Some basic variables

Force Balance and Wind in the Atmosphere

Numerical Weather Prediction (NWP)

Model Output Statistics (MOS)

The Definition of Energy

Energy – the ability to do work

Two familiar types

1. Kinetic energy – the energy of motion: \( K = \frac{1}{2} m V^2 \)

2. Potential energy – stored energy: \( P = mgh \)

Temperature

- The degree of hotness or coldness of an object.
- The higher the temperature the greater the energy of motion of the molecules.
- Temperature is proportional to the average kinetic energy of the air molecules.

Characterizing with Temperature

Standard Atmosphere
- Troposphere (lowest)
- Stratosphere
- Mesosphere
- Thermosphere (highest)
Vertical Structure of Atmosphere

The warm air of the stratosphere acts as a lid on updrafts in the troposphere. The stratosphere is warm because of absorption of UV sunlight by ozone.

Tropopause

Stratosphere puts a lid on the weather

Anvil Cloud

Pressure

The force exerted against a surface by continuous collisions of gas molecules.

Pressure depends on:
1) The speed of the molecules
2) Mass of molecules
3) Frequency of their impacts
Pressure

The force exerted on a surface is equal to the weight of the air in a column from that surface to the top of the atmosphere.

Pressure

A column of air 1 m² (11 sq ft) weighs about 100 kilonewtons (equivalent to a mass of 10.2 metric tons at the surface).

Pressure Decrease with Altitude

Ideal Gas Law – Pressure is proportional to the density of the air times the temperature of the air.

\[ P = \text{Constant} \times T \times D \text{ or } P = \rho RT \]

Charles Law – At constant volume (e.g., a closed can) pressure is proportional to temperature.

\[ P = \text{Constant} \times T \]

For demo link:
http://intro.chem.okstate.edu/1314F00/Laboratory/GLP.htm
Relationship between Pressure & Temperature in the Atmosphere

Warm air molecules move faster than cold air molecules, therefore, they take up more space in the atmosphere (because the atmosphere is not a closed container).

Since surface pressure is the weight of all the overlying air molecules, areas of warm air relative to their surroundings will have lower surface pressure.

Heat

- Heat is a transfer of energy from a warmer object to a colder object.
- Heat makes things warmer.
- Heat is measured in units called calories.
- A calorie is the heat (energy) required to raise one cubic centimeter of water by 1°C.

Heat in the Atmosphere

There are four ways in which heat is transferred.

1. Radiation – heat transfer by electromagnetic waves, which are emitted by all objects.
2. Conduction – heat transfer by direct contact.
Moist Convection

Almost a daily occurrence in Hawaii over the mountains — caused by surface heating, rising buoyant plumes, and the release of latent heat in clouds.

Important Heat Concepts

Heat capacity — amount of heat that must be added to a gram of substance to achieve a 1°C change in its temperature. (e.g., water has a higher heat capacity than air)

Sensible heat — heat that can be measured (sensed) by a thermometer.

Latent heat — heat required/released when a substance changes from one state to another. (Latent means hidden in Latin, e.g., heat when added/removed from a substance does not change its temperature when a change in state occurs.)

Wind

Motion in the atmosphere is the result of the action of forces.

The equation that describes the forces that cause the acceleration of the wind is the momentum equation.

Many consider the momentum equation to be the most important equation in meteorology.

Forces and Wind

Wind in the Atmosphere is measured by:
- Anemometer (10 m tower)
- Doppler Radar (rain)
- Radiosonde (balloon)
Forces and Wind

Wind data are plotted using the convention above.

Forces and Winds

• Pressure gradients produce air movement
• Multiple forces act simultaneously to cause the wind direction to differ from the direction of decreasing pressure
• Newton’s laws of motion describe the relationship between forces and motion

Forces and Wind

A force is a pushing or pulling that will result in motion if it is unopposed.

Newton’s Laws of Motion
1. In the absence of forces an object at rest will remain at rest and an object in motion will remain so with the same velocity.
2. Force equals mass times acceleration \( F = m \cdot a \)
3. To every action there’s an equal and opposite reaction.

What are the macro-scale forces that operate in the Earth’s atmosphere?

There are only five:
1. Pressure Gradient \( \text{PG} \)
2. Gravity \( \text{g} \)
3. Coriolis \( \text{Co} \)
4. Friction \( \text{Fr} \)
5. Centrifugal \( \text{Ce} \)
Five Forces

Pressure gradient – a change in pressure over a distance.
Gravity – acceleration due to the Earth’s gravitational field; directed downward.
Coriolis – a fictitious force due to the rotation of the Earth underneath the air as it moves. $C_o$ is proportional to the velocity and varies from a maximum at the poles and zero on the equator.
Friction – acts to dissipate motion into heat, converting motion into heat through turbulence. Friction is greatest at the ground and it influence decreases upward until it vanishes at ~ 1 km height.
Centrifugal – a fictitious force due to the fact that an object will continue to move in a straight line unless acted on by an unbalanced force.

Coriolis Force

$$F_c = 2\Omega v \sin \phi$$

- $C_o = \text{Coriolis Force}$
- $\Omega = \text{Earth rotation}$
- $V = \text{Wind Speed}$
- $\phi = \text{Latitude}$

Apparent force due to rotation of the earth

Rotation speed due to rotation of the earth
Momentum Equation

\( \frac{D\mathbf{V}}{Dt} = -2\Omega \times \mathbf{V} - \frac{1}{\rho} \nabla p + g + F_r \)

1. Coriolis \( C_o \)
2. Pressure Gradient \( \text{PG} \)
3. Gravity \( g \)
4. Friction \( F_r \)

*Where is Centrifugal force?*

Momentum Equation – Vector Form

\( \frac{D\mathbf{V}}{Dt} = -2\Omega \times \mathbf{V} - \frac{1}{\rho} \nabla p + g + F_r \)

\[-2\Omega \times \mathbf{V} = -2\Omega \begin{bmatrix} i & j & k \\ o & \cos \theta & \sin \theta \\ u & v & w \end{bmatrix} = -(2\Omega w \cos \theta - 2\Omega v \sin \theta)i - 2\Omega u \sin \theta j + 2\Omega u \cos \theta k\]

Momentum Equation – Component Form

\[
\frac{Du}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2\Omega v \sin \theta - 2\Omega w \cos \theta + F_{rx} \\
\frac{Dv}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + 2\Omega u \sin \theta + F_{ry} \\
\frac{Dw}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g + 2\Omega u \cos \theta + F_{rz} 
\]

Characteristic Scales

For synoptic scale systems in the free atmosphere (e.g., ignore friction) the characteristic scales for the field variables can be estimated as follows.

- \( u \sim 10 \text{ m s}^{-1} \) Horizontal velocity
- \( w \sim 10^{-1} \text{ m s}^{-1} \) Vertical velocity
- \( L \sim 10^6 \text{ m} \) Length scale [\( \sim 1(2\pi) \text{wavelength} \)]
- \( D \sim 10^4 \text{ m} \) Depth scale
- \( \Delta P/\rho \sim 10^3 \text{ m}^3 \text{ s}^{-2} \) Horizontal press fluctuation scale
- \( L/u \sim 10^5 \text{ s} \) Time scale (~1 day)
Synoptic Scale Analysis

\[
\frac{du}{dt} = 2\Omega \sin\theta \quad 2\Omega \cos\theta \quad ((1/\rho) \frac{\partial P}{\partial x})
\]

\[
\frac{dv}{dt} = 2\Omega \sin\theta \quad ((1/\rho) \frac{\partial P}{\partial y})
\]

\[
u^2/L \quad \frac{f_0 u}{f_0 w} \quad \frac{\Delta P}{\rho L}
\]

\[10^{-4} \quad 10^{-3} \quad 10^{-6} \quad 10^{-3}\]

\[\ast f_0 \approx 10^{-4} = 2\Omega \sin\theta \text{ at } 45^\circ \text{ N}\]

Vertical momentum equation

\[
\frac{Dw}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g + 2\Omega \nu \cos \theta + F_{rz}
\]

Vertical acceleration = Vertical PGF-gravity+Coriolis+friction

Which terms in this equation are the largest?
How large are they?
If we eliminate small terms, what are we left with?

Hydrostatic Equation

The vertical component of the momentum equation can be rewritten after elimination of small terms.

\[
\frac{\partial p}{\partial z} = -\rho g
\]

Since \( g \) is a constant, this equation tells us that the rate of change of pressure with height is dependent on temperature: it is greater for cold dense air than for warm less dense air.
Hydrostatic Balance

A balance between gravity and the pressure gradient force.

Pressure gradient

Gravity

keeps air from rising due to the upward pressure gradient force.

Consequences of Hydrostatic Balance

Can use Pressure as the Vertical Coordinate

- Since there is a 1-to-1 connection between pressure and height (in a hydrostatic system), one can use pressure as the vertical coordinate and plot z(x,y,P), this is generally applied in meteorology above the surface – e.g. we plot the “500 hPa heights”.
- Pressure is commonly plotted for z = 0, i.e. the sea level pressure (slp, mslp) to show the horizontal variations of pressure by P(x,y,z).

If we assume that the atmosphere is in hydrostatic balance, are we assuming that there is no vertical motion?

Hydrostatic Balance

The GFS model and other forecast models are often “hydrostatic models”. So, if this is true, how can the GFS predict vertical motion and precipitation?

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \]

where \( \omega = \frac{\Delta p}{\Delta t} = -pgw \)

This is the continuity equation, which basically is a statement about the conservation of mass.

Continuity equation can be solved for \( \omega \) in a diagnostic sense if the horizontal wind is known.

GFS model computes new u, v fields, (from horizontal momentum eqs.), uses continuity to determine omega.

Geostrophic Wind

Since accelerations of the wind are an order of magnitude smaller than the other terms in the scale analysis, it is apparent that to a first approximation the Coriolis and pressure gradient terms in the momentum equation are in balance. This is defined as geostrophic balance and the resulting wind is called geostrophic wind.

\[ fv \cong 1/\rho \partial P/\partial x \quad , \quad fu \cong -1/\rho \partial P/\partial y \]
Geostrophic Wind

We can combine these results to give an equation for the geostrophic wind on a surface chart if we know the perpendicular distance \( n \) between isobars. The equation is as follows;

\[
V_g = \frac{1}{\rho} \cdot \frac{g}{f} \cdot \frac{\Delta p}{\Delta n}
\]

For upper level constant pressure charts we can write

\[
V_g = \frac{g}{f} \cdot \left( \frac{\partial \Phi}{\partial n} \right)
\]

Where \( \Phi \) is geopotential height and \( n \) is distance normal to contours.

Geostrophic Wind

• Geostrophic motion occurs when there is an exact balance between the PGF and the \( C_o \), and the air is moving under the action of these two forces only.

• It implies
  • No acceleration
  • e.g., Straight, parallel isobars
  • No other forces
  • e.g., friction
  • No vertical motion
  • e.g., no pressure changes

An Example

What is the Geostrophic wind speed for a pressure gradient of 2 hPa/km and density of 1.2 kg m\(^{-3}\) at a latitude of 20° ? (\( \Omega = 7.292 \times 10^{-5} \))

\[
\text{PGF} = 2\text{hPa/100km} = \frac{200\text{pa}}{100\text{km}} = 2 \times 10^{-4} \text{ pa/m}
\]
\[
\rho = 1.2 \text{ kgm}^{-3}
\]
\[
f = 2 \Omega \sin \phi
\]
\[
\therefore \text{for } 20^\circ \quad V_g = \frac{1}{1.2 \times 2 \Omega \sin 20} \cdot \frac{200}{10^3}
\]
\[
= 3.35 \text{ ms}^{-1}
\]

The Nature of \( V_g \)

Geostrophic wind blows parallel to the Isobars

If you have your back to the wind then Low Pressure is on your left in the Northern Hemisphere
Gradient Wind – $V_{gr}$

Wind that results under conditions of curved flow when the Centrifugal Force is exactly balanced by the Coriolis and Pressure Gradient Forces

$$C_e = C_o - \text{PGF}$$

Three Cases of $V_{gr}$ exist

- Anti-clockwise flow (high pressure center)
- Clockwise flow (low pressure center)
- Straight Flow ($V_{gr} = V_g$ which is a special case)

Gradient Wind Equations

The equations for the gradient wind ($V_{gr}$) depend on whether the flow is cyclonic or anti-cyclonic. Here $V_g$ is the geostrophic wind and $r$ the radius of curvature.

Cyclonic Flow

$$V_{gr} = -rf + \sqrt{\frac{r^2 f^2 + 4rfV_g}{2}}$$

Anticyclonic Flow

$$V_{gr} = rf - \sqrt{\frac{r^2 f^2 - 4rfV_g}{2}}$$
Cyclostrrophic Wind

\[ \frac{V^2}{R} = - \left( \frac{1}{\rho} \right) \left( \frac{\Delta p}{\Delta n} \right) \]

\[ V = \left( - \frac{R}{\rho} \left( \frac{\Delta p}{\Delta n} \right) \right)^{1/2} \]

Frictional Effects

In the surface layer, \( \sim 1 \) km, the impact of friction can not be ignored.

- Frictional effects reduce wind speed
- Wind backs as Coriolis is reduced
- Cross isobar flow towards low pressure region
- Flow outwards from high pressure
- Flow in towards low pressure
- As friction reduces with height, wind flow will veer with height (turn clockwise with height in NH)

Effect of Surface Roughness on Cross-Isobar Flow at 10 m

Over water \( \alpha = 10^\circ \) \( V = \frac{2}{3} V_g \)

Over land \( \alpha = 30^\circ \) \( V = \frac{1}{3} V_g \)

Variations in surface roughness lead to variations in the cross isobar component of the flow, with larger angles (and convergence into lows) over land than over water.

Effect of Friction

Ekman Spiral

Coriolis Force

Wind will accelerate until the Coriolis Force becomes as strong as the Pressure Gradient Force.

Ekman Pumping

Friction induced cross isobar flow in the friction layer results in Ekman pumping that produces convergence into surface lows and divergence out of surface highs.

Review of Forces

1. Pressure Gradient Force* – changes in pressure over a distance causes air to move.
2. Gravity* – only acts in the vertical direction
3. Coriolis Force – due to Earth’s rotation underneath the moving air.
4. Centrifugal Force – whenever there is curved flow (curved isobars)
5. Friction – only important near the Earth’s surface

* Only the pressure gradient force and gravity can cause winds in air that is initially at rest.

Balance of Forces

- Cyclostrophic Balance
  Pressure Gradient Force = Centrifugal Force
- Geostrophic Balance
  Pressure Gradient Force = Coriolis Force
- Gradient Wind Balance
  Pressure Gradient Force = Centrifugal + Coriolis Forces
- Hydrostatic Balance
  Pressure Gradient Force = Gravity
Questions?