# Direction of Hurricane Beta Drift in Horizontally Sheared Flows

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ABSTRACT

The impacts of linear environmental shears on beta drift direction are assessed through numerical experiments with a single-layer, primitive equation model. It is found that cyclonic (anticyclonic) shears turn the beta drift more westward (northward) in the Northern Hemisphere. In addition, the longitudinal shear of meridional flows ($\partial V/\partial x$) is much more effective than the meridional shear of zonal flows ($\partial U/\partial y$) in deflection of the beta drift.

A theoretical model, the beta gyre dynamic system, describing evolution of the beta gyre amplitude and phase angle is advanced to interpret the numerical model results. In this model, the nonlinear energy transfer from the beta gyres to the primary vortex and higher asymmetric modes was partially parameterized by linear damping. The semi-empirical theory predicts that 1) beta drift direction is independent of the planetary vorticity gradient; 2) in a quiescent environment, the drift angle is primarily determined by the outer azimuthal flows of the vortex; and 3) in a sheared environmental flow, the deflection of beta drift induced by environmental shears depends mainly on the longitudinal shear of meridional flows. The authors show that the environmental shear changes beta drift angle by advection of beta gyre vorticity and planetary vorticity, which affects beta gyre orientation.

## 1. Introduction

In a resting atmosphere, a hurricane-like cyclonic vortex would drift approximately northwestward (southwestward) at a speed of a few meters per second in the Northern (Southern) Hemisphere due to the steering of the primary vortex by beta-induced (planetary vorticity gradient-induced) asymmetric gyres (the beta gyres). This beta drift component accounts for a systematic deviation of tropical cyclone motion from the corresponding environmental steering flow over various tropical ocean basins (Carr and Elsberry 1990). Although the meridional gradient of the earth's vorticity and the vortex structure are essential factors determining beta drift, the presence of horizontal shears of environmental flows adds complexity to computation of the vortex beta drift (Elsberry 1987).

Sheared environmental flows influence cyclone motion by two processes. The first (also the principal) process is their direct steering effect on the primary vortex. The second is their interaction with the primary vortex circulation, which changes the strength and orientation of beta gyres, affecting beta drift. Recent studies (Ulrich and Smith 1991; Smith 1991; Williams and Chan 1994; Wang and Li 1995; Smith et al. 1996) have found that zonal environmental flows with meridional shears may have significant impacts on beta drift. The presence of meridional shear can alter beta gyre intensity and thus the beta drift speed. In a previous paper (Li and Wang 1996, hereafter LW 96), the beta gyre development in a general environmental flow with both meridional and longitudinal shears was analyzed. It was found that a positive (negative) shear strain rate accelerates (decelerates) the beta drift by interacting with an embedded cyclone (both its axially symmetric and asymmetric components) and changing the strength of the beta gyres and thus altering beta drift speed.

The environmental shears influence not only the beta drift speed but also the drift angle. Whereas Williams and Chan (1994) found that beta drift direction does not change appreciably in the presence of a *moderate* meridional shear, we found that a longitudinal shear associated with meridional flows may significantly modify beta drift direction (LW 96). The underlying dynamics, however, remain unexplained. In fact, to our knowledge, even without the complexity of the environmental shear effects, the various factors that determine beta drift angle are little known.

The present paper seeks to elaborate how the horizontally sheared environmental flows influence beta drift direction. We will first assess, by means of controlled numerical experiments, the impacts of environmental shears on beta drift angle (section 2). We will show that beta drift angle is primarily affected by the

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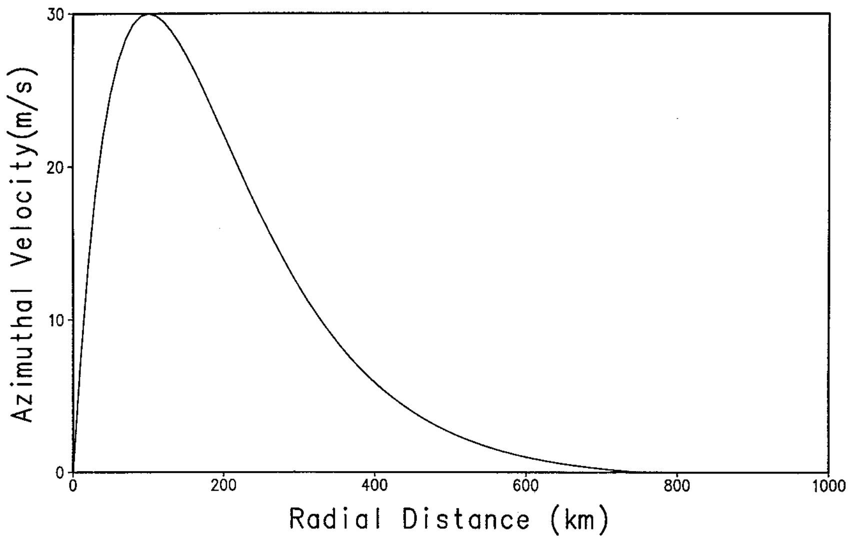


FIG. 1. Azimuthal wind profile of the initial symmetric vortex for experiments E0 and E1 through E10.

longitudinal shear of meridional environmental flow. To interpret the results, we develop a theoretical framework for study of gyration of the beta gyres and change of drift direction (section 3). Next, we advance an analytical model for analysis of beta gyre rotation (section 4). In section 5, we discuss principal processes by which environmental shears influence beta gyre rotation and interpret the results of numerical experiments. The last section presents concluding remarks and discusses the limitation and weakness of the theory.

## 2. Dependence of the drift angle on environmental shears

The impacts of horizontally sheared environmental flows on beta drift direction are assessed through controlled numerical experiments with a single-layer, primitive equation model (shallow-water equations). The model used here is identical to that used in LW 96, in which environmental flows are kept permanently unchanged. For details of the numerical model, readers are referred to LW 96.

TABLE 1. A summary of characteristic parameters of the environmental flows used in numerical experiments. The unit $f_0$ is the Coriolis parameter at 20°N. Here, $U_y$ and $V_x$ are meridional shear of the zonal flow and longitudinal shear of the meridional flow, respectively. Meaning of $b$ is referred to the text.

| Case | $U_y$ | $V_x$ | $b$ |
|---|---|---|---|
| E0 | 0.0 | 0.0 | 0 |
| E1 | $0.12f_0$ | 0.0 | −2.5 |
| E2 | $-0.12f_0$ | 0.0 | −3.2 |
| E3 | 0.0 | $-0.12f_0$ | −4.0 |
| E4 | 0.0 | $0.12f_0$ | −2.2 |
| E5 | $0.06f_0$ | $-0.06f_0$ | −3.1 |
| E6 | $-0.06f_0$ | $0.06f_0$ | −2.7 |
| E7 | $-0.06f_0$ | $-0.06f_0$ | −3.6 |
| E8 | $0.06f_0$ | $0.06f_0$ | −2.3 |
| E9 | $0.24f_0$ | 0.0 | −2.0 |
| E10 | $-0.24f_0$ | 0.0 | −3.7 |

When the planetary vorticity gradient is fixed, the beta drift direction depends on both the shears of environmental flows and the structure of vortex circulation. To identify environmental flow effects, the structure of initial symmetric vortex circulation was deliberately fixed in the experiments. The azimuthal wind profile of the initial vortex is shown in Fig. 1. The vortex has a finite size of 750 km with a maximum wind speed of 30 m s$^{-1}$ that is located at a radial distance of 100 km from the vortex center. The vortex is initially located at 20°N.

We assume that the environmental flow $\mathbf{V}_E$ is nondivergent and includes both meridional and zonal shears,

$$\mathbf{V}_E = \mathbf{i}U_y(y - y_o) + \mathbf{j}V_x(x - x_o), \qquad (2.1)$$

where $U_y$ and $V_x$ represent constant meridional shear of the zonal flow component and zonal shear of the meridional flow component, respectively. The environmental flow (2.1) is general enough to depict a variety of realistic flow patterns surrounding tropical cyclones (LW 96).

To examine the effects of the environmental flow (2.1) on beta drift angle, eight experiments were designed with differing shears. The characteristic parameters of these environmental flow fields are listed in Table 1. Cases E1 and E2 have pure zonal flows with an anticyclonic and cyclonic shear, respectively. Cases E3 and E4, on the other hand, have pure meridional flows with an anticyclonic and cyclonic shear, respectively. The environmental flows in cases E5 through E8 contain both meridional and zonal shears. They represent, respectively, ideal flow fields near a subtropical high (case

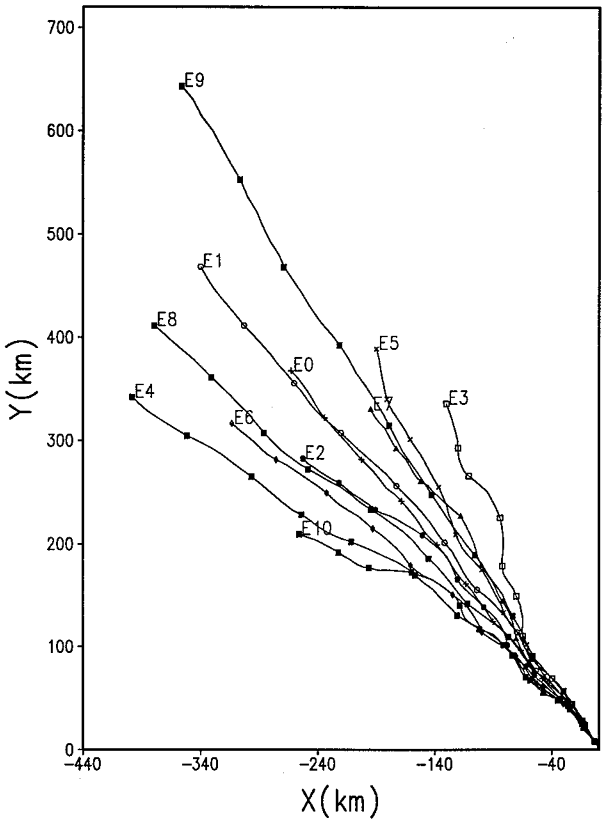


FIG. 2. The beta drift tracks for a resting environment, case E0, and ten different horizontally sheared environmental flows (E1 through E10) whose characteristic parameters are listed in Table 1. The positions of cyclone centers are shown every 6 h. Note that the difference of the beta drift tracks is established after the first 24 h.

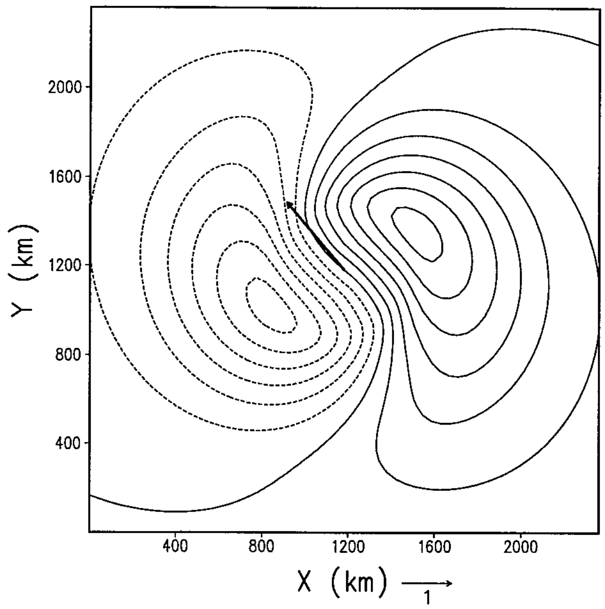


FIG. 3. The streamfunction of the beta gyres averaged over 48 hours from hour 24 to 72 with a contour interval of $1 \times 10^5$ m$^2$ s$^{-1}$ for the case E2. The arrow between two beta gyres denotes the beta drift velocity of the cyclonic vortex.

E5), a monsoon low (case E6), and two saddle flow fields (cases E7 and E8) (see discussions in LW 96).

The beta drift tracks for each case were obtained by removing the environmental steering component. The results are shown in Fig. 2. For a reference, the beta drift in a resting environment (case E0) is also presented. The pure zonal flows in cases E1 and E2 do not seem to have a significant influence on beta drift angle, in agreement with the result of Williams and Chan (1994). In sharp contrast, a pure meridional flow with an anticyclonic (cyclonic) shear deflects beta drift clockwise (counterclockwise) significantly, as previously noticed in LW 96. In a more general environmental flow with both zonal and meridional shears, the drift direction varies primarily with the longitudinal shear of the meridional flow, $V_x$. A positive $V_x$ (E4, E6, E8) causes a counterclockwise rotation of the beta drift track, while a negative $V_x$ (E3, E5, E7) causes a clockwise deflection. Furthermore, the degree of deflection increases as the magnitude of the $V_x$ increases. For instance, larger magnitudes of $V_x$ in case E3 and E4 (Table 1) correspond to a stronger deflection than those in case E5 through E8 (Fig. 1). We conclude that the deflection of beta drift depends principally on the longitudinal shear of meridional flows.

## 3. The vorticity tendency equation for beta gyres

Explanation of the previous numerical results requires a theoretical framework. Such a framework can be established based on the fact that the beta drift closely follows the ventilation (secondary steering) flow associated with the beta gyres. This was demonstrated by Fiorino and Elsberry (1989). Despite numerical inaccuracies and some uncertainties (such as the definition of ventilation flow), a robust feature in numerical experiments is that the beta drift direction changes in harmony with the gyration of the beta gyres. In all eight experiments, beta drift directions are fundamentally consistent with the secondary steering flows and the azimuthal phases of the beta gyres. The example illustrated in Fig. 3 shows the close relationship between the orientation of the beta gyres and beta drift direction.

The key to addressing environmental shear effects on beta drift angle is, therefore, to understand how the shears could change the orientation of beta gyres. Analysis of beta gyre vorticity tendency can provide physical insights into the causes of beta gyre rotation (Carr 1989). For this reason, we will derive a tendency equation for beta gyre vorticity, including a linear environmental flow in a reference frame moving with the vortex.

The nondivergent barotropic motion on a $\beta$-plane conserves absolute vorticity. In a reference frame that moves with the vortex at a translation velocity relative to fixed coordinates, absolute vorticity conservation requires

$$\frac{d}{dt}(\zeta + f) = \frac{\partial'}{\partial t}(\zeta + f) + \mathbf{V}' \cdot \nabla'(\zeta + f) = 0, \quad (3.1)$$

where $\zeta$ is the vertical component of relative vorticity and $f$ the Coriolis parameter; both are the same in the moving and the fixed coordinates. Here, $\nabla'$ denotes a gradient operator defined in the moving coordinates and $\mathbf{V}'$ is the velocity observed in the moving coordinates, which can be written as

$$\mathbf{V}' = \mathbf{V} - \mathbf{C}(t), \quad (3.2)$$

where $\mathbf{V}$ is the velocity observed in the fixed coordinates and $\mathbf{C}(t)$ is the translation velocity of the vortex (or moving coordinates).

Assume that in the fixed coordinates, the total flow $\mathbf{V}$ consists of an environmental ($\mathbf{V}_E$) and a vortex component. The latter is further partitioned into an axially symmetric ($\mathbf{V}_S$) and asymmetric ($\mathbf{V}_A$) component. That is,

$$\mathbf{V} = \mathbf{V}_E + \mathbf{V}_S + \mathbf{V}_A. \quad (3.3)$$

Numerical experiments show that $\mathbf{V}_S$ reaches a quasi-steady state after an initial adjustment. Since the linear environmental flow is steady, use of (3.3) in (3.1) leads to

$$(\partial'/\partial t)(\zeta_A + f) + (\mathbf{V}_A + \mathbf{V}_S + \mathbf{V}_E - \mathbf{C}) \cdot \nabla'(\zeta_A + \zeta_S + f) = 0, \quad (3.4)$$

where $\zeta_A$ and $\zeta_S$ represent the vorticity of the asymmetric and symmetric flow, respectively. In fixed Cartesian coordinates, the environmental flow relative to the vortex center $(x_c, y_c)$ is

$$\mathbf{V}_{E1} = \mathbf{V}_E - \mathbf{V}_E(x_c, y_c). \quad (3.5)$$

From (2.1),

$$\mathbf{V}_{E1} = \mathbf{i}U_y(y - y_c) + \mathbf{j}V_x(x - x_c). \quad (3.6)$$

For a linear environmental flow, the instantaneous environmental flow at the vortex center, $\mathbf{V}_E(x_c, y_c)$, is precisely the environmental steering experienced by the vortex, thus the beta drift velocity

$$\mathbf{C}_\beta(t) = \mathbf{C}(t) - \mathbf{V}_E(x_c, y_c). \quad (3.7)$$

Namely, the total cyclone translation [$\mathbf{C}(t)$] consists of a steering [$\mathbf{V}_E(x_c, y_c)$] and a beta drift [$\mathbf{C}_\beta(t)$] component.

With the aid of (3.5) and (3.7), the tendency equation for asymmetric cyclonic vorticity becomes

$$(\partial'/\partial t)(\zeta_A + f) + (\mathbf{V}_A + \mathbf{V}_S + \mathbf{V}_{E1} - \mathbf{C}_\beta) \cdot \nabla'(\zeta_A + \zeta_S + f) = 0. \quad (3.8)$$

To facilitate analysis, we now introduce a set of cylindrical coordinates that move with the vortex and whose origin is located at the vortex center. Let $r$ and $\lambda$ denote, respectively, the radial distance and the azimuthal angle measured from due north counterclockwise. To focus on beta gyre vorticity tendency, we define the beta gyres as the wavenumber-1 azimuthal Fourier harmonic. The asymmetric vortex circulation can thus be expressed by

$$\mathbf{V}_A = \mathbf{V}_g + \mathbf{V}_{\text{res}}, \quad (3.9)$$

where $\mathbf{V}_g$ and $\mathbf{V}_{\text{res}}$ represent beta gyre flow and the flow associated with residual azimuthal Fourier harmonics. Note that the velocity components associated with vortex circulation, $\mathbf{V}_S$, $\mathbf{V}_g$, and $\mathbf{V}_{\text{res}}$, are all invariant with respect to the transformation from the fixed to the moving coordinates. In the moving cylindrical coordinates, the gradient of the planetary vorticity is

$$\nabla \text{f} = \beta(\mathbf{e}_r \cos\lambda - \mathbf{e}_\lambda \sin\lambda), \quad (3.10)$$

where $\mathbf{e}_r$ and $\mathbf{e}_\lambda$ denote the unit vectors in radial and azimuthal directions, respectively. The symmetric cyclonic circulation

$$\mathbf{V}_S = \upsilon_S \mathbf{e}_\lambda. \quad (3.11)$$

The beta drift velocity in the moving cylindrical coordinates can be written as (see appendix A)

$$\mathbf{C}_\beta = C_\beta(\text{t})[\sin(\lambda - \alpha_g)\mathbf{e}_r + \cos(\lambda - \alpha_g)\mathbf{e}_\lambda], \quad (3.12)$$

where $\alpha_g$ is the azimuthal phase angle of the anticyclonic beta gyre, which signifies the orientation of the beta gyre axis (Fig. A1). The environmental flow ($\mathbf{V}_{E1}$) can be expressed by (see appendix A)

$$\mathbf{V}_{E1} = \mathbf{e}_r[-0.5r(V_x + U_y)\sin 2\lambda] + \mathbf{e}_\lambda[0.5r(V_x - U_y) - 0.5r(V_x + U_y)\cos 2\lambda], \quad (3.13)$$

which indicates that the environmental shear consists of an axially asymmetric component and a wavenumber-2 azimuthal component in the moving cylindrical coordinates.

We note that the beta gyres are dominated by the azimuthal wavenumber-1 component. In order to derive an equation for beta gyre vorticity tendency, we define $\mathbf{\Lambda}_g$ as an operator by which one obtains an azimuthal wavenumber-1 component. Applying the operator $\mathbf{\Lambda}_g$ to (3.8) and using (3.10) through (3.13) yields

$$\partial\zeta_g/\partial t = -(\mathbf{V}_g - \mathbf{C}_\beta) \cdot \nabla\zeta_S - \mathbf{V}_S \cdot \nabla\zeta_g - \mathbf{V}_S \cdot \nabla f - \mathbf{V}_{E1} \cdot \nabla(f + \zeta_g) + \text{RES}, \quad (3.14)$$

where

$$\text{RES} = \mathbf{\Lambda}_g[(\mathbf{V}_g - \mathbf{C}_\beta) \cdot \nabla\zeta_{\text{res}} + \mathbf{V}_{\text{res}} \cdot \nabla(\zeta_g + \zeta_{\text{res}} + f) + \mathbf{V}_E \cdot \nabla\zeta_{\text{res}}]. \quad (3.15)$$

In (3.14) and (3.15), $\zeta_g$ and $\zeta_{\text{res}}$ denote, respectively, the relative vorticity associated with the beta gyres and the residual asymmetric circulation. The operator $\mathbf{\Lambda}_g$ does not appear in the first four terms of the rhs of (3.14) because these terms contain only the azimuthal wavenumber-1 component. The first rhs term of (3.14) results from advection of symmetric vorticity by the gyre flow relative to beta drift; the second rhs term is advection of beta gyre vorticity by the symmetric circulation; the third term arises from meridional advection of planetary vorticity by symmetric circulation; and the fourth rhs term expresses direct effect of the environmental flow on beta gyre vorticity tendency, that is, the advection of planetary vorticity and beta gyre vorticity by the environmental flow relative to the vortex center. The term RES is associated with the residual asymmetric circulation. It includes mutual advection of the beta gyres and the residual asymmetric flows, which results in a dissipative effect on the beta gyre vorticity. For simplicity, we will parameterize this vorticity cascade to higher asymmetric modes simply by a Rayleigh damping to the beta gyre vorticity, and neglect other terms associated with the residual asymmetric flows, that is, let RES $= -\varepsilon\zeta_g$ in Eq. (3.14).

## 4. A simple model for beta gyre rotation

To derive governing equations for beta gyre rotation, we first express the streamfunction of beta gyre flow, in the moving cylindrical coordinates, by the following general form:

$$\psi_g(r, \lambda, t) = \Phi_g(r, t)\cos[\alpha_g(r, t) - \lambda], \quad (4.1)$$

where $\phi_g$ denotes the amplitude and $\alpha_g$ the azimuthal phase of the beta gyre axis (Fig. A1). The vorticity of the beta gyres, $\zeta_g = \nabla^2\psi_g$, is thus

$$\zeta_g(r, \lambda, t) = R_1\cos(\alpha_g - \lambda) - \mathrm{R}_2\sin(\alpha_g - \lambda), \quad (4.2)$$

where

$$R_1 = \frac{\partial}{r\partial r}\left(r\frac{\partial\Phi_g}{\partial r}\right) - \frac{\Phi_g}{r^2} - \Phi_g\left(\frac{\partial\alpha_g}{\partial r}\right)^2, \quad (4.3a)$$

$$R_2 = \Phi_g\frac{\partial^2\alpha_g}{\partial r^2} + 2\frac{\partial\Phi_g}{\partial r}\frac{\partial\alpha_g}{\partial r} + \frac{\Phi_g}{r^2}\frac{\partial\alpha_g}{\partial r}. \quad (4.3b)$$

Differentiating (4.2) with respect to time leads to

$$\frac{\partial\zeta_g}{\partial t} = \left(\frac{\partial R_1}{\partial t} - R_2\frac{\partial\alpha_g}{\partial t}\right)\cos(\alpha_g - \lambda) - \left(\frac{\partial R_2}{\partial t} + R_1\frac{\partial\alpha_g}{\partial t}\right)\sin(\alpha_g - \lambda). \quad (4.4)$$

Multiplying (4.4) by $\sin(\alpha_g - \lambda)$ and $\cos(\alpha_g - \lambda)$, respectively, and integrating the resulting equations from 0 to $2\pi$ yield

$$\frac{\partial R_2}{\partial t} + R_1\frac{\partial\alpha_g}{\partial t} = -\frac{1}{\pi}\int_0^{2\pi}\frac{\partial\zeta_g}{\partial t}\sin(\alpha_g - \lambda)\,d\lambda, \quad (4.5)$$

$$\frac{\partial R_1}{\partial t} - R_2\frac{\partial\alpha_g}{\partial t} = \frac{1}{\pi}\int_0^{2\pi}\frac{\partial\zeta_g}{\partial t}\cos(\alpha_g - \lambda)\,d\lambda. \quad (4.6)$$

Inserting the right-hand side of (3.14) and (3.10)–(3.13) into (4.5) and (4.6), respectively, we obtain, after lengthy and tedious algebra,

$$\frac{\partial R_2}{\partial t} + R_1\frac{\partial\alpha_g}{\partial t} = \frac{\partial\zeta_S}{\partial r}\left(C_\beta - \frac{\phi_g}{r}\right) + R_1\frac{\upsilon_S}{r} + \beta(\upsilon_S + rV_x)\cos\alpha_g + A\cos2\alpha_g - B\sin2\alpha_g + \frac{1}{2}(V_x - U_y)R_1 + \varepsilon R_2, \quad (4.7)$$

$$\frac{\partial R_1}{\partial t} - R_2\frac{\partial\alpha_g}{\partial t} = -R_2\frac{\upsilon_S}{r} + \beta(\upsilon_S + rV_x)\sin\alpha_g + B\cos2\alpha_g + A\sin2\alpha_g - \frac{1}{2}(V_x - U_y)R_2 - \varepsilon R_1, \quad (4.8)$$

where $\upsilon_s$ is the tangential speed of the symmetric vortex circulation and

$$A = \frac{1}{4}(V_x + U_y)\left(-rR_2\frac{\partial\alpha_g}{\partial r} + r\frac{\partial R_1}{\partial r} + R_1\right), \quad (4.9a)$$

$$B = \frac{1}{4}(V_x + U_y)\left(rR_1\frac{\partial\alpha_g}{\partial r} - r\frac{\partial R_2}{\partial r} + R_2\right). \quad (4.9b)$$

The nonlinear equations (4.7) and (4.8), along with (4.9), describe evolutions of the amplitude and phase angle of the beta gyres that interact with the primary circulation and environmental flows. The parameterized damping terms $-\varepsilon R_1$ and $-\varepsilon R_2$ represent the weakening of beta gyres due to the energy cascade to higher asymmetric modes and due to the outward radiation of beta gyre energy by Rossby wave dispersion. Note that even without these dissipation terms the system can reach steady solutions. This is because the interaction between the beta gyres and the primary vortex provides a nonlinear damping, an energy transfer from the beta gyres to the primary vortex, that balances the growth of the beta gyres.

Given adequate initial and boundary conditions, as well as the vortex symmetric circulation and environmental shears, one can solve (4.7) through (4.9) numerically. Steady solutions can also be obtained for the beta gyre amplitude and phase angle. But this numerical solution can hardly offer an understanding more illuminative than what the numerical model experiments can provide. To make progress, crucial approximation has to be made. For all the cases listed in Table 1, numerical computations have shown that the term $R_2 \simeq 0$, and

$$R_1 \approx \frac{\partial}{r\partial r}\left(r\frac{\partial\Phi_g}{\partial r}\right) - \frac{\Phi_g}{r^2} \equiv R_g, \quad (4.10)$$

which represents the amplitude of beta gyre vorticity $\zeta_g$ [Eq. (4.2)]. We will adopt this approximation to simplify the problem. Physically, this approximation amounts to the assumption that the phase angle of the beta gyres does not vary in radial direction. The approximation leads to a partial linearization of the governing Eqs. (4.7) and (4.8). The price is the loss of nonlinear interaction between the primary vortex and the beta gyres. Based on previous studies, we know that this interaction results in a nonlinear energy transfer from the beta gyres to primary vortex (Wang and Li 1992) that occurs primarily near the core region of the cyclone and is characterized by a tilt of the constant azimuthal phase line of the beta gyres in the direction of shear vector of the primary circulation (Carr and Williams 1989). This missing physics (nonlinear damping to the beta gyres), however, may be partially compensated for by considering an adequate increase in the Rayleigh damping coefficient.

With this approximation, Eqs. (4.7) and (4.8) reduce to

$$\frac{\partial \alpha_g}{\partial t} = \frac{\partial \zeta_S}{R_g \partial r}\left(C_\beta - \frac{\phi_g}{r}\right) + \frac{v_S}{r} + \frac{\beta v_S}{R_g}\cos\alpha_g + \frac{r}{R_g}\beta V_x \cos\alpha_g + \frac{1}{4}(V_x + U_y)\cos 2\alpha_g\left(1 + \frac{r}{R_g}\frac{\partial R_g}{\partial r}\right) + \frac{1}{2}(V_x - U_y), \tag{4.11}$$

$$\frac{\partial R_g}{\partial t} = \beta v_S \sin\alpha_g + \beta r V_x \sin\alpha_g + \frac{1}{4}(V_x + U_y)\sin 2\alpha_g\left(R_g + r\frac{\partial R_g}{\partial r}\right) - \varepsilon R_g. \tag{4.12}$$

Equations (4.11) and (4.12) compose a closed set for beta gyre phase $\alpha_g$ and beta gyre vorticity amplitude $R_g$. In Eq. (4.11), a positive (negative) value of $\partial\alpha_g/\partial t$ means a tendency of counterclockwise (clockwise) rotation of the beta gyres. With the approximation $R_2 = 0$, the beta gyre dynamic system (4.11) and (4.12) has much reduced nonlinearity. The dissipation terms become critical for steady beta drift, because they represent the dissipation effects associated with nonlinear energy transfer from the beta gyres to the primary vortex and the higher asymmetric modes.

## 5. Dynamic processes and factors determining beta drift direction

Equation (4.11) indicates a number of processes that may potentially contribute to change of beta gyre orientation. The first rhs term is associated with advection of the vorticity of symmetric circulation by gyre flow relative to the beta drift. This process, however, has little net effect on gyre phase change for the following reasons. Recall that $\phi_g$ is the beta gyre amplitude of the streamfunction. The mean asymmetric flow in the vicinity of the vortex center has a speed $V_a \approx |\partial\Psi_g/\partial r| \approx \phi_g/r$. Thus, the term $\phi_g/r$ can be approximately regarded as the speed of the ventilation flow at the core regions of the vortex. Since the ventilation flow at the vortex core nearly equals the corresponding beta drift speed $C_\beta$ (Fiorino and Elsberry 1989), the first rhs term should be negligibly small.

The second term on the rhs of (4.11) represents the effect of beta gyre vorticity advection by the symmetric vortex circulation. It induces a gyre phase tendency that equals the angular velocity of the symmetric circulation, which tends to rotate the beta gyres counterclockwise. The third rhs term of (4.11) results from planetary vorticity advection by the symmetric circulation that rotates the beta gyres clockwise, because $\cos\alpha_g > 0$ when the anticyclonic gyre is located in the northeast quadrant. It is, therefore, evident that in a quiescent atmosphere beta gyre orientation is primarily a result of the balance between the advection of the beta gyre vorticity and the advection of the planetary vorticity, both by the symmetric vortex circulation.

The environmental shears affect beta gyre gyration by advection of the planetary vorticity [the fourth rhs term in (4.11)] and by advection of gyre vorticity [the last two rhs terms in (4.11)]. Note that the gyre phase tendency induced by environmental advection of planetary vorticity is only determined by the longitudinal shear, whereas that induced by environmental advection of gyre vorticity is produced by both the longitudinal and zonal shears. Obviously, the environmental shears can alter the balance between gyre vorticity advection and planetary vorticity advection by symmetric vortex circulation and thus further deflect the beta drift.

When the beta gyres and beta drift reach a steady state, $\partial\alpha_g/\partial t = \partial R_g/\partial t = 0$, from (4.11) and (4.12), one can show that (appendix B)

$$\tan\alpha_g = \frac{-\varepsilon}{v_s/r + 0.5(V_x - U_y) - 0.5b(V_x + U_y)}, \tag{5.1}$$

where

$$b \equiv \frac{1}{2}\left(1 + \frac{r}{R_g}\frac{\partial R_g}{\partial r}\right). \tag{5.2}$$

Assume that the symmetric vortex circulation is

$$v_s(r) = V_0(r/r_0)\exp(1 - r/r_0), \tag{5.3}$$

where $V_0 = 30$ m s$^{-1}$ and $r_0 = 100$ km represent, respectively, the maximum tangential wind speed and the radial distance where the maximum wind speed is located. It can be shown that $b$ is the real root of the following cubic equation,

$$\frac{1}{16}\varepsilon^2 b^3 + a_1 b^2 + a_2 b + a_3 = 0, \tag{5.4}$$

where

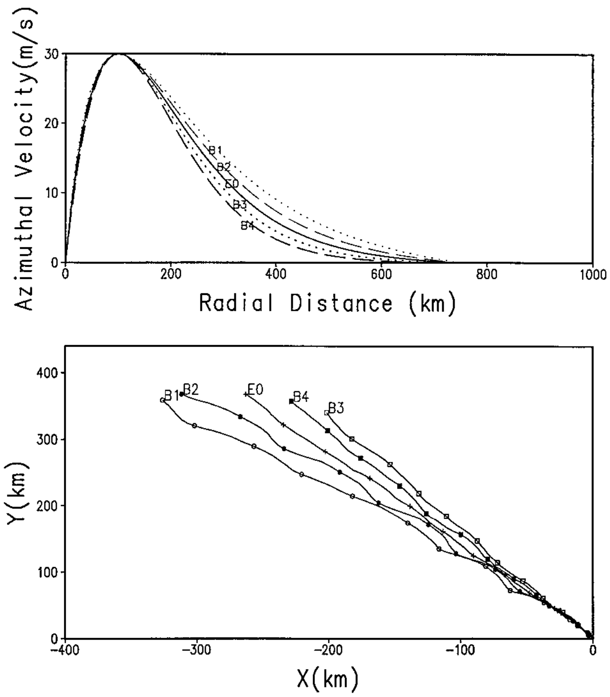


FIG. 4. Dependence of beta drift direction on the vortex structure. (a) Azimuthal wind profiles of the initial symmetric vortices with different azimuthal wind profiles. (b) The corresponding beta drift tracks for the vortices shown in (a).

$$a_1 = \frac{1}{16}(V_x + U_y)^2\left(\frac{v_s}{v_s + rV_x}\frac{r}{r_0} - 2\right), \qquad (5.5a)$$

$$a_2 = \left(\frac{v_s}{r} + \frac{V_x - U_y}{2}\right)^2 + \varepsilon^2 + \frac{v_s}{2r_0}(V_x + U_y), \qquad (5.5b)$$

$$a_3 = \frac{2v_s}{r_0}\left(\frac{v_s}{r} + \frac{V_x - U_y}{2}\right) - \left(\frac{v_s}{v_s + rV_x}\frac{r}{r_0} - 2\right) \times \left[\left(\frac{v_s}{r} + \frac{V_x - U_y}{2}\right)^2 + \varepsilon^2\right]. \qquad (5.5c)$$

Apparently, parameter $b$ can be evaluated given the dissipation parameter $\varepsilon$, the environmental shear $V_x$ and $U_y$, and the vortex azimuthal wind profile $v_s$. The last column in Table 1 shows values of $b$ in the vicinity of the beta gyre center $r = r_A$ ($\approx$400 km, see Fig. 2) for each case. In the computation, we assumed a constant dissipation coefficient of $2.6 \times 10^{-5}$ s$^{-1}$. This value is estimated based on the results obtained from the controlled experiment E0 (no environmental flow) and is kept constant for all cases. It can be seen that, in general, $b \le -1$. Taking $r = r_A$, we have, from (5.1),

$$\alpha_g = 2\pi - \tan^{-1}\left(\frac{\varepsilon}{v_s(r_A)/r_A + M}\right), \qquad (5.6)$$

where

$$M \equiv \frac{1 - b}{2}V_x + \frac{1 + b}{2}U_y \qquad (5.6a)$$

is an environmental shear index. Since the beta drift angle is approximately perpendicular to the axis of the beta gyres (Fig. A1), we may assume a constant phase difference between the gyre phase and the drift angle, that is, $\alpha = \alpha_g - (3/2)\pi$. Thus the beta drift angle is

$$\alpha = \cot^{-1}\left(\frac{\varepsilon}{v_s(r_A)/r_A + M}\right). \qquad (5.7)$$

Equation (5.7) indicates that the steady-state beta drift direction depends on vortex symmetric circulation, environmental flow shear, and dissipation rate of the asymmetric vorticity. It is interesting to note that the drift angle does not depend on the beta parameter. Additional experiments confirm that beta drift direction indeed does not change when the values of beta are doubled or halved (figure not shown) while all other parameters are kept the same. This also agrees with the results of Smith et al. (1996).

In a resting atmosphere, the beta drift angle

$$\alpha_0 = \cot^{-1}\left(\frac{\varepsilon}{v_s(r_A)/r_A}\right) \qquad (5.8)$$

depends on symmetric wind at the beta gyre center or depends on the outer azimuthal flows of the vortex. To verify this, a family of finite-size symmetric vortices with different outer flow profiles are designed (Fig. 4a). The corresponding beta drift tracks are shown in Fig. 4b. Evidently, a vortex with a stronger (weaker) outer flow drifts more westward (poleward). The drift directions obtained in the above numerical experiments are in good agreement with that predicted from (5.8) (see Table 2). It is useful here to recall that previous studies have found that beta drift speed is also sensitively dependent on the outer azimuthal flows of the vortex (e.g., DeMaria 1985; Fiorino and Elsberry 1989).

The presence of environmental shears further deflects beta drift. It is readily shown that the deflection angle

$$\Delta\alpha = \alpha - \alpha_0 = \tan^{-1}\left[\frac{\varepsilon r_A^2 M}{\varepsilon^2 r_A^2 + v_s(v_s + r_A M)}\right]. \qquad (5.9)$$

If $M > 0$, that is, the combined environmental shears are cyclonic, $\Delta\alpha > 0$, implying a counterclockwise deflection of the beta drift (or more westward). Likewise, an anticyclonic shear turns beta drift more northward. The degree of deflection as a function of $M$ is shown in Fig. 5 by the solid line. The agreement between (5.9)

TABLE 2. The theoretical beta drift angles for different azimuthal wind profiles in Fig. 4a in comparison with numerical model results (unit: degree).

| | B1 | B2 | B3 | B4 |
|---|---|---|---|---|
| Theory | 44.6 | 43.6 | 32.1 | 29.8 |
| Model | 45.3 | 42.6 | 31.5 | 30.4 |

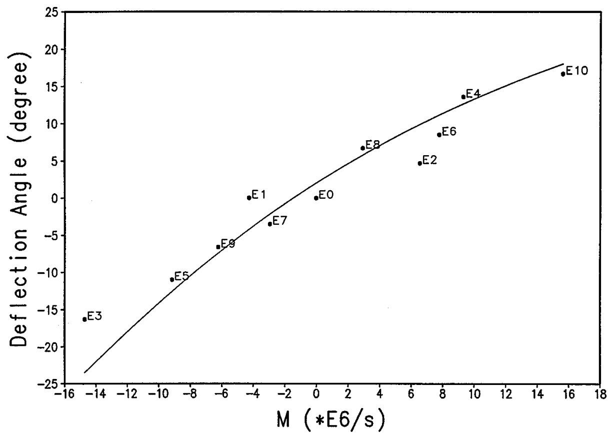


FIG. 5. Deflection angle ($\Delta\alpha$, in units of degree measuring counterclockwise from due north) as a function of environmental shear index $M$ defined by Eq. (5.7). The angle for $M = 0$ corresponds to the beta drift angle in a resting atmosphere. Negative (positive) angle denotes the clockwise (counterclockwise) deflection. The solid curve is derived from theory [Eq. (5.8)], while the closed dots represent results obtained from numerical experiments with the barotropic model.

and experimental results is reasonably good. The disagreements can be caused by various sources, including the approximations we have made in derivation of (5.6), the numerical errors in determination of the central locations, etc.

In view of the fact that $b \leq -1$ (Table 1), Eq. (5.6a) indicates that the shear index is dominated by $V_x$. This implies that $V_x$ is much more effective than $U_y$ in deflection of beta drift. This explains why in our previous experiments (section 2) the deflection is much more sensitive to $V_x$ than to $U_y$. Note, however, although beta drift direction is more sensitive to longitudinal shear, the meridional shear also contributes to change of the drift angle. In experiments E1 and E2, the change of direction is rather moderate. This is partially due to the fact that $U_y$ is small. If we enhance the meridional shear by a factor of 2 as has been done in experiments E9 and E10 (see Table 1), the resulting beta drift deflection becomes more obvious (Fig. 2).

## 6. Conclusions and discussion

Through a series of numerical experiments, we found that although both $\partial V/\partial x$ and $\partial U/\partial y$ of the environmental flow can affect beta drift angle, the longitudinal shear ($\partial V/\partial x$) plays a much more important role in deflecting beta drift. A cyclonic shear (either zonal or meridional or a combination of the two) turns the beta drift to the left (more westward) in the Northern Hemisphere when facing the direction of the drift. An anticyclonic shear has an opposite influence.

Numerical experiments have confirmed that in the presence of horizontally sheared environmental flows the beta drift direction (after removing environmental steering effect) is basically determined by the orientation of the beta gyres. The gyre orientation can be measured by the azimuthal phase angle of the anticyclonic gyre. The change of gyre phase angle is predictable in terms of the tendency of beta gyre vorticity.

With the aid of the beta gyre vorticity tendency equation and a number of simplifications, we derived a simple theoretical model for steady-state phase angle of the beta gyres. In a quiescent environment, the orientation of the beta gyres is a result of the balance between gyre vorticity advection and planetary vorticity advection, both by the symmetric vortex circulation. The environmental shears relative to the moving vortex can further advect gyre vorticity and the planetary vorticity, change the original balance in the resting environment, and thus change the orientation of the beta gyres. The beta drift deviates accordingly. The deflection angle depends on a combined shear index, $M$, which is a weighted mean of the cyclonic shear contributed by both the meridional and zonal components of the environmental flow. The present theory predicts that the longitudinal environmental shear is considerably more effective than the meridional shear in deflecting beta drift. The beta drift angle is a nonlinear function of $M$, but to the lowest approximation, it can be described by a linear relationship. This supports the empirical beta drift law derived from dimensional analysis and numerical fitting by Smith et al. (1997).

In numerical experiments, the beta gyres grow initially by extracting energy from the primary vortex via a term designated as ''beta conversion'' (Li and Wang 1994). This energy source is essentially associated with the meridional transport of planetary vorticity by primary vortex circulation, which continuously generates negative (positive) vorticity to the east (west) of the vortex center, sustaining the beta gyres. There are three energy sink terms that offset this beta generation in a quasi-steady beta drift: the energy transfer from the beta gyres to the primary vortex due to inward transport of eddy momentum associated with the asymmetric gyres, the energy cascade to higher asymmetric modes, and the outward radiation of energy due to Rossby wave dispersion. In the simplified beta gyre dynamic system (4.11) and (4.12), the dissipation is necessary and critical for a steady beta drift, because it is used to represent a simple parameterization of all these nonlinear energy dissipation effects.

The model derivation involves a critical simplification that is based on an ''after the fact'' assumption ($R_2 = 0$) and a parameterization that was used to surrogate neglected nonlinear processes. The damping coefficient used in the model was empirically determined from the numerical experiments. In this sense, the theory is semi-empirical in nature. Other assumptions adopted in the model derivation include the specified vortex structure, a finite size with a positive total angular momentum; the quasi steadiness of the beta drift; the steadiness of the symmetric circulation; and the absence of feedback from vortex to environmental flow. Whereas these assumptions are valid to various degrees, they are sources for any discrepancy between the theory and numerical results. In particular, the beta gyres develop by extracting kinetic energy from the symmetric circulation and the environmental shears in the initial adjustment process (Wang and Li 1995). The energy conversion from sheared flow to gyre depends on the magnitude of the shear strain rate. To avoid instability, the strength of the shears must be constrained. The environmental flow considered in the present study has linear shears. Further study of the effects of environmental relative vorticity gradient is needed, because the latter has been shown to have a significant influence on beta drift direction on certain occasions.

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APPENDIX A

### Derivation of (3.12) and (3.13)

Assume that $\mathbf{e}_r$ and $\mathbf{e}_\lambda$ are, respectively, the unit vectors in the radial and azimuthal direction in the cylindrical coordinates and **i** and **j** are the unit vectors in the Cartesian coordinate. Then

$$\mathbf{i} = -\mathbf{e}_r\sin\lambda - \mathbf{e}_\lambda\cos\lambda, \quad \text{(A.1)}$$

$$\mathbf{j} = \mathbf{e}_r\cos\lambda - \mathbf{e}_\lambda\sin\lambda. \quad \text{(A.2)}$$

To express beta drift velocity in the moving cylindrical coordinates, we assume that beta drift is in the direction of mean secondary steering flow over the vortex center that is perpendicular to the beta gyre axis (Fig. A1). Let $\alpha_g$ be the azimuthal phase angle of the anticyclonic gyre center that signifies the orientation of the beta gyre axis. Then beta drift velocity can be expressed in the cylindrical coordinates as

$$\mathbf{C}_\beta = \mathbf{i}C_x + \mathbf{j}C_y, \quad \text{(A3)}$$

where

$$C_x = -C_\beta\sin(\pi/2 + \alpha_g) = -C_\beta\cos\alpha_g, \quad \text{(A.4)}$$

$$C_y = C_\beta\cos(\pi/2 + \alpha_g) = -C_\beta\sin\alpha_g. \quad \text{(A.5)}$$

Substituting (A.1), (A.2), (A.4), and (A.5) into (A.3) leads to (3.12).

The environmental flow defined by (3.5) can be written as

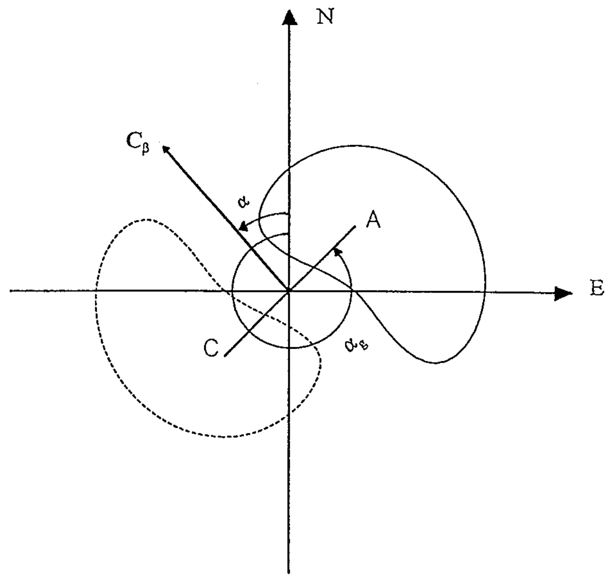


FIG. A1. Schematic diagram of the transformation between Cartesian and cylindrical coordinates. Here, $\mathbf{C}_\beta$ denotes the beta drift velocity of the vortex, which is approximately perpendicular to the gyre axis that links the centers of the cyclonic and anticyclonic gyres. The anticyclonic and cyclonic gyres are denoted by A and C, respectively, and $\alpha_g$ is the phase angle of the center of the anticyclonic gyre, while $\alpha$ is the beta drift direction.

$$\begin{aligned}\mathbf{V}_{E1} &= \mathbf{i}(y - y_c)U_y + \mathbf{j}(x - x_c)V_x \\ &= (-\mathbf{e}_r\sin\lambda - \mathbf{e}_\lambda\cos\lambda)U_y r\cos\lambda \\ &\quad - (\mathbf{e}_r\cos\lambda - \mathbf{e}_\lambda\sin\lambda)V_x r\sin\lambda \\ &= \mathbf{e}_r[-0.5r\sin2\lambda(U_y + V_x) + \mathbf{e}_\lambda[0.5r(V_x - U_y) \\ &\quad - 0.5r\cos2\lambda(U_y + V_x). \end{aligned} \quad \text{(A.6)}$$

APPENDIX B

### Derivation of the Beta Gyre Phase Equations (5.1) and (5.2)

When the beta gyres reach a steady state, Eqs. (4.11) and (4.12) become, after neglecting the first term on the rhs of (4.11) for the reason discussed in the text,

$$\begin{aligned}&\frac{1}{4}(V_x + U_y)\left(\frac{r\partial R_g}{\partial r} + R_g\right)\cos2\alpha_g \\ &\quad = -\beta(v_s + rV_x)\cos\alpha_g - R_g\frac{v_s}{r} \\ &\quad\quad - \frac{R_g}{2}(V_x - U_y), \end{aligned} \quad \text{(B.1)}$$

$$\begin{aligned}&\frac{1}{4}(V_x + U_y)\left(\frac{r\partial R_g}{\partial r} + R_g\right)\sin2\alpha_g \\ &\quad = -\beta(v_s + rV_x)\sin\alpha_g + \varepsilon R_g. \end{aligned} \quad \text{(B.2)}$$

Dividing (B.1) by (B.2) yields

$$R_g = \frac{-\beta(v_s + rV_x)\sin\alpha_g}{\varepsilon \cos 2\alpha_g + [v_s/r + 0.5(V_x - U_y)]\sin 2\alpha_g}. \quad \text{(B.3)}$$

On the other hand, the operations (B.1) $\times \sin 2\alpha_g -$ (B.2) $\times \cos 2\alpha_g$ and (B.1) $\times \cos 2\alpha_g +$ (B.2) $\times \sin 2\alpha_g$ yield, respectively,

$$\beta(v_s + rV_x)\sin\alpha_g = -R_g \sin 2\alpha_g \left(\frac{v_s}{r} + \frac{V_x - U_y}{2}\right) - \varepsilon R_g \cos 2\alpha_g, \quad \text{(B.4)}$$

$$\beta(v_s + rV_x)\cos\alpha_g = -R_g \cos 2\alpha_g \left(\frac{v_s}{r} + \frac{V_x - U_y}{2}\right) + \varepsilon R_g \sin 2\alpha_g - \frac{1}{4}(V_x + U_y) \times \left(\frac{r\partial R_g}{\partial r} + R_g\right). \quad \text{(B.5)}$$

Elimination of the lhs terms of (B.4) and (B.5) leads to

$$\tan\alpha_g = \frac{-\varepsilon}{v_s/r + 0.5(V_x - U_y) - 0.25(V_x + U_y)(r\partial \ln R_g/\partial r + 1)}. \quad \text{(B.6)}$$

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