Forces expressed as Vectors

• Forces have two properties
  – Magnitude (length of arrow)
  – Direction (direction of arrow)
• Force balance

Flow driven by pressure difference

TANK A

H

Higher pressure

Net force

Lower pressure

TANK B
Pressure gradient force
- Directs from high to low pressure
- Magnitude = pressure difference/distance

Vector representation of pressure gradient force (PGF)
The closer the isobar spacing, the greater the pressure gradient
The closer the isobar spacing, the stronger the wind. (Winds blow more or less in parallel with isobars, instead of simply directing from high to low → other forces at work)

Apparent force is introduced to study the balance of force on a frame of reference with acceleration $a$.

$$ F - m \cdot a = 0 $$

$-m \cdot a$: apparent force in this non-inertial frame of reference.

Zero gravity
Apparent force: Centrifugal Force

- When viewed from a fixed reference frame, a ball swung on a string accelerates towards the center of rotation. This centripetal acceleration is provided by the pull of the string. \[ F = m \cdot a \]

- When viewed from a rotating reference frame, this ball has no relative motion. The pull of the string (A) is considered to be opposed by an apparent force (centrifugal force, B).

Centrifugal Force in action

- Planets orbiting the Sun.
- Satellites orbiting the Earth.
- Merry-go-round
- Tether ball
- Golf swing
**Centrifugal Force**

\[ \text{Centrifugal Force} = \frac{\text{velocity}^2}{\text{radius}} \]

- **Magnitude depends on**
  - the radius from the center
  - the speed of the air parcel
- **Direction**
  - outward at right angles to the motion

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**The Coriolis Force:**

An apparent force on a rotating frame like Earth, first fully described in 1835.

The ball's trajectory appears to veer to the right of the anticipated path, by the Coriolis force.

A must-see movie at http://ww2010.atmos.uiuc.edu/(Gh)/guides/mtr/fw/crls.rxml
The pendulum is turned to the right of the “anticipated” straight path by the Coriolis force.

A must-see movie at
http://ww2010.atmos.uiuc.edu/(Gh)/guides/mtr/fw/crls.rxml

The ball does not go straight but always wants to turn to the left of its velocity. Note that the platform rotates in a clockwise sense analogous to the Southern Hemisphere.

The Focault’s Pendulum:
Demonstration of the Coriolis Force

“Come to see the Earth turn”
Foucault’s invitation of February 2, 1851
In lobby of UNITED NATIONS General Assembly building, Foucault’s pendulum (gift of Queen Juliana of the Netherlands) has a 200 lb. bob, a 75 foot wire, and rotates about 235° in 24 hours.

Foucault’s pendulum in 1851 in Pantheon, Paris, was 67 m long.

Rotation period / Pendulum period = 5

Coriolis Force

\[ = 2 \times \text{Earth Rotation Rate} \times \sin(\text{latitude}) \times \text{velocity} \]

- **Apparent force due to rotation of the earth**
- **Magnitude** depends upon the latitude and the wind speed
  - The higher the latitude, the larger the Coriolis force
    - zero at the equator, maximum at the poles
  - The faster the speed, the larger the Coriolis force
- **Direction**: always acts at right angles to the direction of movement
  - To the right in the Northern Hemisphere
  - To the left in the Southern Hemisphere
Coriolis Force

The explanation in the textbook is problematic, invalid after the plane crosses the equator.

← Velocity relative to rotating Earth.
Centrifugal force = velocity² / radius

As the object begins to move relative to the rotating platform in the direction of rotation, the tension increases in the string to balance the increased centrifugal force. This increased centrifugal force is the origin of the Coriolis force in this case.

→ Measure the Coriolis force

Erratum 3
Example, though partially correct, does not explain how.

Apparent forces are introduced for easy application of Newton’s laws on a rotating frame of reference

Without relative motion, balance between the centripetal & centrifugal forces

With relative motion, balance among the centripetal, centrifugal & Coriolis forces

The Coriolis force is in action only when there is relative motion.