals (the depth averaging is an inherent limitation to the ADCP; our averaging interval was chosen to obtain independent shear estimates). An evaluation of the ADCP measurements made from the R/V WECOMA during the TROPIC HEAT experiment may be found in

Chereskin, Halpern and Regier (1986). K_m and $-F_x$ were calculated from the one-hour average dissipation rates and shears, and then averaged over the entire 12-day period. GPWOS calculated 4-day average values for ϵ, and then used an estimate of the mean shear based on 8 XCP casts.

MC's dissipation rate measurements begin at 10 m depth and extend to 110 m. The 12-day average dissipation rate is nearly constant in the upper 30 m, but falls off exponentially in the 30 m to 70 m depth range (Fig. 2). The upper ocean is continuously stratified. The 12-day average buoyancy frequency is smallest near the surface, and increases from 1 cph at 10 m depth to about 7 cph at 60 m depth; from 60 m to 110 m, the buoyancy frequency is roughly constant at 7 to 8 cph. The estimates of K_m and $-F_x$ using eq.(8) begin at 33 m because that is the shallowest depth at which the shear could be independently determined from ADCP measurements. The momentum flux and eddy viscosity are largest nearest the surface, and monotonically decline with increasing depth (Fig. 3,4). GPWOS dissipation rates are somewhat larger than those measured by MC; the precise reason for the difference is unknown at present, but may be a result of different algorithms for calculating the dissipation rate.

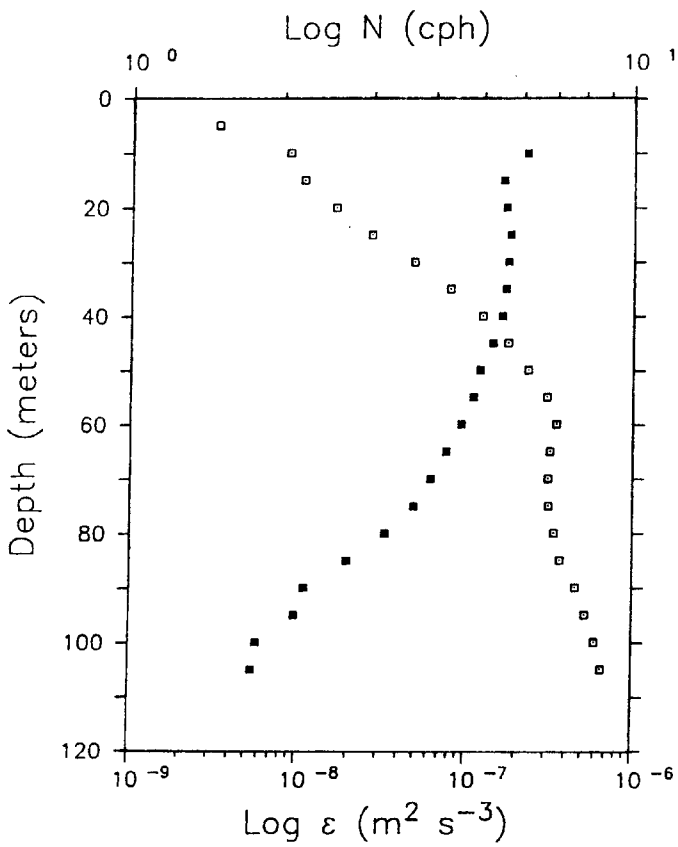

Fig. 2. Turbulent kinetic energy dissipation rates (solid square) and buoyancy frequency (open square) averaged over 12 days using Moum and Caldwell's (1985) data.

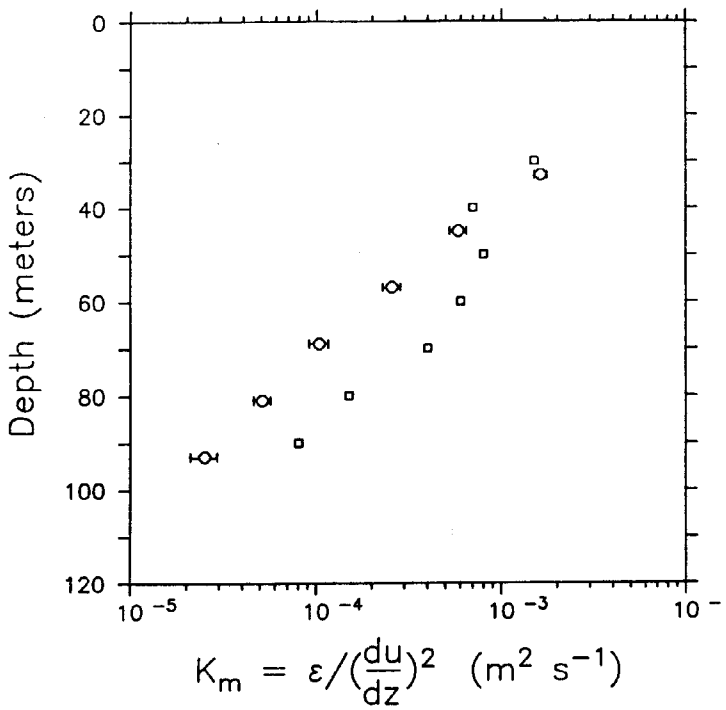

Fig. 3. Eddy viscosity estimates, K_m, vs depth. Boxes are from GPWOS. The open circles were estimated from eq.(8) using hourly averages of dissipation and velocity shears, and averaged over the 12-day station. The confidence limits of ±1 standard deviation were found from a bootstrap test using the parent population of the 288 hourly-averaged samples at each depth bin. Distributions of bootstrapped means are approximately Gaussian, and 68% of the values lie within the plotted confidence limits.

The shears used by GPWOS are from 8 XCP profiles made during their 4-day station and these do not agree in detail with the overlapping 4-day average from the MC data set. MC's eddy viscosities are surprisingly small; BB's estimate at 75 m is twenty times larger than the eddy viscosity estimated from MC's data using eq.(9).

Wind stress was calculated from R/V WECOMA observations. Wind speed and direction were recorded at 2 minute intervals, and first averaged over an hour. Atmospheric stability was determined from surface heat flux estimates using measured total downward solar radiation, the measured sea surface temperature, and humidity (measured with a dew-point hygrometer). Large and Pond's (1981) iterative bulk-aerodynamic procedure, which provides drag coefficients corrected for atmospheric stability and sensor height, was used to calculate the zonal stress. The hourly stress was then used to calculate τ_0, the average zonal stress. For the 12-day station, $\tau_0 = 0.097$ N/m^2. The standard deviation was 0.030 N/m^2, and the sampling error in the mean is 0.002 N/m^2.

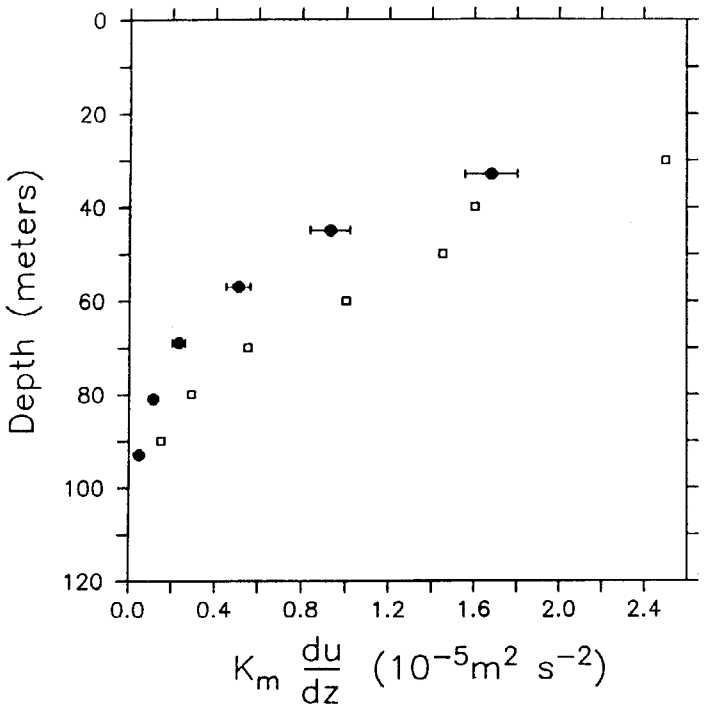

Fig. 4. Zonal component of the turbulent momentum flux estimated using eq.(9). Boxes are from GPWOS. Solid circles are from hourly averages of dissipation and vertical shears averaged over the 12-day station. Confidence limits are as in Figure 3.

Large and Pond (1981) found good agreement between stability-corrected bulk stress estimates, and direct eddy correlation measurements. They list an uncertainty of 30% for daily average bulk estimates, with the largest discrepancies noted when winds were rapidly rising or falling; others have noted deficiencies in the bulk method when the relative wind/wave direction changes. The wind speed was nearly constant over the 12 days spent on station, ranging between 6 and 12 m/s (see Moum and Caldwell [1985] for a plot), with no rapid changes of direction, and an infinite fetch. We therefore expect the WECOMA bulk estimates to have an uncertainty smaller than 30%; conservatively, we place the systematic uncertainty at 20%, and round the stress estimate to 0.10 N/m^2.

Surface stress must be continuous across a fluid boundary, assuming surface tension is negligible. Much of the surface stress can be taken up by the surface wave field in cases where the fetch is small. If surface wave transport were an appreciable part of the momentum budget in the infinite fetch case, however, the growth of the wave field would be unbounded. When the fetch is large, as in the tropical Pacific, surface wave transport must play a small role in the average zonal momentum budget.

The 12-day average stress $F_x(z)$ is reasonably well described by

$$-F_x(z) = \tau_0 \, e^{z/z_\tau} \tag{10}$$

in the 30 m to 90 m depth range. The shallowest (33 m) estimate of the turbulent momentum flux is much smaller than τ_0. The shear above 100 m does not vary much with depth, and the eddy viscosity and dissipation rate, as well as F_x, have an approximately exponential depth dependence. The scale depth for the divergence of F_x is z_τ = 18 m, surprisingly small. We have no dynamical reason why ϵ, K_m, and F_x should be approximately exponential; rather, an exponential profile is simply a convenient way of summarizing the measurements. The fact that eq.(10) intercepts the surface at the measured surface stress is probably circumstantial. The exponential profile in eq.(10) simply illustrates a surprisingly small effective penetration depth for the surface stress.

4. A MOMENTUM IMBALANCE?

Mean zonal pressure gradient and advective accelerations were calculated by BB from a diagnostic model of equatorial circulation. The horizontal eddy flux of zonal momentum was estimated by Bryden, Brady and Halpern (1986; hereafter, BBH) using moored current meter arrays near 152°W from April 1979 to June 1980, and near 110°W from January 1979 to October 1981. Covariances were calculated using daily-average velocity measurements extending from the upper 15-20m to 250m depth. The BB and BBH estimates were made based on measurements of more than a year duration, and we interpret them here as "typical" estimates of large-scale quantities. The BB and BBH estimates are presented with the turbulence estimates in Table 1.

The dominant term estimated by BB and BBH is ZPG, the vertical integral of the zonal pressure gradient. The contribution to zonal momentum from upwelling (UW) is about half as large as the ZPG, and is positive in the upper 90 m. The contribution from large-scale eddies, EF, is about 2/3 of the ZPG, and is negative, reinforcing the wind stress and countering the ZPG. The only term small relative to the ZPG is ZD, the mean zonal divergence of zonal momentum.

MC's turbulence measurements extend from roughly 30m to 90m, and we have divided the upper ocean into 3 sections: 0-30m, 30m-60m, and 60m-90m (Table 1). We cannot expect to balance eq.(2) exactly, and so rewrite it with a residual term, R, representing the imbalance:

$$\mathrm{ZPG} + \mathrm{ZD} + \mathrm{UW} + \mathrm{EF} + \tau(b)/\rho - \tau(a)/\rho = R \tag{11}$$

Here, $\tau_0(z)/\rho = -F_x(z)$ represents wind stress at the surface z = 0, and is estimated as $K_m\langle\partial u/\partial z\rangle$ in the fluid interior; a represents the depth of the top of each layer, and b represents the bottom. The right side of eq.(11) includes all of the terms we are unable to estimate, including the local acceleration $\partial u/\partial t$, as well as local anomalies in the zonal

Table 1. Momentum budget for the upper 90 m of the equatorial ocean in three 30 m segments. Units are 10^{-6} m^2/s^2. ZPG, ZD, UW, and EF, taken from Bryden and Brady (1985) and Bryden, Brady and Halpern (1986), are "typical" estimates of large-scale terms. The stress estimates are from Moum and Caldwell's (1985) TROPIC HEAT dissipation rate measurements. The uppermost 30 m segment has a large residual compared to the ZPG.

TABLE 1

Layer Depth Range	0 - 30m	30m - 60m	60m - 90m
$ZPG = \int -\frac{1}{\rho}\frac{\partial P}{\partial x}\,dz$	14	12	8
$ZD = \int -U\frac{\partial U}{\partial x}\,dz$	-0.9	-0.6	2
$UW = \int -W\frac{\partial U}{\partial z}\,dz$	5	6	1.5
$EF = -\int \frac{\partial}{\partial x}\overline{(U'U')} + \frac{\partial}{\partial y}\overline{(U'V')}\,dz$	-9	-6	-3
ZPG + ZD + UW + EF	9	11	9
τ/ρ, layer top	-100	-18	-4
τ/ρ, layer bottom	-18	-4	-0.6
$\delta\tau/\rho$, (top - bottom)	-82	-14	-3.4
Residual: $R = ZPG + ZC + UW + EF + \delta\tau/\rho$ (all units 10^{-6} m^2/s^2)	-72	-3	5.6
R/ZPG x 100%	-514%	-25%	70%

pressure gradient, advective accelerations, and, possibly, contributions to the stress which we are unable to measure (for example, internal wave horizontal-vertical velocity covariances in the frequency range of 1/day to the local buoyancy frequency); by "anomalous" we mean "different from BB's 'typical' estimates".

The residual for the 60-90m depth range is large, amounting to 70% of the ZPG for that depth range. This is unexpectedly large, but not alarming. One must expect any 12 day period to have substantial differences from long term means. In the 30-60m depth range, the residual is only 25% of the ZPG.

The residual in the 0-30m depth range is extraordinarily large, 5 times larger than the ZPG. The imbalance is so large that we must suspect the dynamics near the surface are different than we have assumed, i.e., our formulation of the problem may be fundamentally wrong. But if our assumptions are so far off the mark in the upper depth range, we are not justified in making the same assumptions at deeper depths. We must consider the smaller residuals in deeper depth ranges as coincidental, and possibly having no physical significance.

5. DISCUSSION

A basic premise of equatorial dynamics is that the westward wind stress is transmitted from the sea surface to deeper waters by small-scale vertical transport, there to be balanced by larger-scale forces resulting from the mean zonal pressure gradient, the upwelling of eastward momentum, the convergence of zonal momentum, and the mesoscale eddy flux. (e.g. Gill,1982). An equally basic premise of small-scale turbulence dynamics is that the local rate of production of turbulent kinetic energy is balanced by the rate at which it is locally dissipated, allowing the turbulent stress to be calculated from dissipation rate estimates. We have demonstrated that "typical" large-scale forces cannot balance the resultant of wind stress and turbulent stress forces in the upper 30 m at 140°W. The eastward turbulent stress at 30 m depth is too small to account for more than 20% of the westward surface wind stress, and typical large-scale forces can account for only 10% of the wind stress. Clearly, something is seriously in error.

Two possibilities should be explored to resolve the momentum imbalance: (A) the large-scale horizontal forces at the time of the TROPIC HEAT experiment might have been much larger than our "typical" estimates; or (B) our estimate of the momentum flux may be unrealistically low. We examine each in turn.

A. Large-Scale Anomalies:

The average wind stress during the 12-day turbulence experiment was 0.10 N/m^2, approximately a factor of 2 larger than the mean annual surface stress at 140°W (Weare and Strub, 1981; Weare, Strub and Samuel, 1981). The detailed response of the equatorial ocean to anomalous changes in the wind is unknown, and it may be overly optimistic to expect a rapid

response in the zonal pressure gradient or other large-scale processes to fluctuations in the wind. Large-scale, annual average circulation terms can only balance the large-scale, annual average wind stress. If the mean annual surface stress (BB use 56×10^{-6} m^2/s^2) is used instead of the measured wind stress, the residual R drops from -72×10^{-6} m^2/s^2 to -28×10^{-6} m^2/s^2, still twice as large as the ZPG. In a perfect world, we would use the same time scale to average the wind stress, the turbulent stresses, and the large-scale terms. Since such measurements are not available, we cast the problem in a different sense: If estimates of typical large-scale terms can explain only annually averaged surface stress, can the local wind stress be balanced by anomalies in the ZPG, the upwelling velocity, or the mesoscale eddy flux?

If our estimate of the turbulent momentum flux is correct, momentum supplied by the local wind was not transmitted very deeply by turbulent stirring. Can the disposition of the momentum supplied by the wind but not transmitted below 30 m be explained by a local, unmeasured anomaly with a depth scale of 30 m or less? Candidate anomalies which might explain the large residual are (a) the local acceleration of the top 30 m of the ocean; (b) an anomalous zonal pressure gradient that is much larger than the mean pressure gradient; (c) the local meridional advection of zonal momentum (i.e., an anomalous mesoscale eddy flux); (d) a local anomaly in upwelling. Each of these is treated below:

(a) We have entirely neglected the local acceleration of the top 30 m of the ocean in eq.(11); we now ask, "how large would the acceleration have to be to explain the residual?" If $R = \int \{\partial u/\partial t\} dz$, the average acceleration of the top 30 m would be 2.4×10^{-6} m/s^2, equivalent to a gain in westward velocity of 2.5m/s over the 12-day period. No such acceleration was observed, and we consider this explanation unlikely.

(b) Suppose that the residual is in fact balanced by an anomalous pressure gradient. To evaluate $R = -\int \rho^{-1} \partial P_a/\partial x \, dz$, where P_a is the pressure anomaly, one must estimate the horizontal scale of the anomaly. Since "mean" zonal pressure gradients have the scale of thousands of km (typically, the mean zonal pressure gradient is calculated from sections taken at 150°W and 110°W, a distance of 4400 km), an "anomaly" should have a smaller horizontal scale, say, 500 km. R could be explained by a difference in dynamic height of 0.12 m over the 500 km distance, and such a difference has been noted by Lemasson and Piton (1968) over the longitudes 135°W to 145°W, in the region of large thermocline slope. However, the required anomalous pressure gradient must be concentrated entirely in the upper 30 m. The mean zonal pressure gradient has a depth scale of 100 m or more (Mangum and Hayes, 1984; Fig. 5); the anomalies shown by Lemasson and Piton (1968) are vertically variable, but have depth scales comparable to the mean zonal pressure gradient. An anomalous pressure distribution with a 30 m depth scale can be caused by an anomalous temperature distribution, but in this case, the required expansion is

0.12m/30m = 4 parts per thousand. A typical expansivity for sea water is 0.2 parts per thousand per degree, so the pressure gradient anomaly would require a temperature change of about 20 degrees over 500 km. No such temperature distribution has been observed on the equator, and we consider this explanation unlikely.

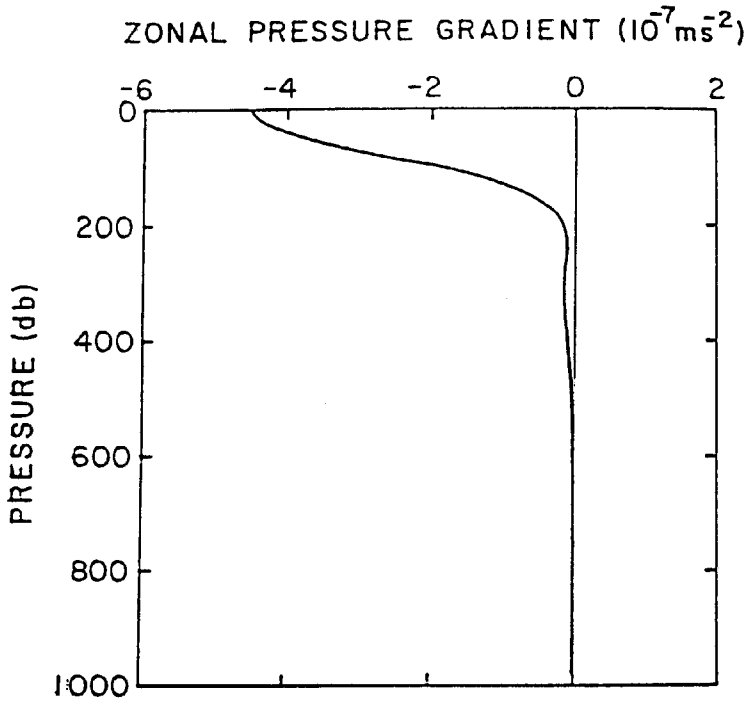

Fig. 5. Mean zonal pressure gradient from 1979 to 1981 (from Mangum and Hayes, 1984), calculated from NORPAX and EPOCS CTD sections at 150°W and 110°W, has a depth scale of greater than 100 m.

(c) Suppose the residual is balanced by a meridional advection anomaly. If $R = \int v(\partial u/\partial y)\, dz$, one must estimate the meridional scale. ADCP measurements from the MC vessel show a 6 day time interval midway through the measurement period when the meridional velocity averaged about 0.4 m/s northward (Chereskin et al, 1986; this appears to be related to the surface-intensified 21-day waveform discussed by Philander et al, 1985). To balance the residual requires a 260 km meridional length scale. The change δu in zonal velocity over a distance of δy = 260 km would have to be 1 m/s. Changes in zonal velocity of this size (to within a factor of 2 or so) are sometimes observed near the equator (Moum et al, 1986), with one crucial difference: <u>such fluctuations have a vertical coherence scale of order 50 m</u>, not less than 30 m as demanded by the residual. The sign is also wrong: BBH demonstrated that <u>the meridional advection of zonal momentum</u>

transports momentum westward, not eastward. We think it unlikely that this term could explain the residual, but because it is the large-scale term which has the shallowest depth scale, it should be investigated more carefully.

(d) If the residual is balanced by an upwelling anomaly, that is, $R = \int w(\partial u/\partial z)\, dz$, where w is an anomalous vertical velocity, one can estimate the size of w necessary for the balance. The vertical shear of zonal velocity during the 12 days of the turbulence experiment was of order 0.01 s^{-1} (Chereskin et al, 1986). Using this, we estimate that $w = 2.4 \times 10^{-4}$ m/s = 20 m per day. This is approximately 24 times as large as the mean upwelling velocity estimated by BB at 30m depth, and we think it unlikely that upwelling anomalies could explain the residual.

We can find no evidence that the large zonal momentum flux residual can be explained by anomalies in the large-scale effects. It is unlikely that zonal pressure gradient, horizontal advection, or upwelling anomalies are large enough to balance the residual; eq.(11) is so far out of balance that other explanations must be considered.

B. THE DISSIPATION METHOD REVISITED

The turbulent stress at 30 m depth is 20% of the surface stress. If we use the dissipation method for calculating eddy viscosity, we find that most of the momentum supplied by the local wind was not transmitted below 30 m depth, yet we know that momentum put into the ocean by the annual wind stress must penetrate deeply in order to balance the ZPG. We can only conclude that if the dissipation method is correct, local turbulent stirring in the upper 30 m does not respond rapidly to changes in the local surface stress. But there is direct evidence which indicates that turbulent mixing is a strong function of surface conditions: both MC and GPWOS found a very strong diurnal signal in the strength of turbulence above the undercurrent. Perhaps the truth is that the momentum supplied by the local wind is penetrating deeply, but our estimate of the momentum flux is wrong.

There are three potentially large sources of error which should be questioned: (a) Was the sampling adequate to determine the mean kinetic energy dissipation rate? (b) Is some process other than turbulent mixing responsible for vertically transporting most of the momentum at the equator? (c) Is the method of estimating turbulent stresses from kinetic energy dissipation rates formally flawed? We address each of these questions in turn:

(a) Is it possible that we have underestimated the dissipation rate by a factor of 4 because of sampling error? MC and GPWOS together made over 2,000 casts, each extending to 100 m depth or more. In all, well over 200 km of water was sampled; there is little hope of improving this record by another factor of 10 in the near future. We performed a bootstrap test of the significance of our mean turbulent stress calculation, using averages of the turbulent

stress over one hour and 10 m vertically to define a parent probability distribution having 288 elements in each 10 m depth bin. One thousand example averages were calculated by randomly drawing 288 samples from the parent distribution, and a histogram of the example averages was formed. It was found that 90% of the example averages were within 20% of the average of the parent distribution. We conclude that sampling error is an unlikely source of a factor of 4 error.

It is worth noting that GPWOS use the Cox number method (Osborn and Cox, 1972) to estimate the mean vertical heat flux, finding about -140 W/m^2 at 30 m depth. This estimate seems reasonable (at least within a factor of 2 or so), because the net surface heat flux was of roughly the same size. If the kinetic energy dissipation rates were grossly undersampled and underestimated, the Cox number would also be undersampled and underestimated. Both the vertical heat flux and the momentum flux would be seen as much too small if the turbulence were severely undersampled. Instead, the heat flux is roughly correct, but the momentum flux is much too small. If sampling error is invoked to explain the large residual, one must explain how the momentum flux could be increased without making the interior heat flux larger than the surface heat flux. Estimates of the vertical heat flux from the MC data set and dissipation measurements (Osborn, 1980) agree reasonably well (though perhaps only coincidentally) with the Cox number estimates of GPWOS and our own unpublished Cox number estimates.

(b) We have not considered vertical momentum transport by internal wave interactions. Internal waves can be excited by turbulence (Townsend, 1968; Wu, 1969), and can sometimes transfer momentum vertically (Muller, 1976; Booker and Bretherton, 1967). The present measurements are insufficient to determine whether or not internal wave radiation is the major momentum transport process. We only note that wave-like processes transport kinetic energy easily, but do not transport momentum very well, because horizontal and vertical velocity fluctuations are not well correlated in the internal wave band. While we cannot confidently evaluate internal wave momentum transport, in (c) below we demonstrate that internal wave kinetic energy transport by itself may be a major problem.

(c) It may not be possible to parameterize the vertical flux of horizontal momentum using the kinetic energy dissipation rate. The prescriptions for eddy viscosity and turbulent stress, eqs.(7), (8), and (9), all depend on eq.(6), the assumption that the local rate of production of turbulent kinetic energy is balanced by the local dissipation rate. Third-order correlations and pressure-velocity correlations have both been neglected in deriving eq.(6). Neither term has been measured in the ocean and there is no method to make such measurements directly. Wyngaard and Cote (1971) measured vertical velocity-turbulent kinetic

energy correlations in the atmospheric boundary layer, and found it to be a small quantity, sometimes positive and sometimes negative, when the stratification was stable. Wyngaard and Cote (1971) estimated the divergence of pressure-vertical velocity fluctuations as a residual in the unstable atmospheric boundary layer, but did not estimate it when the stratification was stable (presumably because there were too few examples when the atmospheric boundary layer was stable). We thus have no a priori reason to suppose that a local production-dissipation balance holds when the ocean is stably stratified.
There is some qualitative evidence that the pressure-velocity correlation divergence may be a significant term in the turbulent kinetic energy budget. C. Paulson (personal communication) observed a significant increase in internal wave isotherm amplitudes during the night, when turbulence was most energetic and dissipation rates were largest. We do not intend to discuss diurnal variations in equatorial turbulence, but simply note that increases in the internal wave energy content is observed to be correlated with increases in turbulent kinetic energy.

It may be possible for turbulent overturns to drive the internal wave field, because the rate of increase of wave energy is controlled by pressure-velocity correlations (Lighthill, 1978):

$$\frac{\partial E}{\partial t} = - \nabla \cdot (p\underline{u}) \quad , \tag{12}$$

where E is internal wave energy and $\underline{u}$ is the vector velocity. The mechanism described by eq.(12) is the radiative loss of turbulent energy to an internal wave field.

If the pressure-velocity correlation is large and confined to a restricted region of the fluid, internal waves can be excited, and energy can be radiated away, eventually to be dissipated elsewhere. If the energy is radiated away from the region it is generated, a local production-dissipation balance cannot hold, and kinetic energy dissipation rates may be only loosely related to the eddy viscosity and turbulent stress. In view of our inability to explain the large momentum budget residual with any other mechanism, we must consider the pressure-velocity correlation divergence as a possibly significant source of error, and we must treat estimates of the turbulent stress based on dissipation rates with skepticism.

6. SUMMARY AND CONCLUSION

We have shown that the momentum budget at 30 m depth and above on the equator at 140°W cannot be balanced using typical estimates of large-scale forces (zonal pressure gradients, upwelling of eastward momentum, convergence of westward momentum, and meridional eddy transport) and

conventional estimates of the turbulent stress. One of two conclusions must be drawn from this study: Either the large-scale processes during the time of measurement were highly atypical, or estimates of the momentum flux based on a production-dissipation balance are seriously flawed. For the present, we believe eddy viscosities estimated from measured dissipation rates must be treated with caution.
Estimates of anomalies in the large-scale forces which could explain the imbalance are unrealistically large, and do not have a realistic depth structure. Most of the problem may be attributable to the turbulent stress estimates because the stress estimate at 30 m depth is much smaller than the wind stress, and decreases exponentially below 30 m. It is possible that internal wave interactions play a crucial role in near-surface equatorial dynamics, either by transporting momentum directly, or by radiating energy and invalidating the hypothesis of a local production-dissipation balance.

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