Evolution of Vortical Flow Structure in an Ocean Boundary Process

G. Pawlak
University of Hawaii at Manoa, Honolulu, Hawaii, USA

P. MacCready and R. McCabe
University of Washington, Seattle, Washington, USA

Abstract. Flow across a rough boundary generates vortical structure over a broad range of spatial scales. The change in the pressure field associated with these structures is manifested as topographic drag. Relevant roughness scales can extend from less than a meter, for surface wave-driven flows, up to kilometers, for tidal currents. Here, we explore the relationship between the energy invested into and subsequently dissipated by the vorticity field and the work done by drag forces along the boundary. The case of oscillatory flow, as induced by tides or waves, further enables interactions between long-lived vortical structure of opposite signs. These interactions can introduce secondary circulations that provide a mechanism for the transport of interior fluid to the boundary as well as for redistribution of turbulent energy originating at the boundary and its eventual dissipation. We present analysis of observations and numerical simulations of vortex evolution in oscillatory flow and examine the dissipation mechanisms that lead to decay of the vorticity field. The flow structure and its longevity are found to be strongly affected by baroclinic mechanisms including lee wave generation and internal tides. Vortex longevity and interactions on successive oscillation periods, in turn, determine the fate of boundary turbulent energy.

Introduction

Flow along the ocean boundaries encounters a range of roughness scales, with the relevant scales for a particular flow set by the associated timescales. For surface waves in shallow water, relevant length scales are on the order of a meter or less. For tidal flows, the length scales of importance can extend to several kilometers. For longer timescale, quasi-steady flows, the roughness length scales are determined by rotational constraints. For all of these scenarios, the common characteristic that defines roughness is separation of the near boundary flow. This separation creates large scale vortical structure, radiates and dissipates energy, and introduces secondary circulations that can redistribute mass and momentum between the boundary and the flow interior. In addition, the separated flow is associated with significant drag on the flow (Kundu and Cohen [2002], Vosper et al. [1999], MacCready et al. [2003]). Very often these roughness scales fall below the scales resolved in numerical models. Accurate parameterizations of the associated processes are thus vital to realistic modelling of the flow.

Oscillatory flow across roughness, in particular, can generate strong residual circulations as a result of interactions between long-lived vortical structures on successive oscillation cycles (Pawlak and MacCready [2001]). For an idealized isolated roughness element, these types of interactions lead to a mean flow towards the boundary, for example, with a corresponding flow along the boundary away from the element. Long-lived eddies can further serve to transport momentum and mixed fluid away from the boundary and into the fluid interior.

From a depth-averaged consideration of the vorticity equation (c. f. Signell and Geyer [1991]), one can obtain characteristic time scales for the decay of vortical structure. The effects of bottom friction can be parameterized as

$$t_{bf} = \frac{h}{C_D U}$$

(1)

where $h$ is the flow depth, $C_D$ is a skin friction drag coefficient (a value of $2.5 \times 10^{-3}$ is typically used), and $U$ is the characteristic flow velocity associated with the vorticity field. Turbulent diffusion results in vorticity
decay over time scales given by

\[ t_{\text{turb}} = \frac{\delta^2}{A_H} \]  

with \( \delta \) representing the eddy length scale and \( A_H \) as the horizontal turbulent diffusivity.

For shallow flows, we can expect bottom friction to be the dominant mechanism for spin-down (Geyer [1993], Signell and Geyer [1991], Pawlak and MacCready [2001]). Observations of tidal headland eddies in shallow water (Geyer and Signell [1990]), where bottom friction effects are important, indicate that eddy longevity is limited. Using a typical value for bottom friction coefficient of \( C_D = 2.5(10)^{-3} \) and a tidal velocity of \( U = 30 \) cm/s, we can achieve an 8 hour spin-down time only for depths greater than 20 m. For deeper flows, equations (1) and (2) suggest that horizontal diffusion will be dominant. Diffusion, however, cannot affect the circulation associated with an eddy, and, thus, has only an indirect effect on formation of residual flows.

The timescales for vorticity decay derived from a depth-averaged analysis do not extend accurately to deep, stratified flows, however, due to the introduction of baroclinic mechanisms. Ocean boundaries are generally characterized by relatively low slopes of between 1:100 and 1:10. Very steep coastal regions may have slopes as high as 1:5. These numbers all suggest that separated flows along these boundaries will be characterized by significant tilt in the vorticity field. The effect of this tilt is not correctly represented in the depth-averaged flow equations. It is clear that a vortex inclined strongly relative to the vertical will tend to induce vertical motion. The extent to which it will lead to overturning will depend on the associated Richardson number (or, equivalently, Froude number). At high Richardson numbers (high stratification) the induced vertical motion will not be able to overcome the effects of the vertical gradients in density and will lead to mostly horizontal motion. It is not clear at this point how an inclined vortex can be reconciled with generally horizontal motion, however. We will discuss potential scenarios and examine the topology of the resulting vorticity field further below.

The establishment of residual flows from interactions between boundary generated vortical structure can lead to a net transport of energy between the fluid and the boundary. This energy can be considered either as a sink of flow energy, when considered from a reference frame fixed on the boundary, or as a source, taking a frame of reference moving with the fluid. For the latter case the energy source is the work carried out by the boundary on the fluid, primarily through form drag. The work associated with form drag will be manifested in the vortical flow field, which then transfers the energy to the flow interior as it dissipates. The energy transfer occurs during the initial stages of vortex development and understanding of the dynamics during this phase is crucial to predicting how much energy is transferred and consequently, what its fate is.

In this paper we will examine the evolution of vortical structure associated with flow along an individual roughness element along an ocean boundary. First we present results from field observations which yield an estimate of the lifespan of the flow structure and illuminate the spatial structure of the flow. We will discuss results from a numerical model and use these to extrapolate field observations and examine the three-dimensional nature of the flow. Finally we examine the implications of the observed and modelled vorticity fields and discuss possible mechanisms for baroclinic dissipation of vortical energy.

To begin with, we will explore the relationship between drag and vorticity. Given our understanding of drag over roughness (c.f. MacCready et al. [2003]), this will be useful in interpreting the effects of the boundary on energy and momentum transport along with the role of vortical structure in this transport.

### Vorticity and Drag

It has been observed that drag coefficients across roughness elements with significant flow separation have drag coefficients of order one (Moum and Nash [2000], Nash and Moum [2001], MacCready et al. [2003], Vosper et al. [1999]). It would be of use then to relate the drag to the vorticity field in some manner. The relationship between drag and circulation in the flow can be considered through a simple argument. Consider first a disc moving through a fluid at a constant velocity, \( U \) (figure 1). The disc has a cross-sectional area, \( A \), and drag coefficient, \( C_D \). The drag on the disc then, equal also to

\[ \text{Figure 1. Disc moving through a fluid at a constant velocity, } U. \text{ At time, } T, \text{ the disc has swept out a region of length } L = UT \text{ and, for } C_D = 1 \text{ has a wake of length, } 1/2UT. \]
EVOLUTION OF VORTICAL FLOW STRUCTURE

the rate of change of momentum in the fluid, is

\[ D = \frac{1}{2} \rho U^2 A C_D = \frac{d}{dt} (MV) \quad (3) \]

After a time, \( T \), then

\[ MV = [\rho U] \left[ C_D A \frac{1}{2} UT \right] \quad (4) \]

and the disc has accelerated a region of fluid of volume, \( C_D A \frac{1}{2} UT \), to a velocity, \( U \). We can examine the circulation associated with this motion, taking a path of integration as illustrated in figure 1. The circulation along this path is

\[ \Gamma = \frac{1}{2} C_D U^2 T \quad (5) \]

and the drag is evidently related to the circulation by

\[ D = \rho A \frac{d\Gamma}{dt} \quad (6) \]

The drag exerted by the boundary on the moving fluid, then, is manifested in the vorticity field via the total circulation. Similarly, work done by the drag force is also manifested in the vorticity field and subsequently distributed in the fluid through residual circulations.

Observations of Vortical Structure

Four sets of field observations were carried out in 2001 and 2002 at Three Tree Point in Puget Sound, Washington. The primary aim of the observations has been to examine the flow associated with a rough oceanic boundary, characterize the lifespan of vortical structure and diagnose the mechanisms for its decay. Three Tree Point is a pronounced headland with along and offshore length scales of approximately one kilometer. The bathymetry associated with the headland (figure 2) is fairly uniform with depth extending through the 230 m depth of the Main Basin of Puget Sound, along a steep slope (by oceanic standards) of approximately 1:5.

Here we present results from the field observations, focusing on data from one set of measurements carried out over nine days in June, 2002. These observations coincided with the spring tide portion of the fortnightly modulation, with a significant diurnal inequality in the tidal forcing (see figure 3). This diurnal inequality plays a key role in evolution of vortical structure as will be discussed further, below. The field study focused on the formation of the eddy on the south side of the headland associated with the strong flood tide. The measurements discussed below include Lagrangian trajectories from GPS-tracked drogued drifters, along with three-dimensional velocity fields obtained from vessel-mounted ADCP surveys.

Drifters

A set of 10 GPS-tracked drifters was released daily at varying locations (figure 3) around the headland and at varying phases of the tide. The drifters were drogued at a depth of 20 m using a 'holey sock' drag device in order to minimize the effects of surface wind-driven flows. GPS positions were obtained at 1 minute intervals over typical release durations of about 24 hours. Figure 4 shows the collective set of ninety drifter tracks from the nine days of releases at various phases of the tide. Each day’s deployment is referenced to the tidal phase, with the time relative to the maximum cross-channel averaged flood current, as predicted by a tide model for Three Tree Point (Lavelle et al. [1988]). Time series of drifter positions were smoothed using a 10 minute running Hanning filter. For the highest velocities observed (≈ 50 cm s\(^{-1}\)), this is equivalent to a 300 m spatial scale. More typical velocities (≈ 20 cm s\(^{-1}\)) equated to spatial scales of about 120 m. As seen from similar analysis of drifter tracks from August, 2001 (Pawlak et al. [2002]), the drifter tracks reveal a coherent structure that is repeated on subsequent tidal cycles.

An estimate of the magnitude of vorticity versus time can be obtained from drifter tracks using a least-squares method to obtain \( \partial \upsilon / \partial y \) and \( \partial \upsilon / \partial y \). It is apparent from the tracks in figure 4, that not all of the drifters are captured by the flood eddy and are thus not representative of the vorticity field associated with that eddy. The vorticity analysis then limited to only drifters that do not leave a given latitude band (roughly coincident with the right boundary of the plots and the leftmost transect line in figure 2). The calculation was further limited to a minimum of 15 latitude restricted drifters lying within a 2 km radius of the average location from this subset. These restrictions result in an average of 25 drifters per calculation.

Figure 5a shows a plot of the vorticity estimated using the method described above. The quantity shown represents, then, a collective estimate of the average vorticity from the cumulative drifter data, within the region covered by the drifters included in the calculation. The vorticity rises sharply to a peak at about 2 hours prior to max flood and then decays slowly over the following 17 hours. A least-squares fit of an exponential decaying curve to the data after the point of maximum vorticity gives an e-folding timescale of 7.7 hours. It could be argued that the vorticity estimate becomes inaccurate for times greater than about 6 hours from maximum flood since the drifter tracks from different days begin to diverge. The timescale estimate, however, is insensitive to truncating the data by a few hours on either end. We will compare this timescale to estimates derived from the depth-averaged vorticity.
From the drifters included in the calculation, we can calculate a mean separation and an average drifter location. The former quantity, plotted in figure 5b, will be compared later to estimates of vortex size from ADCP surveys and numerical modelling, and gives a measure of how well we are sampling the eddy. The latter quantity is related to the eddy position although it is not an exact measure since drifters are not necessarily spread evenly around the eddy. The average drifter location is plotted in figure 6. The markers along the plot correspond to those in figure 3. The track shows that the eddy travels northwest cross-channel after shedding and then back to the south (right) on the following flood. The latter stages of the drifter analysis are complicated by increased scatter in the day-to-day eddy center as is evident in figure 4c.

**Shipboard Surveys**

A set of seven surveys was conducted on successive days during the observational period using a 150 kHz vessel-mounted ADCP. Each individual survey consisted of a pair of cross-channel transects offset by 150 to 300 m in the along-channel direction (figure 2). The transect pairs were circulated repeatedly over durations of 8-12 hours extending from the start of the strong flood through a portion of the following strong ebb. The temporal coverage of the surveys is shown in figure 3. ADCP data was collected in 4 m vertical bins at 30 second intervals. Vessel velocity was maintained at roughly 6-7 knots when possible, giving an effective spatial averaging scale of about 100 m for the velocity measurements.

Data from each survey was referenced to a common time vector with $t = 0$ corresponding to the time of maximum flood current. Velocities for each survey were scaled linearly, using the ratio of maximum cross-channel averaged tidal current (from the Lavelle et al. model) for a given survey to the average maximum tidal current for the seven surveys.

Spatial velocity fields at selected depths were then calculated at 1 hour intervals using objective analysis of the collective data from the seven surveys. These velocity fields, then, represent a phase-averaged view of the flow field at a given depth. The spatial vorticity field was subsequently calculated from the phase-averaged velocities using centered-differencing. Figure 7 shows a vorticity sequence at a depth of 20 m. The phase-averaged vorticity is in qualitative agreement with the collective drifter tracks in figure 4. Also apparent in the later stages of the sequence is the formation of the ebb eddy on the north side of the headland. This feature is seen developing into a dipole with the releasing flood tide eddy in panels 7 and 8.
EVOLUTION OF VORTICAL FLOW STRUCTURE

Figure 3. Modelled average tidal variation at Three Tree Point over the period of observations. Time is referenced relative to peak maximum flood current for the strong flood tide. Green / red / blue line shows the temporal extent of ADCP survey / numerical model / drifter vorticity analysis. Circles / triangles represent one hour increments. Numerical model increments (diamonds) are in lunar hours (24.84 lunar hours per day).

Numerical Model Analysis

A numerical model of the flow at Three Tree Point was developed using the Hallberg Isopycnic Model (HIM, Hallberg and Rhines [1996]). Details of the model configuration are given in MacCready et al. [2003]. The model was started from rest and run for 100 lunar hours (1 lunar hour = 12.42/12 hours). Tidal forcing was consistent with that predicted by the Lavelle et al. model for the June field observations. At 100 lunar hours, the modelled tides corresponded to 5.5 hours prior to maximum flood current, with amplitudes matching those in the middle of the study period. Three dimensional velocity fields for the next 25 lunar hours were collected and interpolated to constant depths at 20 m intervals at 100 m horizontal resolution (equivalent to the model horizontal resolution). Vorticity fields at each depth level were then calculated using a centered-differencing scheme.

Figure 8 shows an eight panel vorticity sequence from the model beginning just prior to the maximum of the strong flood tide. The formation of a flood tide eddy is apparent in the first few panels, with the vortex from the preceding tide completing a dipole pair. Individual vortices can be seen persisting throughout the period shown in the sequence. It is interesting to note that the development is significantly altered on the successive flood tides. On the strong flood, the developing eddy forms a dipole with the preceding ebb eddy (panels 1-2), but as the following ebb develops, the flood eddy pairs with the succeeding ebb eddy and the dipole propagates across the channel (panels 3-8). On the weak flood.
**Figure 5.** Comparison of eddy characteristics from drifter data (blue), numerical model (red), and shipboard survey data (green). (a) Mean vorticity; (b) Eddy length scale (drifter data represents mean drifter separation); c) Circulation

**Figure 6.** Eddy location as identified by ADCP survey least-squares analysis (green), numerical model least-squares analysis (red) and drifter average center (blue). Symbols indicate time steps as described in figure 3.

**Figure 7.** Phase-averaged vorticity and velocity sequence from objectively analyzed shipboard ADCP survey data. Tidal phase represented by each plot is indicated by red bar in inset plot and time shown above is relative to time of maximum strong flood current.
then, the previous ebb eddy has already paired with the strong flood eddy so the new flood eddy does not form a dipole until it pairs with the succeeding (strong) ebb eddy. Over the complete diurnal cycle then, there will be a significant net transport across the channel from the strong flood, weak ebb pair. It is also qualitatively apparent that the weak ebb eddy is stronger than the strong ebb eddy, comparing panels 1 and 6. This is an indication that the flow history plays a significant role in the development and evolution of vortical structure. This will be examined quantitatively in the following section.

**Vortex Tracking**

Individual eddies in the vorticity fields from the numerical model and the shipboard surveys were analyzed quantitatively using a nonlinear least-squares fit of an idealized multi-parameter vortex profile. The vorticity field associated with a single vortex is approximated by a two-dimensional Gaussian profile given by

\[ \omega = \Omega e^{-(\alpha^2 + \beta^2)} \]  \hspace{1cm} (7)

where \( \Omega \) is the peak vorticity in the Gaussian distribution and \( \alpha \) and \( \beta \) are defined by

\[ \alpha = \frac{(x - x_0) \cos \theta - (y - y_0) \sin \theta}{L_A} \]

\[ \beta = \frac{(x - x_0) \sin \theta + (y - y_0) \cos \theta}{L_B}. \]  \hspace{1cm} (8)

Here \( x_0 \) and \( y_0 \) represent the location of the vortex center, \( L_A \) and \( L_B \) are the length scales for the vortex in orthogonal directions, and \( \theta \) is the angle of rotation of these directions. A fitting algorithm was developed to optimize the 6 parameters (\( \Omega, x_0, y_0, L_A, L_B, \theta \)) in equations 7 and 8 in a least-squares sense.

At a given depth, the fitting routine first searches for vorticity exceeding a threshold value in the vicinity of the headland to identify a new vortex. Once identified, the least-squares fit is performed within a region in the vicinity of the headland. The data in the region is masked to eliminate vorticity of the opposite sign to that of interest. For the following time step, the fit region is selected based on the vortex position and velocity field from the previous step.

**20 m Model Vorticity**

The analysis described above was carried out on model output at depths from 20 to 140 meters (at increments of 20 m). We will focus here on the 20 m analysis and return to the depth dependence of the flow.
later. A total of 5 eddies were tracked (3 negative and 2 positive) at each depth over the 25 lunar hour simulation. Nondimensional peak vorticity, $\Omega/\omega_0$, for each of the tracked vortices is shown in figure 9 versus time relative to the local maximum tidal velocity associated with that eddy’s formation. The vorticity scale, $\omega_0$, is defined by

$$\omega_0 = \frac{U_{\text{max}}}{\delta}$$

in which $U_{\text{max}}$ is the predicted local maximum of the cross-channel averaged tidal velocity magnitude and $\delta$ is the cross-channel length scale of the headland, given at 1 km. This vorticity scale is characteristic for separated flow behind a sharp step or obstacle with scale, $\delta$ (Pawlak and MacCready [2001]).

We can expect that the peak vorticity will be somewhat higher than $\omega_0$ as is apparent for most of the eddies represented in figure 9, although it is interesting to note that the eddy associated with the weak ebb tide is considerably stronger (20 times) than the characteristic scale. Its dimensional value is the strongest among the three ebb eddies analyzed and is comparable the strong flood eddy on which the field observations are focused. This is illustrative of the idea discussed earlier, that flow history plays a key role in the vortex dynamics of the flow. While the mean tidal velocity associated with the weak ebb might lead, in itself, to a weak eddy, the velocity at the tip of the headland is substantially greater, due to the strong flood eddy (figure 8, panels 4-6). This suggests that a more relevant vorticity scale might be based on the difference in maximum velocity between the tide of interest and the earlier tide defined as

$$\bar{U} = |U_{\text{max}}| - |U_{\text{pmx}}|$$

where $|U_{\text{pmx}}|$ is the maximum tidal current magnitude on the previous tidal phase. The eddy from the earlier tide does not always influence the successive tide, however, as is apparent for the weak flood eddy in figure 8, panels 7-9, where the earlier ebb eddy has formed a dipole with the strong flood eddy and propagated away. The alternative vorticity scaling also would overestimate the strength of the strong flood eddy (following the strong ebb) relative to the weak ebb eddy which has the same magnitude.

We can also account for the observed vortex paths by considering the relative strengths of the vortices. A strong vortex will tend to pull a weaker one around it and a dipole formed from unequal eddies will follow a curved path (curving toward the strong side). This is illustrated by the interactions between the eddies in panels 1-3 and panels 9-10 (followed by panel 1). For a diminishing inequality, we would expect the weak ebb eddy to develop further and pull the strong flood eddy around further, thus resulting in a dipole path directed more northward (left in the figure). The inequality in the tides then leads to significant changes in vortex paths and cross-channel circulation.

We will focus further attention on the positive vorticity eddy formed on the strong flood with the field observations. Figure 3 shows the temporal coverage of the vortex tracking analysis for the model data relative to the tide and to the drifter and survey observations. The estimated eddy center is plotted in figure 6. Markers along the path are spaced at 1 lunar hour and correspond to those shown in figure 3. The initial path of the vortex follows the mean drifter locations closely then deviates as the eddy is released on the following ebb. There is considerable deviation in the paths over time as the modelled eddy moves north and is trapped in the boundary layer along the west side of the channel.

The top plot in figure 5 shows the normalized average vorticity as derived from the maximum vorticity estimated using the least-squares method, to allow a comparison with the drifter observations of the average vorticity. The average vorticity for a Gaussian distribution given by (7) over an e-folding distance is given by

$$\bar{\omega} = \Omega (1 - e^{-1})$$

Along with the average vorticity defined by (11), the average vorticity was calculated from the model data within an ellipse centered at $(x_0, y_0)$ and with axes defined by $(L_A, L_B)$. This comparison quantity represented by the circles in the plot, serves to validate the estimate using the Gaussian fit. The model vorticity shows reasonable agreement with the drifter estimates, exhibiting a decay in intensity throughout the flood tide. The vorticity shows an increase during the last 5 hours tracked as the eddy is captured in the boundary layer along the west channel wall.

The spatial scale of the modelled eddy can be summarized from the two length scales $L_A$ and $L_B$ defined in (7) and (8) by $L = \sqrt{L_AL_B}$. This quantity is plotted in figure 5b. The length scale from the least-squares fit is comparable to the headland scale, as expected, increasing initially and then remaining constant over time. This is in contrast to the drifter separation which increases with time. The increase in drifter separation can be expected due to both loss of drifters from the eddy core and day to day variability in eddy center.

Finally, from the least-squares analysis we can estimate the circulation associated with an individual eddy. Again considering the idealized Gaussian profile given by (7) and (8), we find the corresponding circulation as

$$\Gamma = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \omega dx dy = \pi L_AL_B \Omega.$$
The quantity in equation 12 is actually obtained by integrating the Gaussian distribution over an area of $(\Delta x, \Delta y) = (2L_x, 2L_y)$ around the calculated vortex center to permit masking of regions that fall outside the channel area. The result is plotted in figure 5c, normalized by $\pi \omega_0 \delta^2$. The circulation increases through the point of maximum flood current and slowly decreases over time. Also plotted in figure 5c (circles) is the circulation obtained by integrating the vorticity over an ellipse centered at $(x_0, y_0)$ and with axes defined by $(2L_A, 2L_B)$.

20 m Survey Vorticity

The least-squares analysis was also carried out for the phase-averaged shipboard survey data at 20 m depth. The estimated path of the center is shown in figure 6. The path follows that of the drifter mean location closely, although closer examination of the time markers indicates that the drifter center is offset somewhat. The average vorticity (figure 5a) shows surprisingly little variation over the survey period, with little or no decay evident, in comparison to the drifter vorticity estimate. The least-squares fit becomes more suspect after about $t = 1$ hr, however, since the survey fails to cover the entire region of vorticity.

The length scale from the least-squares fit indicates that the model apparently underestimates the eddy scale. The estimate from the survey data is based on an average of multiple days, however, and variability in eddy location would lead to an overestimate in eddy scale. Again, the sharp drop after $t = 1$ is questionable due to the lack of data coverage.

The circulation, as defined by (12), is plotted in 5c. As for the model data, we also compare the least-squares fit estimate with the integral of the vorticity over an ellipse centered at $(x_0, y_0)$ and with axes defined by $(2L_A, 2L_B)$ (circles). The comparison is weakest around the time of max flood when the coverage of the eddy is poorest. Although the estimate is again affected by the incomplete coverage of the eddy, it is clear that the model underestimates the circulation as obtained from the phase-averaged survey data. This is mostly a reflection of the difference in scale estimate, which appears in the circulation to the second power.

The comparisons with the phase-averaged survey and drifter data show indicate, nonetheless, that numerical model does a reasonable job of describing the position, scale and intensity of the flood eddy. We will use the model data further, below, to consider the depth variation in the vorticity field and to examine the three-dimensional flow structure of the strong flood eddy.

Model Vorticity versus Depth

The least-squares analysis was carried out on the numerical model data at 20 m intervals in depth through a depth of 140 m. Again, we will focus our analysis on the development of the strong flood eddy. Least-squares estimates of vorticity and circulation are summarized in figure 10. The peak vorticity (fig. 10a) is fairly constant with depth during the growth phase of the eddy. As the flood tide weakens, a sharp decay in vorticity is seen followed by a shallower decline after the end of the flood. It appears, as well, that the decay is more significant for the deeper layers, although the rate of decay is similar at all depths. The higher decay at depth might be argued as being consistent with decay due to bottom friction, although it should be noted that peak vorticity is not an integral quantity. To consider the development and decay of the vortex, the circulation (fig. 10) is a more appropriate quantity. The circulation at maximum flood is higher in the shallow layers. The decay in circulation begins earlier for the deeper levels, again with the rate of decay similar at all depths.

We can examine the growth of the eddy in closer detail, in relation to drag exerted by the headland on the flow, using equation 5. If we take the crude approximation of a constant velocity using the maximum flood current, $U_{max}$, we should get a conservative estimate for circulation growth versus time. It is clear, however, from figure 10b, that this simple model, using $C_D = 1$, (green line, right plot) significantly underestimates the modelled circulation. If, instead, we use a velocity scale given by the difference between maximum currents for the strong flood and the prior ebb tide (red line), as given by eqn. 10, then the growth rate estimate from equation 5 is remarkably good.

Given estimates of eddy location at each depth, we can also examine the three-dimensional structure of the eddy. Since the eddy has formed along the channel boundary, we can expect its initial orientation to reflect
Figure 10. Least-squares estimates of normalized model vorticity (a) and circulation (b) vs. time and depth. Plots on right hand side show evolution at selected depths versus time. Straight lines in (b) (right plot) show predictions for circulation growth using eqn. 5 using maximum flood current velocity (green) and difference between maximum flood and previous maximum ebb velocity magnitude (red).

The initial development of the separation vortex is consistent with our simple relation between momentum transfer due to form drag and vorticity (eqn. (5)), only if we consider the alternate velocity scaling using the difference in velocity magnitudes between the two most recent tides, as given by eqn. (10). This is also apparent from the general magnitude of the maximum vorticity ($\sim 10U_{Imx}$) as well as from the prominence of the weak ebb eddy in the numerical model. The physical rationale for the relevance of this alternate scaling results from the role of the flow history in establishing the velocity field near the headland. As shown in the discussion on the model results, the paths taken by separation eddies vary significantly on distinct tidal cycles, depending on the behavior of the eddies formed earlier. The magnitude of the velocity just offshore of the

Discussion

The initial development of the separation vortex is consistent with our simple relation between momentum transfer due to form drag and vorticity (eqn. (5)), only if we consider the alternate velocity scaling using the difference in velocity magnitudes between the two most recent tides, as given by eqn. (10). This is also apparent from the general magnitude of the maximum vorticity ($\sim 10U_{Imx}$) as well as from the prominence of the weak ebb eddy in the numerical model. The physical rationale for the relevance of this alternate scaling results from the role of the flow history in establishing the velocity field near the headland. As shown in the discussion on the model results, the paths taken by separation eddies vary significantly on distinct tidal cycles, depending on the behavior of the eddies formed earlier. The magnitude of the velocity just offshore of the
EVOLUTION OF VORTICAL FLOW STRUCTURE

headland tip is determined by the tidal current plus the contribution from the vorticity field. The scale of the vorticity field is determined by the flow history, most importantly the tidal magnitude on the earlier cycle. If the eddy from the earlier tidal cycle is not present, as in the case of the weak flood eddy (figure 8) then the flow history is of less importance.

For all the eddies examined in the model and the observational data, it is apparent that vortex growth ceases near the time of maximum flood. Our relationship between drag and circulation would then suggest that drag would reach a maximum in the early stages of flood and decay rapidly near the time of maximum flood. Estimates of drag using model data (MacCready et al. [2003]) seem to support this.

The vortex decay observed both in the model and observational data occurs over a much shorter timescale than that suggested by timescales from depth-averaged considerations. Using parameters characteristic for Three Tree Point in equation (1) gives an unrealistically long decay time of 111 hours. An eddy viscosity argument based on estimates of turbulence downstream of a sharp step gives an upper bound for horizontal turbulent diffusivity, $A_H = 8m^2s^{-1}$ (Pawlak and MacCready [2001]). This gives a decay timescale due to diffusion of 35 hours. Our estimate of 7.7 hours from the drifter data in figure 11 indicates that baroclinic mechanisms may be important.

The tilt of the vortex center illustrated by the model data in figure 11 indicates that baroclinic mechanisms may play a significant role in the decay of the vorticity field. A vortex that is inclined at a 1:5 slope cannot be adequately represented by a vertically averaged approach. The associated horizontal component of the vorticity will introduce isopycnal tilt and, in the limit of weak stratification, overturning. The ability for a vortex to overturn the stratified ambient fluid is determined, of course, by the Richardson number criteria

$$Ri = \frac{N^2}{\omega^2} < 0.25.$$  (13)

For Three Tree Point, typical values of $N$ are 2 to $4(10)^{-3} s^{-1}$, while our highest values for vorticity are $\omega \sim 10 \omega_0 = 2(10)^{-3} s^{-1}$, so we are not in a regime with large scale overturning. The tilted vortex we observe can then tilt isopycnals, but not overturn them. The tilted isopycnals will introduce baroclinic circulation of opposite orientation to the tidal eddy and thus lead to a net loss of circulation.

Two questions are raised by these ideas: how do internal baroclinic mechanisms and the related change in circulation relate to energy loss and how can a tilted vortex core exist with only horizontal motion? The second question might appear initially trivial, but close consideration of the horizontal velocity field necessitated by a strongly stratified environment suggests that a vortex with an inclined core can only exist if vortex lines are not aligned with that core. (This can be seen from considering two thin horizontal layers offset laterally, with horizontal rotational motion in each. The resulting vertical shear between the two layers can account for only half of the vorticity necessary to tilt the vortex core to the angle prescribed by the offset. This equates to a tilt in the vortex lines equal to half of the tilt in the core.) This, in turn, implies that vortex lines end in the fluid, thus violating the Helmholtz theorems. While the vorticity remains attached to a sloping boundary and thus to its source of vorticity, there is no dilemma. Once the vorticity separates, however, the tilt poses a topological problem. One possible resolution of the issue is suggested by laboratory experiments of wakes in stratified fluids that reveal the formation of vertically oriented ‘pancake’ vortices separated by regions of strong vertical shear (Lin and Pao [1979], Brown and Roshko [1993], Spedding [1997]). There is also evidence that vortex pairs in stratified fluids are unstable to axial disturbances and break down into these pancake type structures (Billant and Chomaz [2000]). The layers associated with these vertical vortex structures is marked by strong shear at the interfaces between pancakes as well as by enhanced dissipation (Fincham et al. [1996]), thus offering a potential resolution to the first question posed above.

There is some question about how this type of layered structure would be manifested in a layered flow model like that used for the analysis above. The isopycnic model pre-establishes a layered topology and the modeled vortices can be considered to have a default pancake structure. Although not included here, the depth structure in the field data clearly shows a tilt in the vortex core in qualitative agreement with the model. It is not clear, however, how even a highly resolved three-dimensional data set could capture a layered structure using a phase-averaged approach.

The observations and model results presented here summarize the relationship between boundary drag and vortical flow structure. The analysis indicates that eddy growth is consistent with simple momentum balance between form drag force and momentum input into a vortical wake. The relatively rapid timescale for decay suggests that baroclinic mechanisms may be important in vortex evolution and raise questions on flow topology and its implications for dissipation.

Acknowledgments. The authors are indebted to Kate Edwards and her numerous volunteers who participated in the field data collection. We would also like to thank the crews of the R/V Miller and R/V Barnes for their skilled
work and patience during the long hours of field work.

References


This preprint was prepared with AGU’s LATEX macros v4, with the extension package ‘AGU++’ by P. W. Daly, version 1.6a from 1999/05/21.