

## Barotropic Dynamics of the Beta Gyres and Beta Drift

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### ABSTRACT

The movement of a symmetric vortex embedded in a resting environment with a constant planetary vorticity gradient (the beta drift) is investigated with a shallow-water model. The authors demonstrate that, depending on initial vortex structure, the vortex may follow a variety of tracks ranging from a quasi-steady displacement to a wobbling or a cycloidal track due to the evolution of a secondary asymmetric circulation. The principal part of the asymmetric circulation is a pair of counterrotating gyres (referred to as beta gyres), which determine the steering flow at the vortex center. The evolution of the beta gyres is characterized by development/decay, gyration, and radial movement.

The beta gyres develop by extracting kinetic energy from the symmetric circulation of the vortex. This energy conversion is associated with momentum advection and meridional advection of planetary vorticity. The latter (referred to as "beta conversion") is a principal process for the generation of asymmetric circulation. A necessary condition for the development of the beta gyres is that the anticyclonic gyre must be located to the east of a cyclonic vortex center. The rate of asymmetric kinetic energy generation increases with increasing relative angular momentum of the symmetric circulation.

The counterclockwise rotation of inner beta gyres (the gyres located near the radius of maximum wind) is caused by the advection of asymmetric vorticity by symmetric cyclonic flows. On the other hand, the clockwise rotation of outer beta gyres (the gyres near the periphery of the cyclonic azimuthal wind) is determined by concurrent intensification in mutual advection of the beta gyres and symmetric circulation and weakening in the meridional advection of planetary vorticity by symmetric circulation. The outward shift of the outer beta gyres is initiated by advection of symmetric vorticity by beta gyres relative to the drifting velocity of the vortex.

### 1. Introduction

Tropical cyclone motion is primarily controlled by nonlinear vorticity advection. Theoretically, two distinctive mechanisms can be identified that affect adiabatic motion of a barotropic vortex: steering by environmental flows, and advection by an asymmetric flow induced by interaction of the vortex circulation with the environmental absolute potential vorticity gradient. An ideal example of the environmental steering is given by Adem and Lezama (1960), who proved that a barotropic symmetric vortex embedded in a uniform flow on an  $f$  plane moves exactly with the velocity of the environmental flow. The presence of the environmental absolute potential vorticity gradient, however, may generate asymmetric circulation through interaction with a symmetric vortex. The secondary asymmetric flow thus generated can further advect symmetric relative vorticity, causing another type of motion, which was termed as propagation by Holland (1983). An ideal problem of propagation was first investigated

by Rossby (1948), who showed that an isolated rigid-body-rotation vortex on a beta plane will undergo a poleward acceleration due to the increase of the Coriolis force with latitude. The movement of a vortex embedded in such a quiescent environment on a beta plane is now commonly referred to as beta drift. Rossby's solution, however, did not consider the effects of secondary circulation and compensating pressure gradient forces and hence was not in full agreement with subsequent numerical solutions (e.g., Anthes and Hoke 1975).

A fundamental theoretical problem is to explain the mechanisms causing beta drift. Adem (1956) first worked out a series solution for a barotropic nondivergent vorticity equation, which suggested a beta effect-induced asymmetric circulation that drives the vortex first westward and then poleward. Holland (1983) argued that the advection of earth vorticity by symmetric azimuthal winds could produce asymmetric vorticity (which, on the one hand, drags a cyclone westward and, on the other hand, creates two counterrotating gyres—counterclockwise to the west and clockwise to the east) that is often referred to as beta gyres. The resulting poleward wind over the vortex center [referred to as "ventilation flow" by Fiorino and Elsberry (1989)] would advect the cyclone poleward. The roles

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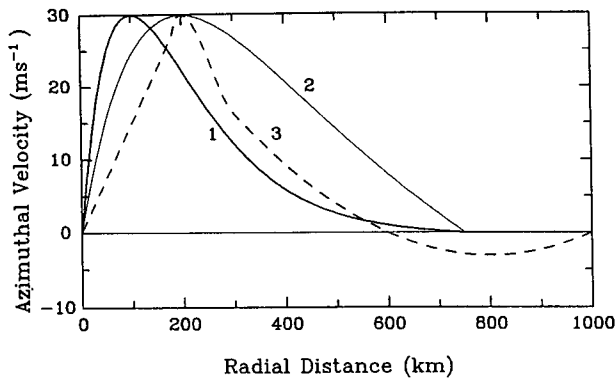


FIG. 1. Azimuthal wind profiles of initial symmetric vortices in three experiments. Thin solid, thick solid, and dashed lines are for cases 1, 2, and 3, respectively.

of asymmetric flows on vortex propagation have received much attention in recent years (Chan and Williams 1987; Willoughby 1988; Fiorino and Elsberry 1989; Peng and Williams 1990; Shapiro and Ooyama 1990; Smith et al. 1990; and others). The propagation of a barotropic vortex is found to be intimately linked to the orientation and strength of the beta gyres in both barotropic (e.g., Fiorino and Elsberry 1989) and baroclinic models (Wang and Li 1992). In most previous studies the beta drift was described as a quasi-steady displacement associated with a pair of quasi-steady beta gyres. Yet, the dynamics of the beta gyres has not been systematically studied and thoroughly understood even within a barotropic dynamic framework.

In section 2, we describe results from extended-range time integrations for the beta drift of vortices with different horizontal structures. We demonstrate that a cyclone may take a variety of tracks ranging from a quasi-steady displacement to a snake shape or a cycloidal track, depending upon the symmetric circulations of the initial vortices. It is also established that different tracks of beta drift are results of the distinctive evolution of beta gyres involved in each individual case. The unsteady beta gyres exhibit intensification (or decay), azimuthal movement (gyration), and radial movement (outward expansion). The primary objective of the present study is to answer the following questions: How do beta gyres change their intensity? What are necessary and favorable conditions for beta gyres to grow or decay? What physical processes determine the rotation and radial movement of beta gyres? We attempt to address these questions in sections 3 and 4, respectively, via analyses of energetics and streamfunction tendencies associated with the beta gyres. In the last section, we summarize major findings regarding the barotropic dynamics of beta gyres.

## 2. Unsteady beta drift and associated beta gyres

The numerical model used in this study is a shallow-water model developed by Li and Zhu (1991). A

square horizontal domain of  $6000 \times 6000 \text{ km}^2$  was chosen with a square mesh of 40 km. Sponge layers were used to furnish lateral boundary conditions. Initial vortices are axially symmetric and centered at  $20^\circ\text{N}$ . Integrations were all carried out for 180 hours.

Our interest is to examine the nature of beta drift and associated beta gyres for vortices with differing horizontal structures. For this purpose, three types of cyclones were designed whose initial symmetric azimuthal wind profiles are shown in Fig. 1. In cases 1 and 2, the initial vortices have an isolated cyclonic circulation within a radius of 750 km and a positive total relative angular momentum (TRAM). In case 3, the initial vortex consists of cyclonic flows inside the ra-

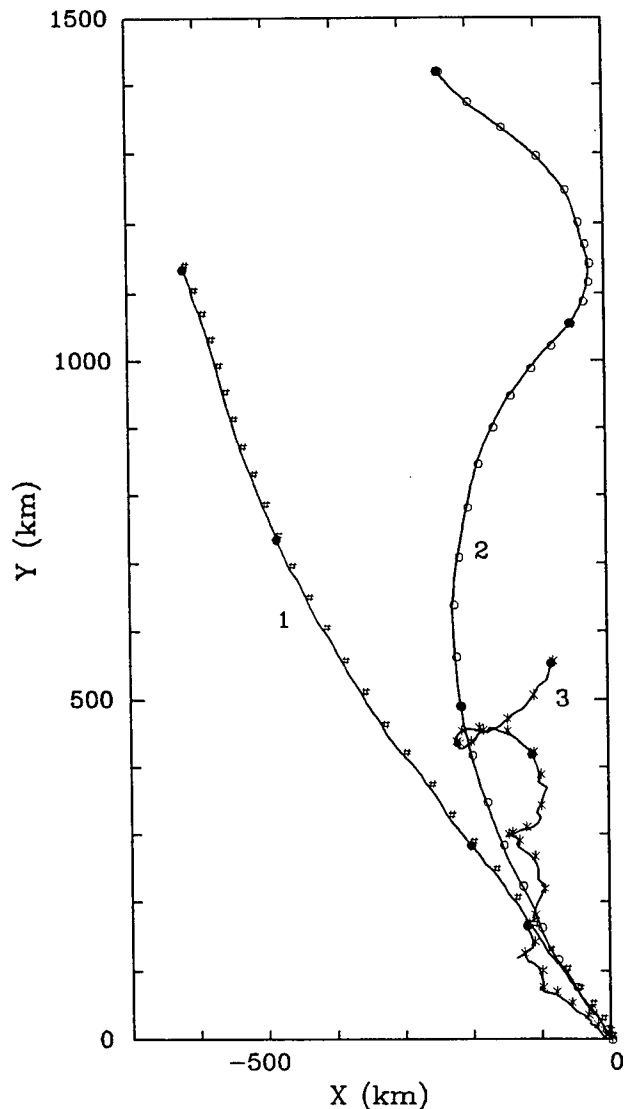


FIG. 2. The vortex tracks at 6-hour intervals for cases 1 (#), 2 (O), and 3 (\*). The closed dots denote positions of the vortex center every 60 hours.

dius of 600 km and weak anticyclonic flows in the annulus between 600 and 1000 km, so that the TRAM of the vortex is zero.

Figure 2 displays vortex tracks for the three cases. The vortex in case 1 shows a quasi-steady northwestward drift. In contrast, the vortices in cases 2 and 3 exhibit a wobbling (snake-shaped) and a looping (cycloidal) track, respectively. One may ask how asymmetric circulations differ in the three cases and how they are related to vortex movement.

Analysis of asymmetric streamfunctions reveals that in all three cases the asymmetric circulation is dominated by a wavenumber 1 azimuthal mode, which maximizes (minimizes) at a case-dependent radial distance from the vortex center, forming a pair of beta gyres. In case 1, the beta gyres slowly intensify from the beginning to hour 108 (Fig. 3). The gyre centers were oriented southwest–northeast with an anticyclonic gyre to the northeast so that the ventilation flow over the cyclone center is northwestward. When the beta gyres started to rotate clockwise at hour 108 they weakened and shifted outward. At the same time a new pair of beta gyres immediately began to form and develop near the vortex center, maintaining the northwestward steering over the cyclone center and the northwestward drift of the vortex. The ventilation flow and beta-drift velocity are both quasi-steady and in good agreement during the entire period of integration. Note that a weak trend of clockwise rotation of the beta-drift velocity and ventilation flow is discernible.

In case 2, the snake-shaped track is also in good agreement with the evolution of the ventilation flow generated by the beta gyres (Fig. 4). In the first 36 hours, the northwestward ventilation flow near the cyclone center steered the cyclone northwestward. Subsequent clockwise rotation of the beta gyres gradually turned ventilation flow to the northeast so that the cyclone recurved from the northwest to northeast (from hour 36 to 84). As the gyres continued to rotate clockwise, they decayed and moved away from the cyclone center. Around hour 156, a new pair of beta gyres developed, restoring the northwestward ventilation flow around the cyclone center and causing a recurvature from northeast to northwest.

Consistent with the looping track in case 3, the evolution of beta gyres displays a more complicated picture. The evolution of the beta gyres during a major looping period from hour 120 to 162 is illustrated in terms of azimuthal wavenumber 1 asymmetric streamfunction (Fig. 5). During this period, two sets of counterrotating gyres that have centers at different radii appear to alternatively dominate the ventilation flow. We call them beta gyres because energetics analysis indicates that the beta effect is responsible for the establishment and development of the gyres. To see more clearly how the two beta gyres evolve, we partitioned azimuthal wavenumber 1 asymmetric circulation into two radial components using a Bessel function expan-

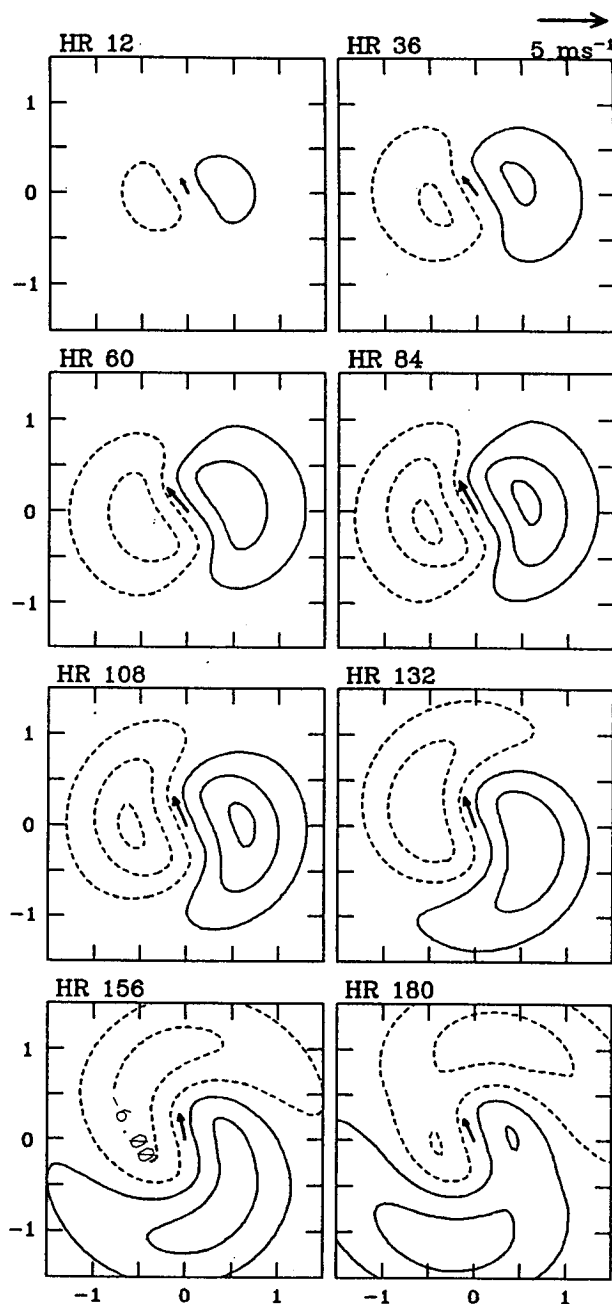


FIG. 3. The streamfunctions of the beta gyres at every 12 hours with a contour interval of  $4 \times 10^5 \text{ m}^2 \text{ s}^{-1}$  for case 1. The arrow at the vortex center denotes the drifting velocity of the vortex. Abscissa and ordinate units are 1000 km.

sion: the first radial mode and the residual (see the Appendix for details). The former with a maximum amplitude at about 600-km radius represents outer beta gyres (Fig. 6a), whereas the latter is basically dominated by inner beta gyres (Fig. 6b). Figure 6a shows that the outer gyres are stationary with an anticyclonic gyre to the southeast and a cyclonic gyre to the north-

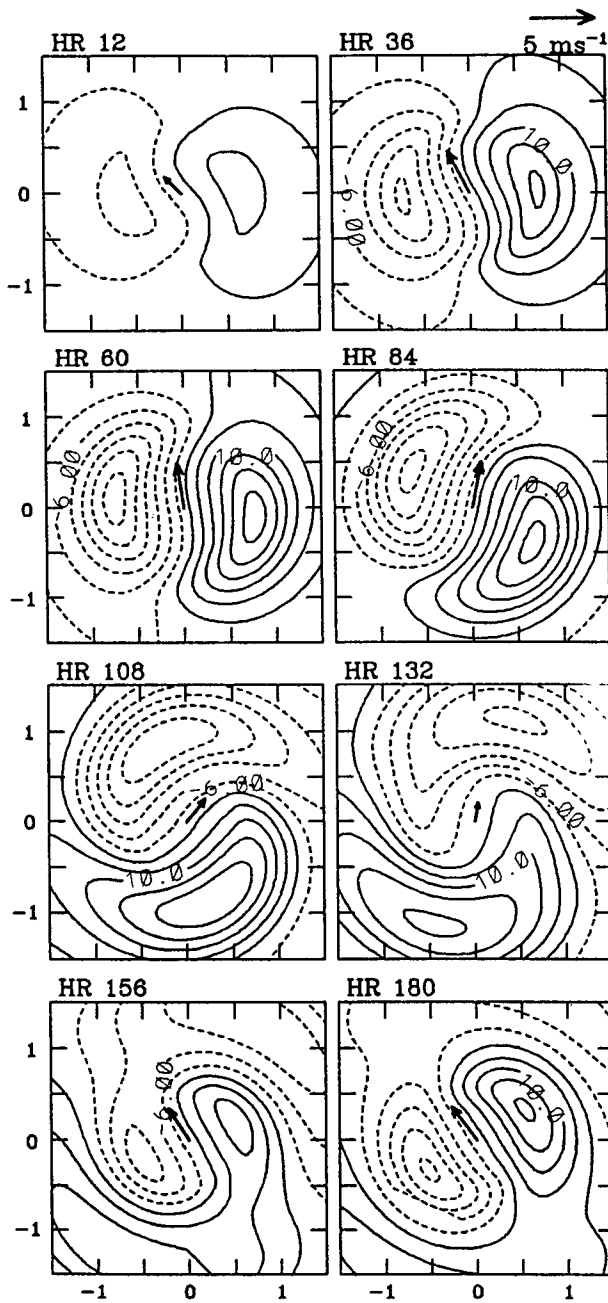


FIG. 4. As in Fig. 3 except for case 2.

west so that the corresponding ventilation flow is northeastward. In sharp contrast, the inner gyres within the radius of 300 km rotate counterclockwise from hour 120 to 156 (Fig. 6b). Around hour 132, the inner beta gyres can be identified with an anticyclone to the north and a cyclone to the south in total asymmetric streamfunction (Fig. 5). The direction of the ventilation flow averaged over the circular area within 200-km radius tends to always coincide with the direction of the vortex

drift (figure not shown). Before the looping period, the ventilation flow that is mainly determined by the stationary outer beta gyres is northeastward. When the inner beta gyres develop and dominate the asymmetric circulation near the cyclone center, the ventilation flow turns to the west at hour 126. The counterclockwise rotation of the inner beta gyres further results in a

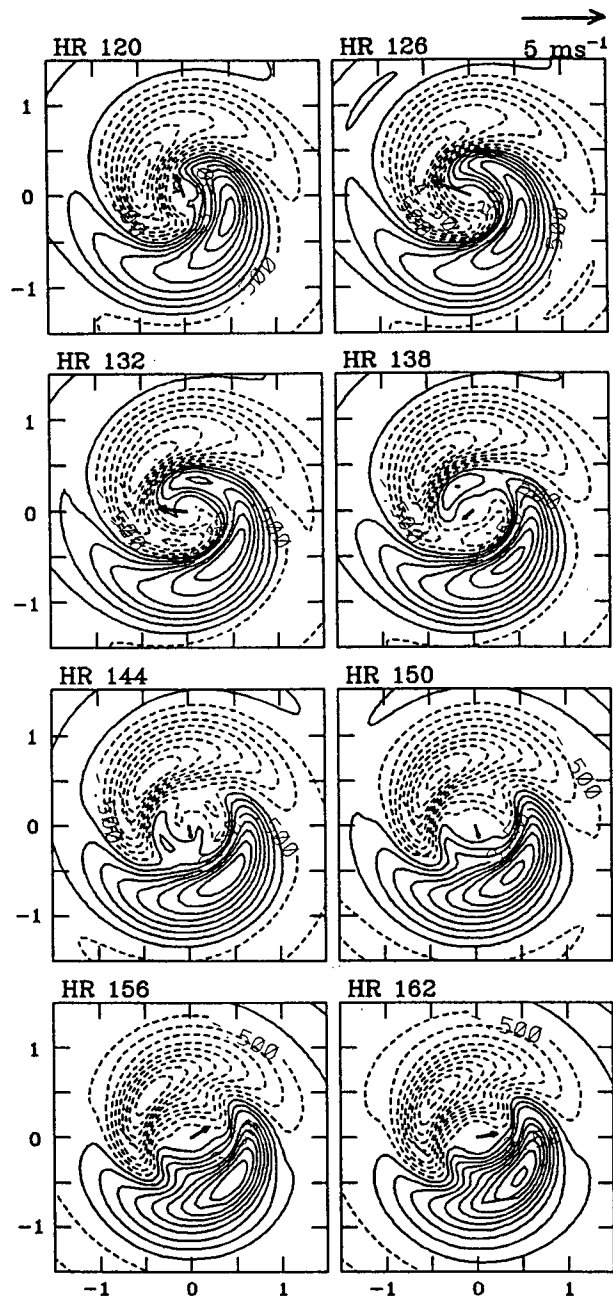


FIG. 5. The streamfunctions of the beta-gyres for case 3 from hour 120 to 162 with a contour interval of 500 m<sup>2</sup> s<sup>-1</sup>. The arrow at the vortex center denotes the drifting velocity of the vortex. Abscissa and ordinate units are 1000 km.

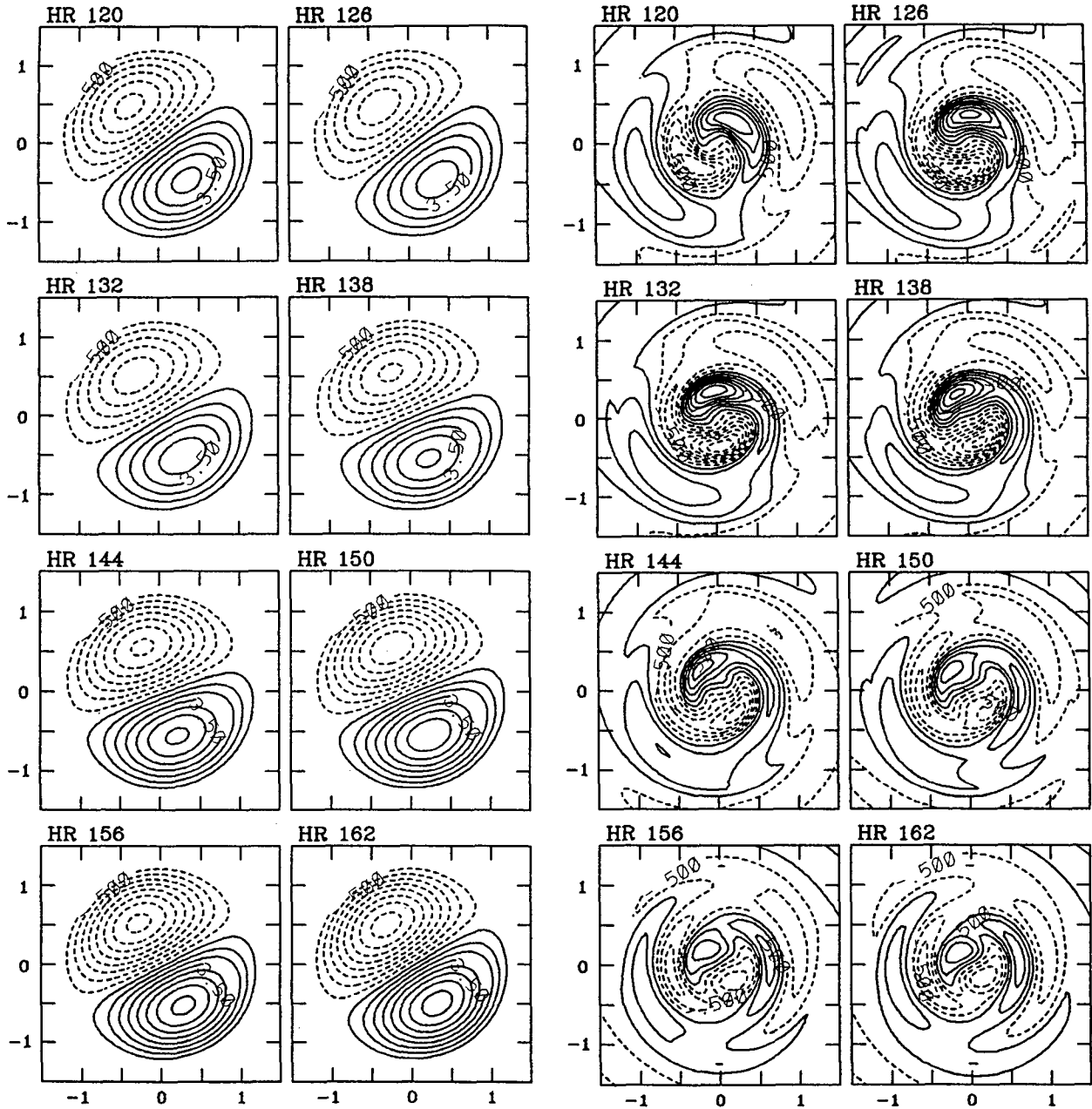


FIG. 6. As in Fig. 5 except for the first radial mode (left panel) and the residual of the beta gyres (right panel).

change of the beta-drift direction from the west to southwest at hour 138. We note that the inner beta gyres start to decay as the anticyclonic gyre center passed due north. Finally, the stationary outer gyres reestablish a northeastward ventilation flow at hour 156, steering the vortex northeastward after the looping. The looping track from hour 120 to 162 is closely associated with the evolution of the inner beta gyres.

It is clear that the difference in the beta drift between cases 2 and 3 results from different behavior of

the beta gyres: the clockwise rotation and decay of the original beta gyres and subsequent development of a new pair of beta gyres result in a wobbling, whereas the rapid growth, counterclockwise rotation, and decay of the inner beta gyres in case 3 causes a looping. Since the beta gyres control vortex drift, it is fundamental to understand the mechanisms that govern the evolution of beta gyres, in particular, their intensity change, azimuthal rotation, and radial displacement. The physical processes responsible for the develop-

ment and movement of the beta gyres are explored in the next two sections.

### 3. Development of the beta gyres

From the energetics point of view, the development and decay of beta gyres are determined by the generation rate of the kinetic energy of asymmetric circulation. It is natural to examine the development of the beta gyres based on an energetics analysis. A kinetic energy equation for asymmetric circulation is first derived. Particular attention will be given to the comparison of the energetics between cases 2 and 3 because they differ remarkably in TRAM and outer flow structure.

#### a. Asymmetric kinetic energy equation

Because in the present study the divergent wind is negligible compared to rotational wind, we will consider only the rotational wind in the following discussions. The rotational wind can be advantageously decomposed into a symmetric and an asymmetric component. Upon neglecting the divergence term, a vorticity equation becomes

$$\frac{\partial \zeta}{\partial t} = -\mathbf{V} \cdot \nabla (\zeta + f) + \mathbf{k} \cdot \nabla \times \mathbf{F}, \quad (3.1)$$

where  $\zeta$  is the vertical component of relative vorticity and  $\mathbf{F}$  denotes friction.

The kinetic energy components for symmetric and asymmetric rotational wind are defined by

$$K_{\psi_s} = \left\langle \frac{\nabla \psi_s \cdot \nabla \psi_s}{2} \right\rangle \quad (3.2a)$$

$$K_{\psi_a} = \left\langle \frac{\nabla \psi_a \cdot \nabla \psi_a}{2} \right\rangle, \quad (3.2b)$$

where  $\psi_s$  and  $\psi_a$  represent symmetric and asymmetric streamfunction, respectively, and the angle bracket implies an area integration:

$$\langle ( ) \rangle = \int_0^{r_o} r dr \int_0^{2\pi} ( ) d\lambda. \quad (3.3)$$

Here  $r_o$  is the radius of integration domain and we chose  $r_o = 1200$  km.

An equation for the asymmetric kinetic energy,  $K_{\psi_a}$ , can be derived by multiplying the vorticity equation (3.1) by  $\psi_a$  and applying the integration defined by (3.3) on the resulting equation:

$$\frac{\partial}{\partial t} K_{\psi_a} = F_{\psi_a} + (K_{\psi_s}, K_{\psi_a}) + D_{\psi_a}, \quad (3.4)$$

where  $F_{\psi_a}$  is the energy flux due to radial flow evaluated at the outer radial boundary of the integration domain,  $D_{\psi_a}$  is a dissipation term, and the symbol  $(K_{\psi_s},$

$K_{\psi_a})$  means an energy conversion from symmetric to asymmetric circulation that is defined by

$$(K_{\psi_s}, K_{\psi_a}) = -\langle (\mathbf{V} \cdot \nabla \mathbf{V}_{\psi_s}) \cdot \mathbf{V}_{\psi_a} \rangle + \langle f \mathbf{k} \cdot \nabla \psi_a \times \nabla \psi_s \rangle. \quad (3.5)$$

Energy budget calculations (Table 1) reveal that for both cases 2 and 3 the asymmetric kinetic energy is two or three orders of magnitude smaller than the symmetric kinetic energy at hour 6. The calculations at hour 24 are similar to those at hour 6 (table not shown). The asymmetric circulation develops by extracting kinetic energy from the symmetric circulation against outward energy dispersion and dissipation. Notice that the flux term  $F_{\psi_a}$  is more than one order of magnitude smaller than the conversion from  $K_{\psi_s}$  to  $K_{\psi_a}$  in case 2, whereas it is almost zero in case 3. This indicates that for a cyclone with zero TRAM (case 3), the outward radiation of kinetic energy is negligible. On the other hand, for a cyclone that has positive TRAM (case 2), there is an appreciable kinetic energy leakage due to the outward energy dispersion by Rossby waves (McWilliams and Flierl 1979).

The rate of kinetic energy conversion from symmetric to asymmetric rotational flow,  $(K_{\psi_s}, K_{\psi_a})$ , can be rewritten as

$$(K_{\psi_s}, K_{\psi_a}) = CT_1 + CT_2, \quad (3.6)$$

where

$$CT_1 = -\langle (\mathbf{V} \cdot \nabla \mathbf{V}_{\psi_s}) \cdot \mathbf{V}_{\psi_a} \rangle, \quad (3.7a)$$

$$CT_2 = \langle \beta r \cos \lambda \mathbf{k} \cdot (\nabla \psi_a \times \nabla \psi_s) \rangle. \quad (3.7b)$$

Here  $r$  is a radial distance from the vortex center,  $\lambda$  is an azimuthal angle measured counterclockwise from due north. Term  $CT_1$  represents the conversion associated with the advection of symmetric circulation. Term  $CT_2$  is due to the beta effect. For convenience we refer to  $CT_2$  and  $CT_1$  as beta conversion and nonlinear conversion, respectively. As noticed by Carr and Williams (1989), the nonlinear conversion is determined by the radial phase tilt of the asymmetric circulation and the radial gradient of the symmetric angular wind (defined as the azimuthal wind of the symmetric vortex divided by the radial distance from the vortex center).

Table 2 indicates that for both cases 2 and 3 at hour 6 the major contribution to  $(K_{\psi_s}, K_{\psi_a})$  is the beta conversion; that is, the meridional gradient of the planetary

TABLE 1. The kinetic energy components  $K_{\psi_s}$ ,  $K_{\psi_a}$  (unit:  $10^{12} \text{ m}^4 \text{ s}^{-2}$ ), the asymmetric kinetic energy generation term  $(K_{\psi_s}, K_{\psi_a})$  and the energy flux  $F_{\psi_a}$  in (3.4) (unit:  $10^7 \text{ m}^4 \text{ s}^{-3}$ ) computed at hour 6 for cases 2 and 3.

Case	$K_{\psi_s}$	$K_{\psi_a}$	$(K_{\psi_s}, K_{\psi_a})$	$F_{\psi_a}$
2	237.68	0.78	7.09	-0.51
3	99.36	0.13	1.73	0.00

TABLE 2. The breakdown of the asymmetric kinetic energy generation term ( $K_{\psi_s}$ ,  $K_{\psi_a}$ ) at hour 6 (unit:  $10^7 \text{ m s}^{-3}$ ) for cases 2 and 3. See (3.7) for the definition of  $CT_1$  and  $CT_2$ .

Case	$CT_1$	$CT_2$
2	-1.78	8.87
3	0.32	1.41

vorticity plays a key role in transferring kinetic energy from  $K_{\psi_s}$  to  $K_{\psi_a}$ . We also calculate the energy generation term at hour 24 (table not shown), which shows similar results as displayed at hour 6 except for the change of the sign of the nonlinear conversion in case 3. In both cases, the nonlinear conversion is about one-fifth of the magnitude of the beta conversion. In case 2 it is negative, but in case 3 it may be positive (e.g., at hour 6) or negative (e.g., at hour 24).

### b. Necessary and favorable conditions for the development of beta gyres

Because beta conversion is the principal process by which the beta gyres extract kinetic energy from symmetric circulation, we will focus on the beta conversion in the following discussion. The beta conversion is determined by the beta effect and the vertical component of the vector product of the symmetric and asymmetric rotational flows [Eq. (3.7b)]. In cylindrical coordinates the beta conversion can be expressed as

$$CT_2 = -\beta \left\langle r \cos \lambda \frac{\partial \psi_a}{\partial \lambda} \frac{\partial \psi_s}{\partial r} \right\rangle. \quad (3.8)$$

For simplicity, consider a cyclonic vortex that has a symmetric circulation and an azimuthal wavenumber 1 asymmetric circulation (the beta gyres) expressed by

$$\psi_s = R_s(r), \quad R_s > 0 \quad (3.9a)$$

$$\psi_a = R_a(r) \cos(\alpha - \lambda), \quad R_a > 0, \quad (3.9b)$$

where  $R_s$  and  $R_a$  are amplitudes of the symmetric and beta-gyre circulation, respectively, and  $\alpha$  denotes azimuthal angle of the center of the anticyclonic gyre measured counterclockwise from due north. Substituting (3.9a,b) into (3.8) leads to

$$CT_2 = - \left[ \beta \pi \int_0^{\infty} r \frac{\partial R_s}{\partial r} R_a dr \right] \sin \alpha. \quad (3.10)$$

From (3.10) two important conclusions can be made.

1) A necessary condition for beta gyres to extract kinetic energy from the symmetric circulation of a cyclonic vortex is that the anticyclonic gyre must be located to the east of the vortex center. The opposite is true for an anticyclonic vortex.

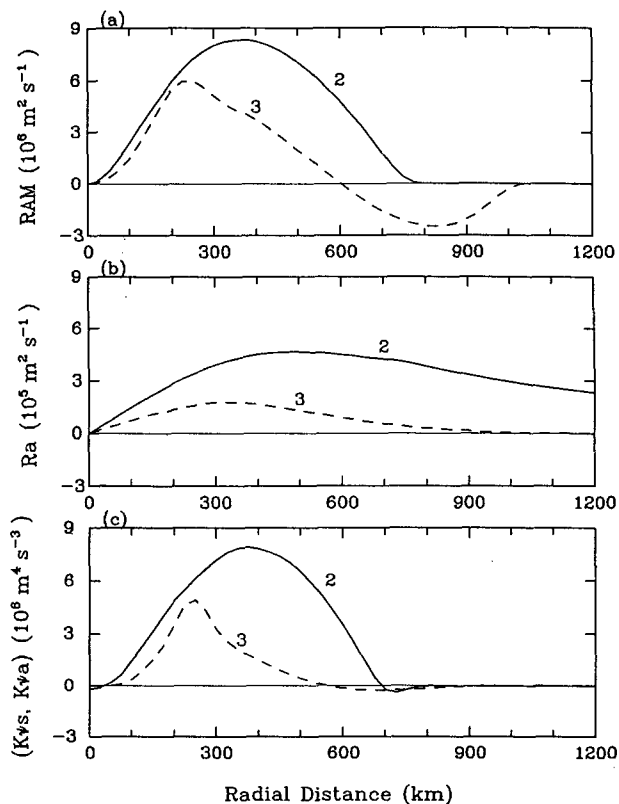


FIG. 7. The radial distributions of (a) relative angular momentum ( $10^6 \text{ m}^2 \text{ s}^{-1}$ ), (b) amplitude of the beta gyres ( $10^5 \text{ m}^2 \text{ s}^{-1}$ ), and (c) conversion from  $K_{\psi_s}$  to  $K_{\psi_a}$  ( $10^6 \text{ m}^4 \text{ s}^{-3}$ ) at hour 6. Solid and dashed lines are for cases 2 and 3, respectively.

According to (3.10), in order for  $CT_2$  to be positive,  $\alpha$  should be within  $(-\pi, 0)$ . This implies that the asymmetric anticyclonic gyre must be located to the east of the cyclone center (or in the northeast or southeast quadrant) in order for kinetic energy to transfer from cyclonic symmetric circulation to beta gyres. This inference can be readily verified using the experiments described in the previous section. For instance, in cases 1 and 2 the beta gyres amplified when the anticyclonic gyre center was located in the eastern quadrants but weakened immediately after the anticyclonic gyre center entered into the southwest quadrant (Figs. 3 and 4). Another example is the inner beta gyres in case 3. They developed as the anticyclonic gyre center was located east of the cyclone center from hour 120 to 126 and decayed once the anticyclonic gyre center entered into the northwest quadrant at hour 132 (Fig. 6b).

2) The rate of beta conversion depends on radial distribution of relative angular momentum (RAM) of the symmetric vortex circulation.

It follows from (3.10) that the magnitude of the beta conversion depends upon the covariance between the local RAM of the symmetric circulation,  $r(\partial R_s / \partial r)$ , and the amplitude of the beta gyres,  $R_a$ , both being

functions of the initial symmetric vortex structure. The radial maxima of the RAM appear around 350 km in case 2 and 200 km in case 3, respectively (Fig. 7a). The radial maxima of the amplitudes of the beta gyres occur around 400–600 km in case 2 and around 300 km in case 3 (Fig. 7b). Because the covariance between the local RAM and the beta gyres determines the radial distribution of the beta conversion (as well as the total asymmetric kinetic energy generation), the radial maximum of the asymmetric kinetic energy generation due to beta conversion is located around 400 km in case 2 while around 250 km in case 3 (Fig. 7c). We note that, in general, the intensity and radial distribution of the beta gyres also depend on the RAM of the symmetric circulation (as will be elaborated in the next paragraph), to the first approximation, the maximum generation of the asymmetric kinetic energy occurs roughly where the RAM peaks. Wang and Li (1992) have shown the dependence of the beta-drift speed on the TRAM of the vortex. The results here indicate that the RAM of the symmetric vortex circulation is indeed closely related to the asymmetric kinetic energy generation.

The initial asymmetric circulation is created by the beta effect acting on the symmetric circulation. It is obvious that a stronger outer flow or a larger mean RAM in case 2 must induce a stronger initial asymmetric circulation (figure not shown). In the subsequent development, the stronger asymmetric circulation in case 2 is expected to interact with the stronger symmetric circulation and to convert more kinetic energy from symmetric to asymmetric circulation. The corresponding beta gyres and ventilation flow would be stronger, causing the vortex drift at a higher speed in case 2. This agrees with the results of the numerical experiments. The total beta-drift speeds at hour 6 are 1.37 and 0.55 m s<sup>-1</sup> for cases 2 and 3, respectively.

#### 4. Movement of the beta gyres

##### a. Gyration

To elucidate the dynamic processes controlling azimuthal rotation of beta gyres, an equation for streamfunction tendency in a cylindrical coordinate is introduced, the origin of the coordinates being collocated with the center of a drifting vortex (Carr 1989),

$$\frac{\partial \psi}{\partial t} = \nabla^{-2} \{ -(\mathbf{V}_s - \mathbf{C}) \cdot \nabla \zeta_a - (\mathbf{V}_a - \mathbf{C}) \cdot \nabla \zeta_s - \mathbf{V}_a \cdot \nabla (\zeta_a + f) - \mathbf{V}_s \cdot \nabla f \}, \quad (4.1)$$

where  $\mathbf{C}$  is a drifting velocity of a vortex independent of the radial distance, and symbol  $\nabla^{-2}$  is an inverse Laplacian operator.

To examine the azimuthal rotation of the beta gyres, total streamfunction was decomposed into a symmetric component, a wavenumber 1 azimuthal harmonic (beta

gyres), and a residual. Projecting all terms in (4.1) onto wavenumber 1 azimuthal mode yields the following tendency equation for the beta-gyre streamfunction:

$$\begin{aligned} \frac{\partial \psi_{a1}}{\partial t} = \nabla^{-2} \{ & -(\mathbf{V}_{a1} - \mathbf{C}) \cdot \nabla \zeta_s - \mathbf{V}_s \cdot \nabla \zeta_{a1} \\ & \text{(TT)} \qquad \qquad \text{(ASVA)} \qquad \qquad \text{(AAVS)} \\ & - \mathbf{V}_s \cdot \nabla f - \Lambda_{a1} [(\mathbf{V}_{a1} - \mathbf{C}) \cdot \nabla \zeta_{ares} \\ & \text{(BETA)} \\ & + \mathbf{V}_{ares} \cdot \nabla (\zeta_{a1} + \zeta_{ares} + f)] \}, \quad (4.2) \\ & \text{(ARES)} \end{aligned}$$

where subscripts “a1” and “ares” denote, respectively, the beta gyres and the residual. Symbol  $\Lambda_{a1}$  is an operator for obtaining an azimuthal wavenumber 1 component. Term TT is thus the streamfunction tendency for the beta gyres. Term ASVA is advection of symmetric vorticity by asymmetric beta gyres relative to the vortex drift, and term AAVS is advection of asymmetric vorticity of the beta gyres by symmetric flow. The term BETA is due to meridional advection of planetary vorticity by symmetric circulation. The term ARES results from advection of residual vorticity by the beta gyres relative to the drifting velocity and the advection of absolute vorticity of asymmetric circulation by the residual flow.

Although the timing of the clockwise rotation in case 1 differs from that in case 2, the dynamic processes responsible for beta-gyre rotation are similar. Thus, only case 2 is analyzed. To better understand the clockwise rotation of the beta gyres, we plotted the phase and amplitude evolution for the beta gyres and the terms TT, BETA, and the sum of the other three terms in (4.2) at 750-km radius (Fig. 8) where the centers of the beta gyres are located. The total tendency TT is almost in phase with the beta gyres in the first 24 hours so that the beta gyres were stationary in that period. After hour 24 the phase of the TT starts to lead clockwise that of the beta gyres, yet due to the decrease in the magnitude of the TT, the phase change of the beta gyres is not significant before hour 60. From hour 60 on, the amplitude of the TT increases with time; meanwhile, the phase of the TT leads that of the beta gyres clockwise by about 90°. This resulted in a continuous clockwise rotation of the beta gyres after hour 60. The BETA term is controlled by the strength of the symmetric vortex. It decreases as the symmetric circulation weakens. The sum of the other terms is mainly due to the mutual advection between the symmetric and asymmetric circulations. It increases as the asymmetric circulation develops regardless of the weakening of symmetric circulation, suggesting its primary dependence on the asymmetric circulation. In case 2, the stronger outer flow induces a larger rate of the kinetic energy conversion from symmetric to asymmetric circulation, causing a more rapid decay of the symmetric circulation, which, in turn, causes a larger decline in the term



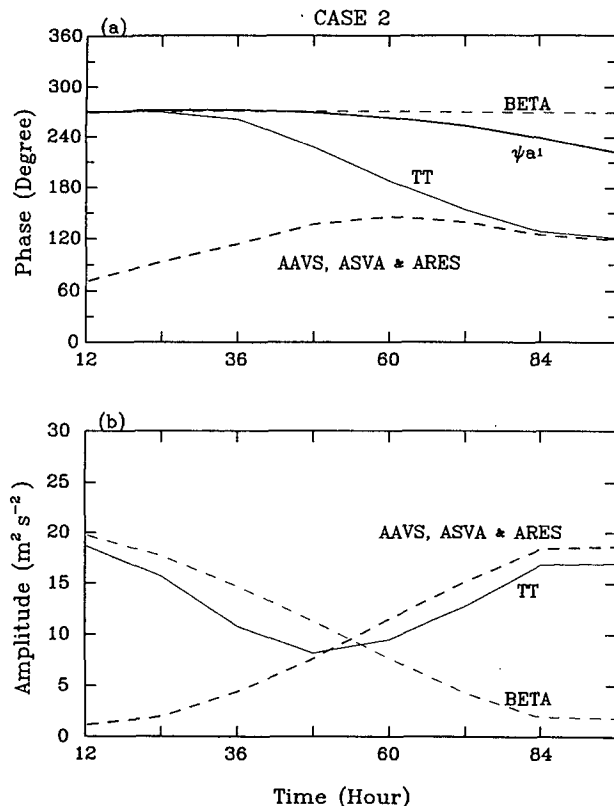


FIG. 8. (a) The phases (degree) and (b) the amplitudes ( $\text{m}^2 \text{s}^{-2}$ ) of the terms TT, BETA and the sum of AAVS, ASVA, and ARES at 750-km radius from hour 12 to 96 in case 2. The thick solid line in (a) denotes the phase of the beta gyres. See Eq. (4.2) for the meaning of abbreviations.

BETA. The more rapid development of the asymmetric circulation, on the other hand, results in a significant increase of this sum, causing a notable increase in the total tendency. Ross and Kurihara (1992) found that the inclusion of the time-dependent symmetric circulation is important for the beta drift. The results presented here indicate that the changes in the symmetric circulation affect the rotation of the beta gyres through the weakening of the BETA term.

In case 3 the centers of the inner beta gyres can be well identified from hour 132 to 143 around 300-km radius. The phase and amplitude evolution of the inner beta gyres, as well as terms TT, AAVS, ASVS, BETA, and ARES at 300-km radius, are thus displayed for that period (Fig. 9). The phase of the inner beta gyres increases with time, implying a counterclockwise rotation of the inner gyres. The average rotation rate is about  $10^\circ$  per hour. The phase of the TT is greater than that of the inner gyres so that the total tendency predicts the counterclockwise rotation of the inner gyres. Further, the term BETA is constant and the term AAVS has a substantially larger amplitude than the remaining

terms. Since the TT is dominated by the AAVS in amplitude, Eq. (4.2) may be simplified as

$$\frac{\partial \zeta_{a1}}{\partial t} = -\mathbf{V}_s \cdot \nabla \zeta_{a1}, \quad (4.3)$$

which has a general solution

$$\zeta_{a1} = A_k(r) \exp\{i[\lambda - (v_{\lambda s}/r)t]\}, \quad (4.4)$$

where  $A_k$  is the amplitude of  $\zeta_{a1}$  and  $v_{\lambda s}$  is the azimuthal wind of the symmetric vortex circulation. Equation (4.4) shows that the inner beta gyres rotate counterclockwise, and their rotation rate is determined by  $v_{\lambda s}/r$ . From hour 132 to 143,  $v_{\lambda s}$  at 300-km radius is about  $15 \text{ m s}^{-1}$ . The predicted rotation rate is thus about  $10^\circ$  per hour, which is very close to that of the simulation.

### b. Radial movement

To interpret the outward movement of the beta gyres in cases 1 and 2, radial distributions of the amplitudes of the beta gyres and the terms TT, ASVA, ARES, and the sum of AAVS and BETA are plotted for case 2

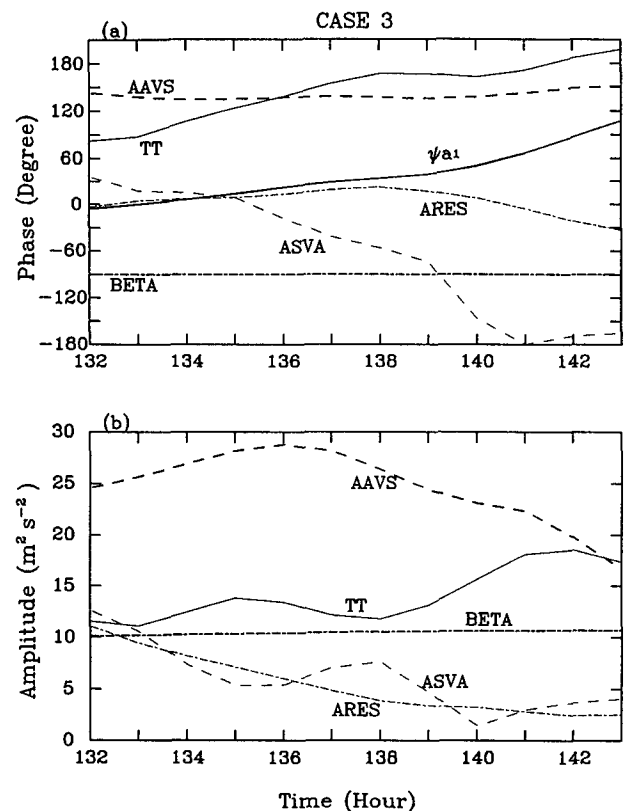


FIG. 9. (a) The phases (degree) and (b) the amplitudes ( $\text{m}^2 \text{s}^{-2}$ ) of terms TT, AAVS, ASVA, BETA, and ARES at 300-km radius from hour 132 to hour 143 in case 3. The thick solid line in (a) denotes the phase of the inner beta gyres. See Eq. (4.2) for the meaning of abbreviations.

(Fig. 10). The radius of the maximum amplitude of the beta gyres is less than that of the TT for both hour 36 and 84, indicating a tendency of outward movement of the beta-gyre centers. At hour 36, the AAVS and the BETA have maximum amplitudes around 350-km radius and are almost out of phase (figure not shown). As a result, their sum is small and almost constant outside 400-km radius. The ASVA contributes the most to the TT maximum and controls the outward movement. At hour 84, outside of 800-km radius, the AAVS and BETA are in phase (figure not shown) so that the sum has an amplitude that is compatible with the ASVA. The three terms are equally important for the outward movement of the beta gyres.

## 5. Summary

We reexamined the classical beta-drift problem with a shallow-water model. An extended-range time integration reveals that vortices with different initial symmetric structures may take quite different tracks. The evolution of counterrotating gyres (the beta gyres), whose flow over the vortex center advects symmetric relative vorticity, is a good indication of vortex movement in all cases. A key to understanding the beta drift is to explain the dynamics of the beta gyres.

The kinetic energy for the development of beta gyres is converted from symmetric vortex circulation. This energy conversion involves two processes: horizontal advection of relative vorticity and meridional advection of planetary vorticity. The latter, referred to as the beta conversion, is a dominant process responsible for the generation of asymmetric kinetic energy. Further analysis of the beta conversion reveals that 1) the development of the beta gyres requires that the anticyclonic gyre must be located to the east of the cyclone center in the Northern Hemisphere and 2) the rate of the beta conversion depends on the covariance between the amplitude of the beta gyres and the RAM of the symmetric vortex. Because the radial distribution of the beta-gyre intensity is also dependent on the radial distribution of the RAM, the rate of beta conversion is determined by the radial distribution of RAM or the symmetric vortex structure. As mean RAM increases, more asymmetric kinetic energy is generated, resulting in stronger beta gyres and a faster beta drift. This supports the empirical relationship between the mean RAM and the beta-drift speed numerically established by Wang and Li (1992).

The azimuthal and radial movement of beta gyres can be explained by vorticity dynamics. The total asymmetric streamfunction tendency is the sum of the advection of symmetric vorticity by the beta gyres relative to the vortex drift (ASVA), the advection of asymmetric vorticity of the beta gyres by symmetric flow (AAVS), the advection of planetary vorticity by symmetric flow (BETA), and the terms arising from the advection of residual vorticity by the beta gyres

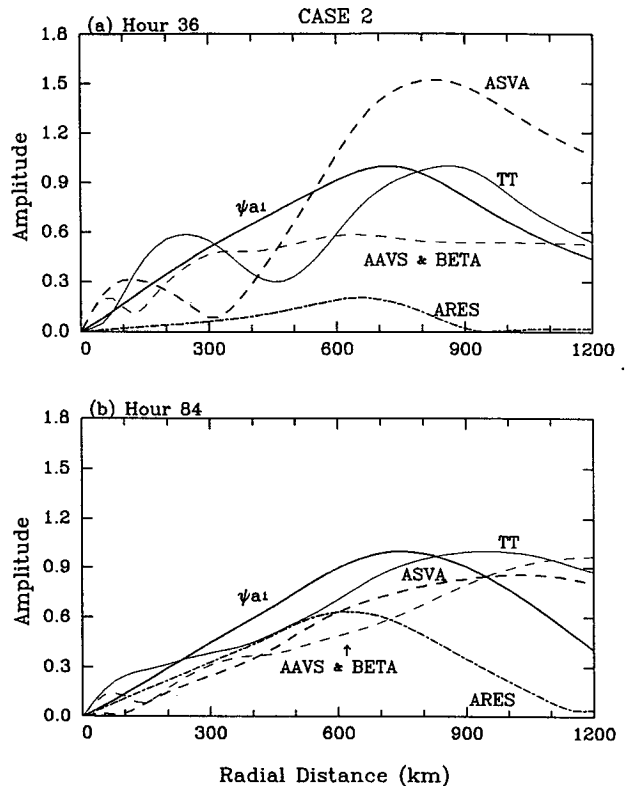


FIG. 10. Amplitudes of the beta gyres (normalized by its maximum), the terms TT, ASVA, ARES, and the sum of AAVS and BETA (divided by the TT maximum) as functions of radial distance (km) in case 2 at (a) hour 36 and (b) hour 84. See Eq. (4.2) for the meaning of abbreviations.

relative to the vortex drift and the advection of absolute vorticity of asymmetric circulation by the residual flow (ARES). For the outer beta gyres whose centers are around the periphery of the cyclonic azimuthal wind, their clockwise rotation found in numerical experiments is caused by a decrease of BETA and a sharp increase in the sum of AAVS, ASVA, and ARES. Their outward movement is mainly caused by the advection of symmetric vorticity by the beta gyres relative to the vortex drift. For the inner beta gyres whose centers are near the radius of maximum cyclonic wind, the advection of beta-gyre vorticity by symmetric flows dominates the total tendency and is responsible for their counterclockwise rotation. The rotation rate can be estimated from the angular velocity of the azimuthal wind of the symmetric vortex at the radius of the gyre centers.

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## APPENDIX

## Definitions of the First Radial Mode and Residual

Supposing  $\psi_a$  is a streamfunction of an azimuthal wavenumber 1 harmonic (beta gyres), the first radial mode is defined as

$$\psi_{a1} = \sum_{k=1}^K J_1(\sigma_{11}r) [\psi_{1c}^{(a)} \cos \lambda + \psi_{1s}^{(a)} \sin \lambda], \quad (\text{A1})$$

where  $r$  is a nondimensional radial distance normalized by  $r_0$  (here  $r_0 = 1200$  km),  $J_1$  represents the first-order Bessel function,  $\sigma_{11}$  is the first zero of  $J_1$ , and  $\psi_{1c}^{(a)}$ ,  $\psi_{1s}^{(a)}$  denote transform coefficients given by

$$\psi_{1c}^{(a)} = \frac{2}{\pi [J_2(\sigma_{11})]^2} \int_0^1 \int_0^{2\pi} \psi_a J_1(\sigma_{11}r) \cos \lambda r dr d\lambda, \quad (\text{A2a})$$

$$\psi_{1s}^{(a)} = \frac{2}{\pi [J_2(\sigma_{11})]^2} \int_0^1 \int_0^{2\pi} \psi_a J_1(\sigma_{11}r) \sin \lambda r dr d\lambda. \quad (\text{A2b})$$

The residual ( $\psi_{\text{ares}}$ ) is defined as the difference between  $\psi_a$  and  $\psi_{a1}$ :

$$\psi_{\text{ares}} = \psi_a - \psi_{a1}. \quad (\text{A3})$$

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