

KELVIN WAVES

B Wang, University of Hawaii, Honolulu, HI, USA

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Wang, B

University of Hawaii,
Department of Meteorology,
2525 Correa Road,
Honolulu, HI 96822, USA

Introduction

The Kelvin wave is a large-scale wave motion of great practical importance in the Earth's atmosphere and ocean. Discovered by Sir William Thompson (who later became Lord Kelvin) in 1879, the Kelvin wave is a special type of gravity wave that is affected by the Earth's rotation and trapped at the Equator or along lateral vertical boundaries such as coastlines or mountain ranges. The existence of the Kelvin wave relies on (a) gravity and stable stratification for sustaining a gravitational oscillation, (b) significant Coriolis acceleration, and (c) the presence of vertical boundaries or the equator. The unique feature of the Kelvin wave is its unidirectional propagation. The Kelvin wave moves equatorward along a western boundary, poleward along an eastern boundary, and cyclonically around a closed boundary (counterclockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere). The wave amplitude is largest at the boundary and decays exponentially with distance from it. At the Equator, Kelvin waves always propagate eastward, reaching their maximum magnitude at the Equator and decaying exponentially with increasing latitude.

There are two basic types of Kelvin waves: boundary trapped and equatorially trapped. Each type of Kelvin wave may be further subdivided into surface and internal Kelvin waves. Surface, or barotropic, waves penetrate the entire depth of the fluid. Kelvin waves also appear within the stably stratified ocean and atmosphere, and are called internal, or baroclinic, Kelvin waves. Internal Kelvin waves are often found in a layer with large density gradients; the density gradient acts as an interface that allows the existence of internal gravity waves. Examples of such density gradients are the oceanic thermocline (a layer of large vertical temperature gradient separating a shallow layer of warm surface water about 50–200 m deep and a much deeper layer of cold water below) and the

lower edge of an atmospheric inversion, a layer in which temperature increases with height. Like gravity waves, Kelvin waves can also propagate vertically in a continuously stratified geophysical fluid.

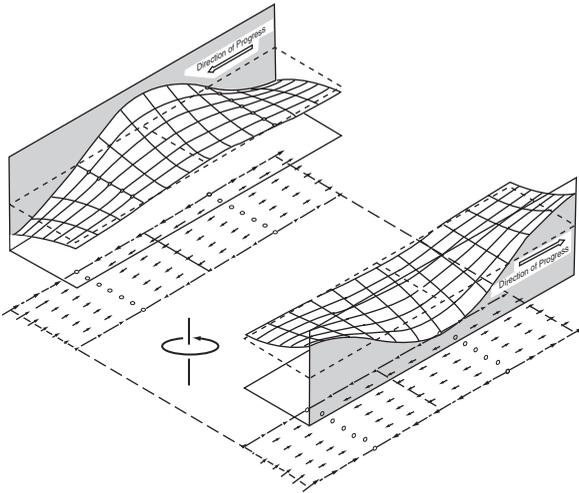
Atmospheric Kelvin waves play an important role in the adjustment of the tropical atmosphere to convective latent heat release, in the stratospheric quasi-biennial oscillation, and in the generation and maintenance of the Madden–Julian Oscillation. Oceanic Kelvin waves play a critical role in tidal motion, in the adjustment of the tropical ocean to wind stress forcing, and in generating and sustaining the El Niño Southern Oscillation.

Boundary-Trapped Kelvin Waves

Surface Kelvin Waves

The mechanism and properties of the Kelvin wave can be illustrated by considering a horizontally propagating Kelvin wave in a rotating fluid of uniform finite depth H , where H is small compared to the horizontal extent of the fluid. The fluid has homogeneous density and a free surface, and is confined by a vertical lateral boundary. Such an idealized model is referred to in geophysical fluid dynamics as a shallow water model.

The lateral bounding wall prohibits flow across the boundary, and this absence of transverse motion with respect to the lateral boundary is a defining characteristic of Kelvin waves. Fluid parcels (elements) are constrained to move in a vertical plane parallel to the lateral boundary in the neighborhood of the boundary. Thus, the horizontal longshore (along the boundary) component of the Coriolis force must vanish. Consequently, the wave motion at the lateral boundary, and in any parallel vertical plane, is exactly the same as that in a nonrotating system, i.e., the shallow water gravity wave (Figure 1). The wave travels along the boundary with the shallow water gravity wave speed C determined by the square root of the product of the gravitational acceleration (g) and the depth of the fluid, $C = (gH)^{1/2}$. The shape of the wave in the longshore direction is arbitrary and is conserved as the wave travels. This implies that the surface Kelvin wave is nondispersive, and that the wave energy is transmitted at the speed of the shallow water gravity wave. Because the Kelvin wave solution in any vertical plane parallel to the lateral boundary is identical to that of the nonrotating case, the energy of a Kelvin wave is partitioned equally between kinetic and potential energy.



0191-F0001 **Figure 1** Northern hemisphere Kelvin waves on opposite sides of a channel that is wide compared with the Rossby radius. In each vertical plane parallel to the coast, the currents (shown by arrows) are entirely within the plane and are exactly the same as those for a long gravity wave in a nonrotating channel. However, the surface elevation varies exponentially with distance from the coast in order to give a geostrophic balance. This means Kelvin waves move with the coast on their right in the Northern Hemisphere and on their left in the Southern Hemisphere. (From Mortimer, 1977.)

0191-P0030 A fundamental difference between the Kelvin wave and the two-dimensional gravity wave is that the Kelvin wave can propagate in only one direction, rather than in two opposite directions. This is due to the constraints of the Earth's rotation and the presence of the lateral boundary. Rotation modifies the flow by piling up fluid against the lateral boundary through Ekman transport, producing an offshore (normal to the boundary) pressure gradient force associated with the surface elevation. Since offshore motion is prohibited by the presence of the boundary, the offshore pressure gradient force must balance the Coriolis force associated with the longshore flow. For this reason, the motion is referred to as semigeostrophic (i.e., geostrophic balance is reached in only one direction). A direct consequence of this geostrophic balance is the exponential decay of the longshore velocity and surface height with distance from the lateral boundary (Figure 1). The Kelvin wave amplitude is significant only within an e -folding distance of the order of the Rossby radius of deformation (R) from the lateral boundary. This important length scale is defined by the ratio of the gravity wave speed, C , over the absolute value of the Coriolis parameter (f). Over this characteristic distance, the tendency of the gravitational force to flatten the free surface is balanced by the tendency of the Coriolis force to deform the surface. This is possible only for a wave traveling in the direction along which the lateral boundary (where the wave has maximum amplitude) is always on the right

in the Northern Hemisphere and on the left in the Southern Hemisphere. Therefore, the effects of rotation and the lateral boundary determine the unidirectional propagation and the trapped behavior of the Kelvin wave.

Internal Kelvin Waves

The horizontally propagating internal, or baroclinic, Kelvin wave behaves in the same manner as the surface wave except that the motion varies with depth. Most frequently, it occurs at a stable interface (or a thin layer with large vertical density gradients) separating two relatively homogeneous layers. For internal Kelvin waves, the pressure gradient force normal to the lateral boundary arises from the tilt of the interface and is balanced by the Coriolis force associated with the vertical differential flow parallel to the boundary. The internal Kelvin wave speed depends on the density difference across the interface and is normally much slower than that of surface Kelvin waves. In the ocean, the typical speed for internal coastal Kelvin waves is of the order of 1 m s^{-1} and the Rossby radius of deformation is of the order of 10 km in the mid-latitudes. Evidence for coastal Kelvin wave propagation along the eastern boundary of the Pacific has been observed in coastal sea level and temperature records.

In the atmosphere, boundary trapped Kelvin waves occur primarily in the form of internal waves. They are often found along the edge of a plateau or a mountain range, such as the coast of South Africa, the west coast of California, and the eastern flank of the Tibetan Plateau. These internal Kelvin waves are created near a stable inversion layer (often located at the top of the boundary layer) against steep topography. The elevated plateau or mountain range rises above the inversion, forming a lateral boundary for the air of the lower layer. Energy is prevented from escaping vertically by the inversion and prevented from escaping laterally by the topography. The maximum disturbance intensity of these waves is found near the coast or plateau and decreases exponentially in intensity away from the coast or plateau. The offshore extent of the waves depends on both the thermal structure and topography. When the topography is steep, the depth of the inversion determines the Rossby radius for the atmospheric Kelvin wave. A typical value for atmospheric internal Kelvin waves is on an order of 1000 km.

Equatorially Trapped Kelvin Waves

Matsuno in 1966 showed that the eastward-propagating Kelvin wave is a possible free solution to the perturbation equations of the shallow water model on

an equatorial β -plane (β is the meridional gradient of the Coriolis parameter), provided that the meridional velocity vanishes. This type of wave is called an equatorial Kelvin wave, so named because it is extremely similar in character to coastally trapped Kelvin waves, with the Equator serving as a boundary. Like the coastal Kelvin wave, the propagation of an equatorial Kelvin wave is unidirectional, i.e., eastward only. In each vertical plane parallel to the equatorial vertical plane, the motion of fluid particles is precisely the same as that in a shallow water gravity wave (Figure 2). The Kelvin wave propagates without dispersion at the speed $C = (gH)^{1/2}$, as for nonrotating gravity waves. Because the Coriolis parameter changes sign at the Equator, eastward flow occurring on both sides of the Equator would induce equatorward Ekman mass transport, piling up fluid at the Equator and generating a meridional pressure gradient force. In this sense, the Equator acts as a lateral wall. The Earth's rotation links the motion in each latitudinal plane, because momentum conservation in the

north–south direction requires a geostrophic balance between the eastward velocity and the meridional pressure gradient force. This geostrophic balance results in the perturbation zonal velocity reaching a maximum on the Equator and decaying with increasing distance from the Equator (Figure 2). This is possible only for eastward-traveling waves. Thus, equatorial Kelvin waves are eastward-propagating and have zonal velocity and pressure perturbations that vary with latitude as Gaussian functions centered on the Equator (Figure 2). The e-folding distance for decay with increasing latitude is given by $Rc = (C/2\beta)^{1/2}$, where $C = (gH)^{1/2}$ is the gravity wave speed and β is the meridional gradient of the Coriolis parameter at the equator. Rc is called the equatorial Rossby radius of deformation, because of its relationship with the decay scale for the case of constant Coriolis parameter. The same analysis can be applied to baroclinic waves in both atmosphere and ocean, with H being interpreted as the equivalent depth. Typical values of the internal gravity wave speed, C , for the tropical atmosphere are $20\text{--}80\text{ m s}^{-1}$, giving an equatorial Rossby radius of between 6 and 12 degrees of latitude. For baroclinic ocean waves, appropriate values of C are typically in the range of $2\text{--}3\text{ m s}^{-1}$, so that the equatorial Rossby radius is $200\text{--}250\text{ km}$.

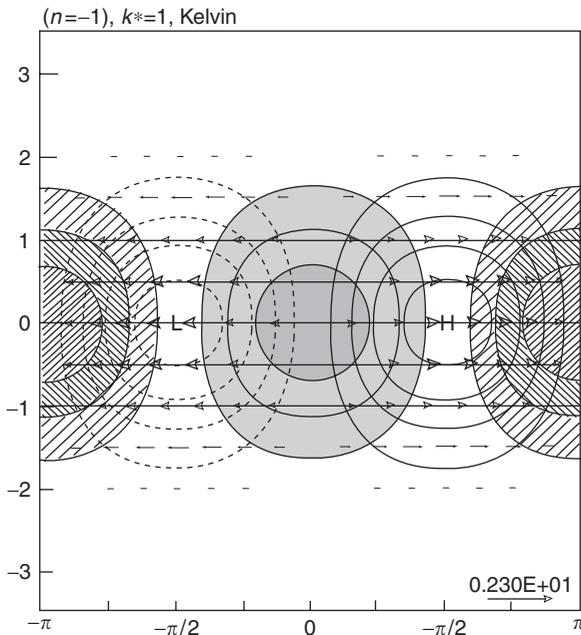


Figure 2 The theoretical equatorially trapped Kelvin wave solution to the linear shallow water equations on an equatorial β -plane for a nondimensional zonal wavenumber 1. Hatching is for convergence and shading for divergence, with a 0.6 unit interval between successive levels of hatching or shading, and with the zero divergence contour omitted. Unshaded contours are for geopotential, with a contour interval of 0.5 units. Negative contours are dashed and the zero contour is omitted. The largest wind vector is 2.3 units, as marked. The dimensional scales are as in Matsuno (1966). (Reproduced with permission from Wheeler M, *et al.* (2000) Large-scale dynamical fields associated with collectively coupled equatorial waves. *Journal of the Atmospheric Sciences* 57(5): 613–640.)

Vertically Propagating Kelvin Waves

In general, the Earth's rotation traps planetary-scale gravity waves in the troposphere unless the frequency of the wave is greater than the Coriolis frequency (about $(1\text{ day})^{-1}$). For this reason, mid-latitude synoptic waves are generally unable to penetrate significantly into the stratosphere. However, near the Equator, the dramatic decrease in the Coriolis parameter allows these longer-period waves to propagate vertically. Vertically propagating Kelvin waves have been identified in both the equatorial atmosphere and ocean.

Vertically propagating Kelvin waves can be illustrated by considering a continuously stratified fluid with a constant buoyancy frequency in a semi-infinite vertical domain near a lateral boundary or in the vicinity of the Equator. For simplicity, consider a linear equatorial β -plane model. Solutions can be obtained using the normal-mode technique by neglecting meridional perturbations. In a vertical section along the equator, or in any parallel vertical plane, the motion of vertically propagating equatorial Kelvin waves shares the properties of an ordinary vertically propagating gravity wave. A vertical cross-section of the perturbation motion, pressure, and temperature structure for vertically propagating Kelvin waves is shown in Figure

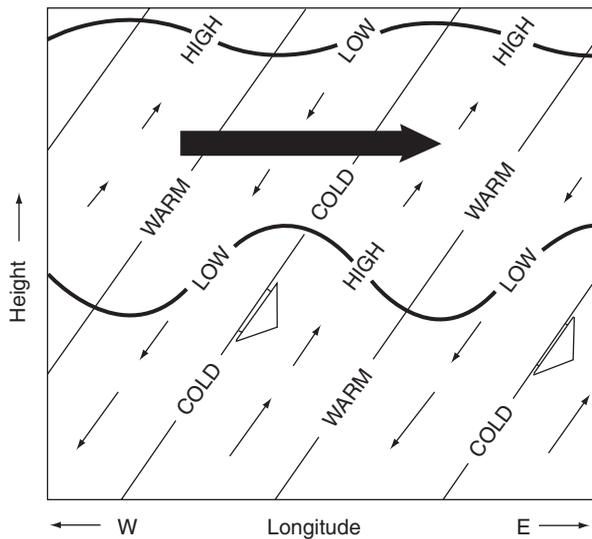


Figure 3 Longitude–height section along the Equator showing pressure, temperature, and wind perturbations for a thermally damped Kelvin wave. Heavy wavy lines indicate material lines; short blunt arrows show phase propagation. Areas of high pressure are shaded. Length of the small thin arrows is proportional to the wave amplitude, which decreases with height owing to damping. The large shaded arrow indicates the net mean flow acceleration owing to the wave stress divergence. (Reproduced with permission from Holton JR (1992) *Introduction to Dynamic Meteorology*, 3rd edn. San Diego: Academic Press.)

3. The local change in temperature is due to adiabatic warming or cooling, so that the temperature oscillation leads the vertical (zonal) wind and pressure oscillations by a quarter-cycle.

Since stratospheric Kelvin waves are forced from below by disturbances in the troposphere, the wave energy propagation must have an upward component. According to theory, Kelvin waves become dispersive in the presence of vertical propagation, and the vertical component of their phase velocity is always opposite to that of their group velocity (the velocity at which the wave energy is transmitted). Thus, the phase velocity must have a downward component. The condition of eastward propagation due to equatorial trapping requires that the vertical wavenumber be negative. Hence, an eastward- and downward-propagating equatorial Kelvin wave has constant phase lines that tilt eastward with height (Figure 3).

Because the variation in zonal wind depends on the pressure gradient force, the highest zonal pressure gradient precedes the largest westerly acceleration by a quarter-wavelength, and the zonal wind and pressure waves coincide. This creates an upward flux of wave energy. An individual parcel moves up along the tilted phase line, bringing westerly momentum upward, and moves down, bringing easterly momentum downward. Thus, the Kelvin wave transports westerly momentum upward.

Significance of Kelvin Waves in the Atmosphere and Ocean

Oceanic Kelvin Waves

Kelvin waves are essential in the description of ocean tides. For a deep ocean ($H = 5$ km) at 30° N, the Rossby radius for barotropic Kelvin waves is about 3000 km. Continental shelf regions normally extend about a hundred kilometers seaward; hence, a steep continental slope is practically indistinguishable from a vertical boundary at the scale of the Rossby radius. Thus, a barotropic Kelvin wave extends far from the coast and occupies a substantial fraction of a typical ocean. Much of the energy of tide waves traveling along continents is transmitted in the form of barotropic Kelvin waves with a speed of about 200 m s^{-1} . For instance, along the coast of California more than two-thirds of the semidiurnal and half the diurnal tidal amplitudes can be accounted for by traveling barotropic Kelvin waves. For shallow seas and coastal waters, the Rossby radius is about 200 km. When a Kelvin wave moves through a region in which the fluid depth or the Coriolis parameter varies and the wave energy flux remains constant, the amplitude of the wave varies in proportion to $(f/H)^{1/2}$. Thus, wave amplitude increases when Kelvin waves move into shallow water. In coastal regions, Kelvin waves can also be generated as storm surges are diffracted by vertical boundaries and scattered by irregular coastlines. Variable longshore winds and atmospheric pressure gradients acting on the sea surface are also possible energy sources for oceanic Kelvin waves.

Internal coastal Kelvin waves can be generated by wind-induced, time-dependent coastal upwelling. Coastal upwelling (downwelling) is caused by an Ekman mass flux transported offshore (onshore) and forced by longshore winds. The disturbances can then propagate along the coast as boundary-trapped internal Kelvin waves. Therefore, the amount of upwelling depends not only on local wind forcing but also on the forcing that generated the waves at an earlier time. In a lake, during strong wind forcing, the region of upwelling progresses cyclonically (in the Northern Hemisphere) at the speed of an internal Kelvin wave.

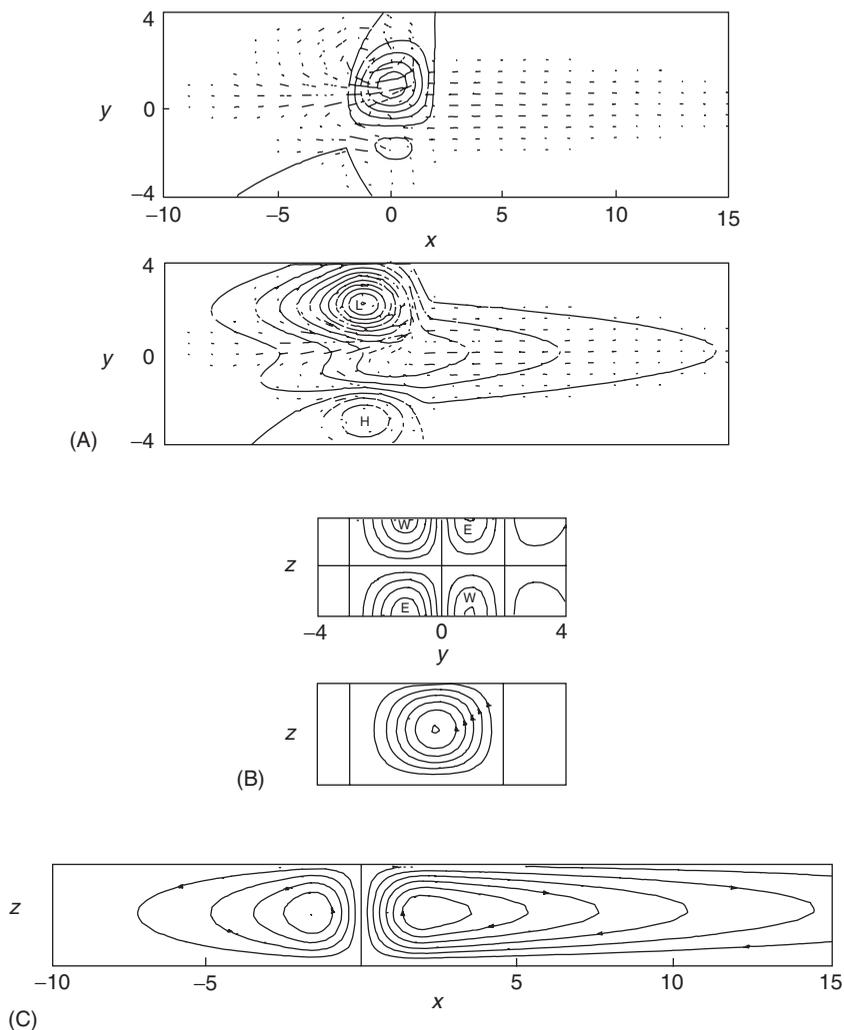
Equatorial Kelvin waves play a critical part in thermocline adjustment. As the equatorial ocean circulation responds to wind stress forcing, equatorial Kelvin waves transmit signals rapidly from the western to the eastern extremities of the ocean basin. Internal Kelvin waves propagating along the thermocline take about two months to cross the entire equatorial Pacific. These equatorial Kelvin waves play an essential role in the El Niño Southern Oscillation (ENSO) cycle. First, they support a positive feedback

between the central Pacific zonal wind and eastern Pacific sea surface temperature (SST) anomalies. A westerly wind anomaly excites downwelling Kelvin waves, which propagate into the eastern Pacific, suppressing the thermocline and causing the SST to rise; this, in turn, enhances the central Pacific westerly wind anomaly by increasing the eastward pressure gradient force in the atmosphere. This positive feedback provides a development mechanism for ENSO SST warming. Second, the cyclonic wind stress curl associated with the central Pacific westerly wind anomaly can induce upwelling oceanic Rossby waves that propagate westward. These waves are eventually reflected at the western ocean boundary, generating

upwelling equatorial Kelvin waves, which propagate into the eastern Pacific and offset the warming by enhancing vertical cold advection. This negative feedback provides a mechanism for turning the coupled system to its opposite (La Niña) phase and sustaining the ENSO cycle. In addition, the atmospheric intraseasonal wind forcing continuously generates equatorial Kelvin waves whose nonlinear rectification to the mean state may also contribute to the eastern Pacific warming.

Atmospheric Equatorial Kelvin Waves

The atmospheric equatorial Kelvin wave is one of the critical wave motions in the response of the tropical



0191-F0004 **Figure 4** The solution of the forced shallow water equations for heating (or evaporation) that is confined to the range of longitudes $|x| < 2a_e$. The distribution with latitude has a maximum north of the Equator. The arrows in (A) give the horizontal velocity field and the solid contours in the upper panel give the vertical velocity, which has a distribution close to that of the heating function. The motion is upward within the contours north of the Equator with a maximum near $x = 0, y = a_e$. The contours in the lower panel are pressure contours, and the axes are labeled in units of a_e . (B) The meridional circulation when the response is interpreted as a baroclinic response to heating with sinusoidal distribution in the vertical. The upper panel gives the zonal flow (E for easterlies, W for westerlies) and the lower panel the meridional flow (Hadley circulation). (C) The meridionally averaged zonal flow (Walker circulation) with the same interpretation, i.e., as a baroclinic response. (Reproduced with permission from Gill AE (1982) *Atmosphere–Ocean Dynamics*. New York: Academic Press.)

atmospheric circulation to a heat source (Figure 4). When an imposed heating centered on the Equator is switched on at some initial time, Kelvin waves carry information rapidly eastward, thereby creating easterly trade winds in that region and forming a Walker cell (rising motion over the heat source region and sinking motion to its east). Internal equatorial Kelvin waves traveling with typical speeds of $20\text{--}80\text{ m s}^{-1}$ are an effective means by which the equatorial atmosphere becomes homogenized in the zonal direction. The easterly winds are in geostrophic equilibrium, so that there is a trough along the Equator, with the winds along the Equator flowing directly down the pressure gradient (Figure 4). The westward-propagating Rossby wave regime to the west of the forcing region is about one-third the size of the Kelvin wave regime because Rossby waves travel at one-third of the Kelvin wave speed. The equatorial westerlies between the symmetric Rossby waves provide inflow into the heating region, and are in geostrophic equilibrium, so that a relative ridge appears along the Equator. Meanwhile, the flow converges toward the Equator. If atmospheric damping is taken into account, this simple model can largely explain the steady-state atmospheric response to an imposed heat source.

Kelvin and mixed Rossby-gravity waves are the predominant disturbances in the equatorial stratosphere, and play a critical role in the stratospheric circulation through their vertical transport of energy and momentum. Stratospheric Kelvin waves are excited by oscillations in the large-scale convective

heating pattern in the troposphere, and are a source of westerly momentum for the QBO. The QBO is a zonal wind oscillation in the equatorial stratosphere, and propagates downward with a period of about 24–30 months. Figure 5 shows an example of the zonal wind oscillations caused by the passage of Kelvin waves near the Equator. The descent of the westerly phase of the QBO is shown in the figure. At each level, there is an increase of the zonal wind with time. Superposed on this secular trend is a fluctuating Kelvin wave component with a period of about 12 days and a vertical wavelength of about 10–12 km. As vertically propagating Kelvin waves carry westerly momentum upward, they are damped by radiative cooling, small-scale turbulence, and critical level interaction. As the waves are damped, they lose momentum and accelerate the westerly mean flow. The damping depends on the Doppler-shifted wave frequency. As the Doppler-shifted frequency decreases, the vertical component of the group velocity also decreases, and a longer time is available for the wave energy to be damped. Hence, westerly Kelvin waves tend to be damped preferentially in the westerly shear zone where their Doppler-shifted frequencies decrease with height. The associated momentum flux convergence produces westerly acceleration of the mean flow, causing the westerly shear zone to descend. A similar argument is valid for the downward propagation of the easterly phase of the QBO through the action of Rossby-gravity waves.

A peak in the variability of the tropical atmosphere appears in the 30- to 60-day period range, and is

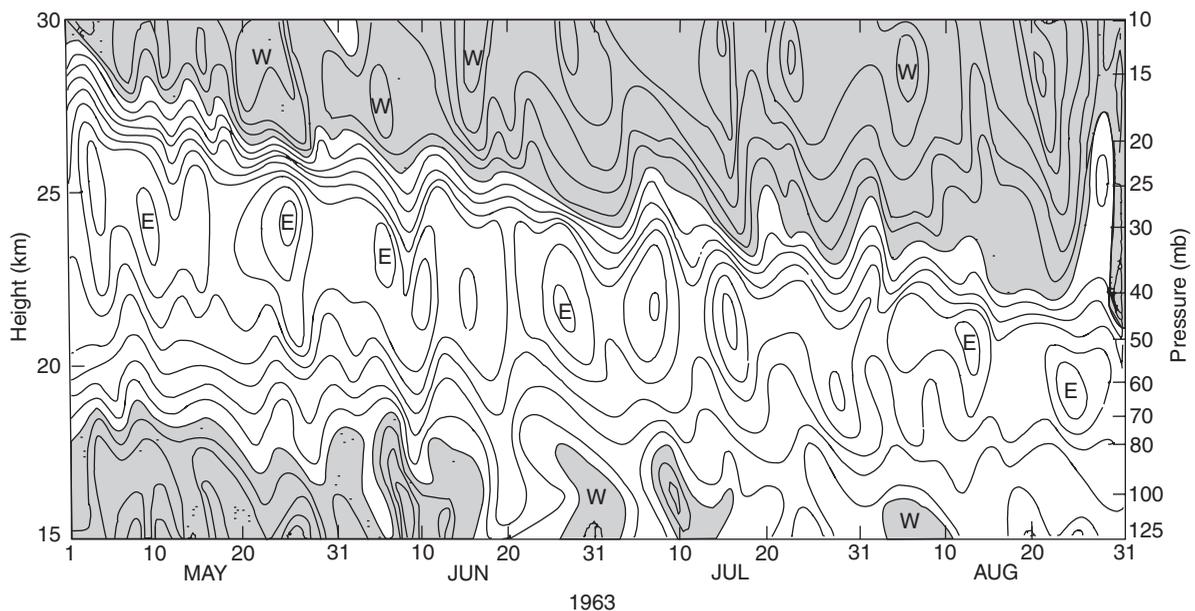


Figure 5 Time–height section of zonal wind at Canton Island (3° S). Isotachs at intervals of 5 m s^{-1} . Westerlies are shaded. (Reproduced with permission from Holton JR (1992) *Introduction to Dynamic Meteorology*, 3rd edn. San Diego: Academic Press; original courtesy of J. M. Wallace and V. E. Kousky.)

known as the Madden–Julian oscillation (MJO). Madden and Julian found that a 30- to 60-day oscillation in zonal winds is in approximate geostrophic balance with varying pressure maxima and minima centered at the Equator. A low-level low-pressure anomaly is accompanied by low-level easterly anomalies. The pressure and zonal wind are out of phase in the upper troposphere. The wave patterns move eastward along the Equator. At the Equator, the meridional winds appear to be insignificant. The amplitude of the oscillation decays with distance away from the Equator. These features are similar to those of internal equatorial Kelvin waves except that the large vertical scale of the MJO implies a faster phase speed than is observed. Arguments involving coupling with equatorial westward-traveling Rossby waves and interaction with the release of latent heat in the disturbances as well as viscous damping have been invoked to explain the observed slow phase speed.

See also

Dynamic Meteorology: Waves and Instabilities (0141).
El Niño and the Southern Oscillation: Observation

(0148); Theory (0149). **Middle Atmosphere:** Quasi-Biennial Oscillation (0232). **Ocean Circulation:** Surface/Wind Driven Circulation (0280). **Rossby Waves** (0346). **Tropical Meteorology:** Equatorial Waves (0414); Intra-seasonal Oscillation (Madden-Julian Oscillation) (0415).

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