Flexure modelling at seamounts with dense cores

Seung-Sep Kim and Paul Wessel

Department of Geology and Geophysics, University of Hawaii, 1680 East-West Road, Honolulu, HI 96822, USA. E-mail: seungsep@hawaii.edu

Accepted 2010 May 6. Received 2010 May 5; in original form 2009 November 30

SUMMARY

The lithospheric response to seamounts and ocean islands has been successfully described by deformation of an elastic plate induced by a given volcanic load. If the shape and mass of a seamount are known, the lithospheric flexure due to the seamount is determined by the thickness of an elastic plate, $T_e$, which depends on the load density and the age of the plate at the time of seamount construction. We can thus infer important thermomechanical properties of the lithosphere from $T_e$ estimates at seamounts and their correlation with other geophysical inferences, such as cooling of the plate. Whereas the bathymetry (i.e. shape) of a seamount is directly observable, the total mass often requires an assumption of the internal seamount structure. The conventional approach considers the seamount to have a uniform density (e.g. density of the crust). This choice, however, tends to bias the total mass acting on an elastic plate. In this study, we will explore a simple approximation to the seamount’s internal structure that considers a dense core and a less dense outer edifice. Although the existence of a core is supported by various gravity and seismic studies, the role of such volcanic cores in flexure modelling has not been fully addressed. Here, we present new analytic solutions for plate flexure due to axisymmetric dense core loads, and use them to examine the effects of dense cores in flexure calculations for a variety of synthetic cases. Comparing analytic solutions with and without a core indicates that the flexure model with uniform density underestimates $T_e$ by at least 25 per cent. This bias increases when the uniform density is taken to be equal to the crustal density. We also propose a practical application of the dense core model by constructing a uniform density load of same mass as the dense core load. This approximation allows us to compute the flexural deflection and gravity anomaly of a seamount in the wavenumber domain and minimize the limitations recognized from the analytic tests. Then, the dense core model is applied to predict the lithospheric flexure beneath Howland Island in the Tokelau seamount chain; these results are compared with the predictions of the uniform density model. Based on age dating of Howland and the age of the seafloor, traditional $T_e$ versus age curves predict the elastic plate thickness beneath the seamount to be around 20 km, which is comparable to the best dense core model of $T_e = 26$ km. However, the best uniform density model is found at $T_e = 12$ km, which is significantly less than the predicted. From our investigations of synthetic and real seamount cases, we conclude that the dense core model approximates the true mass distribution of a seamount better than the uniform density model. Finally, we suggest that the role of underplating in flexure modelling may need to be reexamined because the dense core model predicts substantially less deflections than the uniform density model without requiring additional buoyancy caused by underplated material.

Key words: Geopotential theory; Lithospheric flexure; Mechanics, theory, and modelling.

1 INTRODUCTION

How the lithosphere responds to geological loads (e.g. ice sheets and seamounts) has been modelled using an elastic plate that can flex downwards to support a given load (Watts 2001). The mechanical behaviour of elastic lithosphere is then simply expressed in terms of elastic plate thickness, $T_e$. Numerous studies have been carried out to understand lithospheric deformation of the oceanic crust, especially beneath seamounts and ocean islands (e.g. Walcott 1970; McNutt & Menard 1978; Watts 1978; Calmant 1987; Wolfe & McNutt 1991; Wessel 1993; Collier & Watts 2001; Adam & Bonneville 2008). As $T_e$ depends on the age of the oceanic plate at the time of volcanic construction ($\Delta t$), the relationship between $T_e$ and $\Delta t$ has been widely applied to determine apparent ages of seamounts (e.g. Calmant et al. 1990; Watts et al. 1993).
on an inviscid fluid due to an axisymmetric load, \( q \), which can be fixed as the crustal density, \( \rho_c \), depending on minimizing the misfits. (b) In the two-boundary model (e.g. Wessel 2001), the peripheral density, \( \rho_p \), differs from the seamount density, \( \rho_s \). (c) Dense core model. In this study, we separate the seamount body into the outer edifice (with density \( \rho_d \)) and the inner core (with density \( \rho_c \)), which reflect both geological (see figure 15 in Staudigel & Schmincke 1984) and geophysical (see figure 2 in Minshull & Charvis 2001) observations of internal seamount structure.

In addition, new roles of the lithosphere in influencing the location of intraplate volcanism (Hillier 2007) and limiting the growth of seamounts (Wessel 2001) have been recently proposed.

The flexural moats that flank seamounts and oceanic islands are filled with a mixture of volcanic, erosional and sedimentary materials and hence the peripheral (or moat) materials are less dense than the seamount itself. If one selects a peripheral density, \( \rho_p \), that differs from the seamount density, \( \rho_s \), then the flexed crust boundary exhibits a radial variation of density contrasts (i.e. \( \rho_c - \rho_p \) beneath the seamount and \( \rho_c - \rho_s \) inside the moat). Although this two-boundary model (Fig. 1b) complicates gravity prediction, it is more realistic than the uniform density model (e.g. Smith et al. 1989; Beavan 2006; Hillier 2007).

Many detailed gravity (e.g. Robertson 1967; Kellogg et al. 1987; Hammer et al. 1991; Araña et al. 2000; Camacho et al. 2009) and seismic (e.g. Hildebrand et al. 1989; Watts et al. 1997; Gallart et al. 1999) surveys have provided evidence of dense cores inside seamounts and ocean islands. Interestingly, such dense cores have only been considered when computing gravity anomalies due to the volcanic construct, but not when estimating the flexural deformation beneath it. Yet, lithospheric deformation is often modelled by minimizing the differences between the observed and predicted gravity anomalies. Furthermore, many tools for computing gravity signals due to arbitrary bodies are available (e.g. Talwani & Ewing 1960; Nagy 1966; Plouff 1976; Blakely 1996), but no methodology has been developed for flexure modelling with a dense core. There is a possibility that the uniform seamount assumption distorts the flexure analysis, and may in turn bias the results towards low \( T_e \) estimates (Minshull & Charvis 2001). Exploring such a possibility is long overdue.

In this study, we propose a new dense core seamount model incorporating a core that is denser than the seamount edifice (Fig. 1c) and its practical application to predict flexure and gravity at seamounts. For direct comparison between different seamount models in Fig. 1, we first derive analytic solutions for plate flexure due to axisymmetric dense core loads. Then, we explore the differences between these models using various synthetic loads and a case study of lithospheric flexure beneath Howland Island in the northern Tokelau seamount chain.

**Figure 1.** Various flexure models. (a) Uniform density model assumes all loading parts have a constant density, \( \rho_l \), which can be fixed as the crustal density, \( \rho_c \), depending on minimizing the misfits. (b) In the two-boundary model (e.g. Wessel 2001), the peripheral density, \( \rho_p \), differs from the seamount density, \( \rho_s \). (c) Dense core model. In this study, we separate the seamount body into the outer edifice (with density \( \rho_d \)) and the inner core (with density \( \rho_c \)), which reflect both geological (see figure 15 in Staudigel & Schmincke 1984) and geophysical (see figure 2 in Minshull & Charvis 2001) observations of internal seamount structure.

2 ANALYTIC SOLUTIONS FOR AXISYMMETRIC DENSE CORE LOADS

As a first-order approximation to realistic internal seamount structures (Staudigel & Schmincke 1984; Minshull & Charvis 2001), we first construct axisymmetric disc and parabolic loads that have dense cores at their centres. Then, following previous studies (Brotchie & Silvester 1969; Brotchie 1971; Lambeck & Nakiboglu 1980), the governing equation for the deflection, \( w(r) \), of an elastic plate of flexural rigidity, \( D \), on an inviscid fluid due to an axisymmetric load, \( q(r) \), is given as

\[
D \nabla^4 w(r) + \rho_w g w(r) = q(r).
\]
Table 1. Parameters used in flexure and gravity calculations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_d$</td>
<td>Edifice density</td>
<td>2500</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Peripheral density</td>
<td>2300</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Core density</td>
<td>2900</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Mantle density</td>
<td>3300</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>Crust density</td>
<td>2800</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>Water density</td>
<td>1000</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson's ratio</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>Young's modulus</td>
<td>$10^{11}$</td>
<td>Pa</td>
</tr>
<tr>
<td>$g$</td>
<td>Normal gravity</td>
<td>9.81</td>
<td>m s$^{-2}$</td>
</tr>
<tr>
<td>$G$</td>
<td>Gravitational constant</td>
<td>$6.673 \times 10^{-11}$</td>
<td>m$^3$ kg$^{-1}$ s$^{-2}$</td>
</tr>
</tbody>
</table>

with

$$\nabla^2 \equiv \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr}$$  \hspace{1cm} (2)

and

$$D = \frac{ET^2}{12(1 - \nu^2)}.$$  \hspace{1cm} (3)

The parameters not mentioned in the text are listed in Table 1. In eq. (1), the right-hand side is the driving force flexing the elastic plate downwards, whereas the terms on the left-hand side represent the resisting forces arising from the finite strength of the plate (first term) and the buoyancy of the mantle (second term). Through the balance between these forces, geological features achieve isostasy (Watts 2001).

To simplify the derivation, we assume that the dense core has the same shape as the edifice. The density variations of each part in the dense core disc and parabolic loads are denoted as the core density, $\rho_a$, the edifice density, $\rho_d$ and the peripheral density, $\rho_s$, respectively (Fig. 2a). For further simplification, we assume that the flexural moat is completely filled only up to the originally undeformed plane (i.e. the reference level in Fig. 2a). Also, it is mathematically convenient to divide the load, $q(r)$, into two components: one above the reference

![Figure 2. Axisymmetric dense core loads.](image-url)
level and the other between the reference level and the flexed crust (Fig. 2). To be concise, we use ‘surface’ for any loading materials above the reference level and ‘subsurface’ for materials between the reference level and the flexed crust. The amount of subsurface materials, therefore, becomes a function of the deflection, \( w(r) \). In flexure modelling, however, the term ‘infill’ has been widely used to describe this subsurface material as it infills the flexed crust (Watts 2001). The uniform density model allows the infill density to differ from the density of the topographic feature above the reference level, however the model still requires a uniform density for the infill material (e.g. Hillier 2007).

Although such separation between the surface and subsurface materials can lead to better gravity and flexure predictions, it is incongruent with geological views on seamount growth (Staudigel & Schmincke 1984; Minshull & Charvis 2001). Thus, the subsurface material in the dense core model changes radially from the core material beneath the core, the edifice material beneath the edifice and the material inside the flexural moat (Fig. 2). This radial change of subsurface load requires us to divide \( q(r) \) into three parts: the first for the radial extent of the core at \( 0 \leq r \leq r_a \), the second only under the edifice at \( r_a \leq r \leq r_d \) and the third extending beyond the edge of seaamount at \( r \geq r_d \) (although the uplifted bulge represents yet another domain, we cannot include it in an analytic solution). The dense core disc and parabolic loads, \( q_d(r) \) and \( q_p(r) \), respectively, are defined as follows:

\[
q_d(r) = \begin{cases} 
\rho_d g[h_d + w(r)] + \rho_d g(h_d - h_a) - \rho_w g h_d & 0 \leq r \leq r_a \\
\rho_d g[h_d + w(r)] - \rho_w g h_d & r_a \leq r \leq r_d, \\
\rho_d g w(r) & r \geq r_d 
\end{cases}
\]

and

\[
q_p(r) = \begin{cases} 
\rho_p g[t_p(r) + w(r)] + \rho_p g[t_p(r) - \ell_p(r)] - \rho_w g \ell_p(r) & 0 \leq r \leq r_a \\
\rho_p g[t_p(r) + w(r)] - \rho_w g \ell_p(r) & r_a \leq r \leq r_d, \\
\rho_p g w(r) & r \geq r_d 
\end{cases}
\]

with the parabolas of the core and the edifice defined as

\[
t_i(r) = h_i[1 - r^2/r_i^2] \quad [x = a, d].
\]

The heights of the edifice and the core are measured relative to the reference level (Fig. 2b).

When the shape of the surface load is relatively simple and axisymmetric, the homogenous solution to eq. (1) becomes a combination of Bessel–Kelvin functions (Brotchie & Silvester 1969; Lambeck & Nakiboglu 1980). This solution is also valid for the dense core loads because both the core and the edifice have the same axisymmetric shape. By equating eq. (1) with eqs (4) and (5), we obtain the particular solutions using the method of undetermined coefficients. Customarily, the general solution for a non-homogeneous equation is a sum of the homogeneous and particular solutions that contain every solution satisfying eq. (1) over the interval of interest. In flexure modelling, the general solution (i.e. deflection) for eq. (1) must satisfy the following boundary conditions for continuity at each boundary, where the loading material changes (Fig. 2b):

Condition 1. At \( r = 0 \), \( dw/dr \) and shear stress are finite.

Condition 2. At \( r = r_a \), \( w \), \( dw/dr \), bending moments, and shear stresses are continuous.

Condition 3. At \( r = r_d \), \( w \), \( dw/dr \), bending moments, and shear stresses are continuous.

Condition 4. At infinity, \( w \) and \( dw/dr \) vanish.

Here, the corresponding axisymmetric bending moments, \( M(r) \), and shear stresses, \( Q(r) \), are given by

\[
M(r) = -D \left( \frac{d^2w}{dr^2} + \frac{1}{r} \frac{dw}{dr} \right)
\]

and

\[
Q(r) = -D \frac{d}{dr} \nabla^2 w.
\]

With boundary conditions 1 and 4 applied, the analytic solution to eq. (1) with the dense core disc load is reduced to

\[
w_d(r) = \begin{cases} 
\alpha[1 + C_1 \text{ber}(k_r r) + C_2 \text{bei}(k_r r)] & 0 \leq r \leq r_a \\
\beta[1 + C_1 \text{ber}(k_r r) + C_2 \text{bei}(k_r r) + C_3 \text{ker}(k_s r) + C_4 \text{kei}(k_s r)] & r_a \leq r \leq r_d, \\
\gamma[1 + C_5 \text{ker}(k_s r) + C_6 \text{kei}(k_s r)] & r \geq r_d
\end{cases}
\]

where

\[
\alpha = [h_a(\rho_p - \rho_d) + h_d(\rho_d - \rho_w)]/(\rho_m - \rho_a).
\]

\[
\beta = h_d(\rho_d - \rho_w)/(\rho_m - \rho_d),
\]

and \( C_1 \) - \( C_4 \) are non-zero integration constants. Here, ber, bei, ker and kei are Bessel–Kelvin functions of zero order (Abramowitz & Stegun 1970), and \( k_a, k_d \) and \( k_s \) are the radial flexural wavenumbers defined as

\[
k_x = \left( \frac{(\rho_m - \rho_x) g}{D} \right)^{1/4} \quad [x = a, d, s].
\]
These flexural wavenumbers result from the radial change of subsurface material. The eight arbitrary constants in eq. (9) are evaluated numerically by applying boundary conditions 2 and 3.

Similarly, the analytic solution to eq. (1) with the dense core parabolic load is obtained as

$$w_\mathbf{P}(r) = \begin{cases} \eta [t_\mathbf{a}(r)(\rho_\mathbf{a} - \rho_\mathbf{d}) + t_\mathbf{d}(r)(\rho_\mathbf{d} - \rho_\mathbf{w}) + C_1 \text{ber}(k_\mathbf{a}r) + C_2 \text{bei}(k_\mathbf{a}r)] & 0 \leq r \leq r_\mathbf{a} \\ \zeta [t_\mathbf{d}(r) + C_3 \text{ber}(k_\mathbf{d}r) + C_4 \text{bei}(k_\mathbf{d}r) + C_5 \text{ker}(k_\mathbf{d}r) + C_6 \text{kei}(k_\mathbf{d}r)] & r_\mathbf{a} \leq r \leq r_\mathbf{d} \\ C_7 \text{ker}(k_\mathbf{s}r) + C_8 \text{kei}(k_\mathbf{s}r) & r_\mathbf{d} \leq r \end{cases}$$

(12)

where

$$\eta = \frac{1}{(\rho_\mathbf{m} - \rho_\mathbf{a})}$$

(13)

and

$$\zeta = \frac{(\rho_\mathbf{d} - \rho_\mathbf{w})}{(\rho_\mathbf{m} - \rho_\mathbf{d})}.$$  

(14)

Boundary conditions 2 and 3 are also applied for the numerical determination of $C_1$ – $C_8$. For the sake of minimizing symbols, we keep the same subscript numbers of $C$s in eq. (12) as in eq. (9), but their values are different. Although these constants can be evaluated analytically, the analytic solutions become arduous and it is simpler to assess them numerically.

We verify the analytic solutions for plate flexure due to the dense core loads in three different ways. First, a case of no core (i.e. $\rho_\mathbf{a} = \rho_\mathbf{d}$) is considered. Then, eqs (9) and (12) must be equal to the solutions of Lambeck & Nakiboglu (1980). With no core, the first boundary in the dense core load presents no change in parameters and hence the analytic solution for the first boundary condition is not needed. Although boundary condition 1 requires $C_5$ and $C_6$ to be zero, boundary condition 3 is necessary to determine the other constants. Therefore, we obtain the same solutions as Lambeck & Nakiboglu (1980). This verification proves that our derivation is correct.

Second, when the surface load is very large ($r_\mathbf{d} > 500$ km), the lithosphere becomes very weak compared to the given load and exhibits an Airy-type flexural response regardless of elastic plate thickness (Watts 2001). In an Airy-type flexural response, the maximum deflection directly underneath the dense core loads in Fig. 2 becomes linearly proportional to the mass balance between the surface loads (i.e. the core and the edifice) and the portion of the mantle displaced by the core; it is 17 km for the given dense core loads. To examine if our analytic solutions satisfy this simple physics, we increase only the radii of the edifice and the core and fix all other parameters including $T_e$ as in Fig. 2; the core radius is half the edifice radius. From Figs 3(a) and (c), we find that both analytic solutions result in the expected value. The staircase feature in Fig. 3(a) occurs because the edifice itself is large enough to attain its own Airy-type flexural response.

Last, we compare the analytic solutions directly with the numerical solution of Bodine (1980) using the dense core loads in Fig. 3. For the Bodine solution, the surface load is approximated by stacking the edifice and the core together so that the total mass of the stacked parts is equal to that of the dense core load above the reference level. In addition, the Bodine solution is adapted to include the subsurface

Figure 3. Large dense core loads on an elastic plate of $T_e = 20$ km. Analytic deflections due to the dense core (a) disc and (c) parabolic loads are compared with the numerical deflections of Bodine (1980). The arrows in (a) and (c) indicate the expected Airy-type flexural response for the applied dense core loads. (b) and (d) are the differences between the analytic and Bodine solutions and show a good agreement.
material changes at \( r = r_a \) and \( r = r_d \). Both the analytic and Bodine solutions are constrained by the boundary conditions listed above. Figs 3(b) and (d) show the remainders after subtracting the numerically calculated deflections from the analytical ones. Except the minimal differences \( < 10^{-4} \), both solutions exhibit a good agreement. Therefore, we conclude that the analytic solutions for plate flexure derived with the axisymmetric dense core loads are valid. Next, we will explore how the dense core model differs from the uniform density model when used to predict deflections and the gravity anomalies using various synthetic data.

3 FLEXURE AND GRAVITY CALCULATIONS WITH A DENSE CORE LOAD

3.1 Flexure modelling with a dense core load

The total mass of a uniform density load (i.e. the driving force) depends mainly on the choice of density. Although the detailed internal structure of seamounts can be constrained by, or inferred from, other data (e.g. seismic data), the load density, \( \rho_l \), has often been presumed to equal the crustal density (i.e. Fig. 1a with \( \rho_l = \rho_c \)). However, this choice tends to overestimate the driving force. For example, if we compare the dense core parabolic load in Fig. 2(b) with a uniform parabolic load of same size and of crustal density, the mass above the reference level for the former becomes 40 per cent heavier than that for the latter. Because of the overestimated mass, the uniform density model results in more flexure relative to the dense core model for the same \( T_e \). If we consider the parabolic seamount (i.e. bathymetry) in Fig. 2(b) and the flexure produced by the dense core load as the observed data, the uniform density model requires a thicker elastic plate (larger \( T_e \)) to compensate for the overestimated load (Minshull & Charvis 2001). Therefore, a careful consideration of the load density is necessary to prevent overestimating the driving force.

Rather than adjusting the load density alone (Minshull & Charvis 2001), the load overestimation can be mitigated by lowering the peripheral density relative to the seamount density (Fig. 1b). For instance, Wessel (2001) used a realistic peripheral density \( (\rho_l = 2300 \text{ kg m}^{-3}) \) with a constant seamount density \( (\rho_t = 2800 \text{ kg m}^{-3}) \). We compare this two-boundary model with the dense core and uniform density models for a range of parabolic loads and elastic plate thicknesses. For this comparison, the crustal density is used for the load density. Fig. 4 shows the maximum deflections obtained beneath the centre of a given load as a function of parabolic seamount height (i.e. \( h_d \)). The edifice radius is determined by an empirical relationship between the height and radius of typical seamounts, that is, \( r_d = 4.5 h_d \) (Wessel 2001). The height and radius of the core are set to be half of those of the edifice, which is our conservative choice based on the previous studies (e.g. Robertson 1967; Kellogg et al. 1987; Gallart et al. 1999; Weigel & Grevemeyer 1999). As Wessel (2001) pointed out, the two-boundary model (thick

![Figure 4](image-url)

Figure 4. Maximum deflections for various parabolic seamounts. (a) Elastic plates thinner than 10 km. (b) Elastic plates thicker than 10 km. The dashed lines are for the uniform density model, the thick solid lines for the two-boundary model (Wessel 2001), the thin solid lines for the uniform density model with a load density that is the average of the densities used in the dense core model and the dotted lines for the dense core model. Each group of dashed, thick solid, thin solid and dotted curves represents a distinct elastic thickness \( (T_e) \), shown in bold.
solid lines in Fig. 4) produces less deflection than the uniform model (dashed lines in Fig. 4). However, the two-boundary model predicts more deflections than the dense core model (dotted lines in Fig. 4) simply because the seamount of the two-boundary model is heavier than the dense core seamount (Figs 1b and 1c). The discrepancy between the two-boundary and dense core models increases when the parabolic seamount grows large. For smaller seamounts (Fig. 4a) and thicker plates (Fig. 4b), the differences between the above three models become negligible because the elastic plate is strong enough to support most of the applied loads.

In addition, we examine a uniform load with a load density, 2600 kg m$^{-3}$, which is the average of the densities used in the dense core model (thin solid lines in Fig. 4). Because the average density is estimated by taking into account the volumes of the edifice and the core of the examined parabolic seamounts, the surface mass of the ‘average’ density load is the same as that of the dense core load. However, this uniform case approximates closely the dense core model only for smaller parabolic seamounts ($h_d < 3$ km) on the thinnest plate ($T_e = 1$ km). As the parabolic seamount grows on the thinnest plate, the average density load produces significantly less flexure than the dense core load. Although the total mass and volume of the surface load are the same for both the average density and dense core loads, the dense core load is less equally distributed than the average density load. In other words, the dense core load becomes heavier than the average density load at the axis of the parabolic seamount, where the core material is concentrated. The difference between the subsurface loads of these two models also contributes to the different flexure predictions, because the dense core model infills the flexure with heavier materials than the uniform density model with the average density. For other cases ($T_e > 4$ km), it falls between the predictions of the uniform model with the crustal density and the two-boundary model.

It is self-evident that the uniform density load with the crustal density is heavier than the dense core load because different densities are used for each loading part. If surface mass and peripheral density are the same for both flexure models, would the predicted flexure differ? In Fig. 5, the surface mass of the dense core disc load is equated with the stacked uniform discs of the edifice density. For a simple comparison, the peripheral density is equal to the edifice density. The disc placed on top of the edifice disc has the same mass as the core above the reference level (Fig. 5b). Fig. 5(c) shows relatively insignificant differences between the deflections caused by the stacked uniform density discs and the dense core disc. The dense core model results in slightly larger deflections because the core material is assumed to extend down to the flexed crust and hence acts as an extra driving force. The difference between these models, however, increases when a realistic peripheral density is considered (Wessel 2001), as examined earlier. This particular test illustrates that the surface mass is the largest contributor to flexural deformation of the plate (Menke 1981). However, constraining surface mass is complicated and requires gravity modelling, as discussed in the following section.

### 3.2 Flexure modelling with gravity data

We have shown that the uniform density model tends to overestimate flexural deformation (Fig. 4). Such bias, however, needs to be examined in light of the gravity prediction because free-air gravity and geoid anomalies are the most widely available data that are sensitive to both surface load and lithospheric flexure (Watts 2001). Using the Fourier transform approach of Parker (1972), we derive the gravity anomaly for
a dense core load as a sum of the following attractions due to the edifice (first term), the core (second term), the subsurface materials between the reference level and the flexed crust (third term), and the Moho topography determined by the flexure (fourth term):

\[ F[\Delta g] = 2\pi G \exp(-|k|d) \sum_{n=1}^{\infty} \frac{|k|^n}{n!} \left[ (\rho_d - \rho_w) F[h^d_d(r)] + (\rho_c - \rho_d) F[h^c_d(r)] \right. \\
+ \left. F[\Delta \rho_c(r) w^c_c(r)] + \exp(-|k|d) (\rho_m - \rho_c) F[w^m_m(r)] \right] , \tag{15} \]

where \( k \) is the wavenumber vector, \( r = (x, y) \) is the position vector in space, \( d \) is the regional depth to the seafloor, \( t \) is the thickness of the oceanic crust, \( G \) is the gravitation constant and \( F \) is the 2-D Fourier transform of the given layers of the edifice \( h_d(r) \), the core \( h_c(r) \) and the flexure \( w(r) \). Here, \( n \) accounts for the linear \((n = 1)\) and non-linear higher order terms in eq. (15) and \( \Delta \rho_c(r) \) in the third term denotes the lateral variation in density contrast between the subsurface (i.e. infill) materials and the flexed crust [e.g. it becomes \((\rho_c - \rho_d)\) beneath the core]. If the uniform density model uses the crustal density, then eq. (15) is simplified because the second and third terms cancel. If the load density is different from the crustal density, the third term of eq. (15), with the spatially invariant density contrast, \( \Delta \rho_c(r) = \rho_c - \rho_i \), is necessary for the gravity calculation. Because this uniform model has been used most commonly in previous flexure studies (e.g. Watts et al. 1975; Calmant 1987; Smith et al. 1989; Ito et al. 1995; Lyons et al. 2000; Adam & Bonneville 2008), our comparisons focus on differences in the best \( T_e \) estimated by the dense core and uniform density models constrained by gravity data.

For a simple comparison using synthetic data, we assumed that the dense core parabolic load of Fig. 2(b) is the observed seamount structure. Other key parameters for gravity calculation were a regional water depth of 5 km and a crustal thickness of 7 km. With the density parameters as listed in Table 1, the flexural depression for \( T_e = 20 \) km was calculated analytically using eq. (12); this is what we define as the ‘true’ flexure model. To optimize the speed and accuracy of the Fast Fourier Transform (FFT), a square grid of 512 \( \times \) 512 nodes was used for all calculations in eq. (15); this resulted in the true gravity anomaly over the parabolic seamount. The deflections due to the uniform density loads were estimated analytically using the solution of Lambeck & Nakiboglu (1980). Then, the rms misfits between the predicted gravity anomaly using the uniform density model and the observed data were computed for a range of elastic thicknesses and load densities. We kept all calculations in eq. (15); this resulted in the true gravity anomaly over the parabolic seamount. The deflections due to the uniform density model (Fig. 6a; square and dashed lines) is 25 per cent less than the true model (Fig. 6a; arrow). The overestimated gravitational contribution of the seamount and the negative effect of the flexed crust. As such, flexure modelling with gravity data seeks the best trade-off between the positive and negative contribution from the more flexed crust. This trade-off, which is presented in Fig. 6(c), shows that the uniform density model results in larger deflections than the true flexure. However, Fig. 6(b) shows disturbingly insignificant differences between the true and predicted gravity anomalies with the best uniform density model, reflecting the non-unique nature of gravity modelling.

In addition, Fig. 6(b) shows that the dense core model predicts positive gravity contributions from both the surface and subsurface core materials, while the uniform density model is affected by the negative contribution from the subsurface load material. This is manifested

---

**Figure 6.** Flexure modelling using the gravity anomaly calculated from the dense core parabolic load of Fig. 2(b) as the true data. (a) The rms difference between the true and predicted gravity using the uniform density model. The free-air gravity anomalies over the parabolic seamount placed on the plate of \( T_e = 20 \) km are assumed to be the true data. (b) Residual gravity anomaly by subtracting the predicted gravity by the uniform density model from the true data. (c) Comparison between the true and predicted flexures by the uniform density model.
as a positive difference over the summit; however, differences due to peripheral densities are less distinct in this comparison. Such detailed differences are likely to be masked by noise in real gravity data over seamounts.

To make general comments about the biases observed in flexure modelling using gravity data, we repeated the above exercise for a range of elastic thicknesses. The dense core parabolic load in Fig. 2(b) was again regarded as the observed seamount structure. By placing this parabolic seamount on various elastic plates, we calculated the true gravity anomaly for a given true $T_e$ and then determined the best $T_e$ and $\rho_l$ for the uniform density model that minimized the rms misfits (green lines in Fig. 7). Because many previous $T_e$ estimates were obtained with $\rho_l = \rho_c$, we also found the best $T_e$ when the load density was set to the crustal density (orange line in Fig. 7a). Upper and lower bounds of these $T_e$ estimates (shaded areas in Fig. 7a) were obtained at a 5 per cent tolerance level from the minimum rms (Watts et al. 2006), but sampled along $\rho_c$ and best $\rho_l$. Therefore, this uncertainty bound is narrower than that of a 5 per cent uncertainty ellipse.

If there were no shortcomings in the flexure modelling with the uniform density model, the predicted $T_e$ values should follow the dashed 1:1 line in Fig. 7(a). Instead, we observe a significant underestimation of $T_e$, as was also found by Minshull & Charvis (2001). For thin elastic plates ($T_e < 10 \text{ km}$), the uniform density model with the crustal density predicts $T_e$ close to the true model. This is also reflected in Fig. 7(b) because the uniform density model with variable load density minimizes the misfits with a higher load density for thinner plates. However, the best load density determined by rms minimization is too large so that the amount of the flexure is significantly underestimated (e.g. Fig. 6c). In gravity computation, the uniform density model with variable $\rho_l$ still achieves a balance between the gravity contributions from the load and the flexed crust, but results in largely underestimated $T_e$ relative to the uniform density model with $\rho_l = \rho_c$.

For strong elastic plates, the underestimation of $T_e$ by the uniform density model with $\rho_l = \rho_c$ exacerbates further. Because the surface load is fixed and overestimated, the only way to offset the extra positive gravity signals and minimize the misfits is to decrease $T_e$ (Fig. 7a). The uniform density model with variable $\rho_l$, however, reduces the load density gradually to produce better gravity fits (Fig. 7b) and results in better $T_e$ estimates. Fig. 7(b) shows that a $\sim 100 \text{ kg m}^{-3}$ reduction from the typical crustal density (2800 kg m$^{-3}$) reduces the underestimation of $T_e$ significantly. In addition, this indicates that the role of load density in flexure modelling is more important for seamounts on strong elastic plates.

3.3 Flexure modelling with arbitrary dense core loads

In the preceding discussions, we have dealt with synthetic examples using axisymmetric disc and parabolic seamounts, which may in some cases approximate the actual seamount bathymetry (e.g. Lambeck 1981a). However, such idealized loads are not always adequate for contemporary geophysical studies that utilize high-resolution gridded bathymetry at seamounts. In this respect, the greatest advantage the
uniform density model affords in the flexure and gravity calculations lies in the simplicity of the flexural response function (Watts 2001).

\[ \Phi(k) = (1 + k^2 / k_0)^{-1}. \]  

(16)

with \( k_0 \) defined by eq. (11) with \( x = l \). The lithospheric flexure due to any given shape of topography, \( h_s \), is calculated in the wavenumber domain by

\[ F[w(r)] = \frac{\rho_s - \rho_w}{\rho_w - \rho_l} F[h_s(r)] \Phi(k). \]  

(17)

Calculation of flexure using eq. (17) is fast and reliable. However, we cannot adapt eq. (17) directly for the dense core model to make use of an actual bathymetry grid, because eq. (16) requires a constant \( k_0 \). From the preceding comparisons, we find that the surface mass plays a more important role in flexure modelling than changes to the density of subsurface materials. Also, Menke (1981) proved analytically that a point load equivalent to a distributed load does not alter the amplitude and wavelength of flexure. Therefore, we circumvent the inherent limitation of the dense core model by constructing a uniform load of peripheral density that is equivalent to the sum of the surface core and edifice loads.

\[ h_c(r) = \frac{\rho_d - \rho_w}{\rho_s - \rho_w} h_s(r) + \frac{\rho_d - \rho_s}{\rho_s - \rho_w} h_c(r). \]  

(18)

Although this modification allows us to use eq. (16), it underestimates the amplitude relative to the analytic solutions because the subsurface materials beneath the edifice and the core are now replaced with less dense peripheral material. Comparison of maximum amplitudes calculated using this equivalent load, \( h_c(r) \), with the exact dense core model indicates that such biases are generally less than 10 per cent and increase only when the atypical situations of large seamounts on thin elastic plates are considered. We thus utilize eq. (18) to approximate the dense core surface load and calculate lithospheric flexure from the equivalent load, while the gravity calculation is obtained by eq. (15). In the following, we demonstrate a practical use of the dense core model for Howland Island in the Pacific and examine if the bias inherent in the uniform density model discussed is noticeable when analysing real data.

4 CASE STUDY: HOWLAND ISLAND

Detailed geophysical and geochemical mapping of the Tokelau seamount chain was carried out by R/V Melville in 1999 (Koppers & Staudigel 2005). Howland Island is situated on a 127 ± 1 Ma oceanic plate at the northern end of this trail (Müller et al. 2008), and its formation age is 72.3 ± 2.7 Ma (Koppers et al. 2007). The shipboard depth measurements along the Tokelau chain can be downloaded from the Seamount Catalog at the EarthRef.org. However, we utilize SRTM30_PLUS V5.0 (Becker et al. 2009) for flexure modelling because it provides complete data coverage in the vicinity of Howland by blending the same ship data and the predicted bathymetry (Smith & Sandwell 1997). From the previous 40Ar/39Ar study (Koppers et al. 2007), the predicted elastic plate thickness beneath Howland is 20 ± 2 km using the empirical relationship of Calmant et al. (1990) that closely follows the 400 °C isotherm. By applying both the dense core and uniform density models to Howland, we will differentiate between these two models.

For the synthetic loads (Fig. 2), a constant density was sufficient to define the reference level. Such a choice for actual bathymetry data, however, can misrepresent the reference level and in turn bias the surface load, because a single (e.g. average) depth overlooks bathymetry variations of other geological phenomena, such as swells and the thermal subsidence of the plate. We thus performed a regional-residual separation of the bathymetry around Howland using directional median (DiM) filtering, to estimate the reference plane that defines the surface load (Wessel 1998; Kim & Wessel 2008). To determine the best filter width for an effective separation, we followed the optimal robust separation (ORS) technique (Wessel 1998). We explored a range of filter widths to determine a width that maximized the ratio of the volume to the area of the residual bathymetry. In this study, however, the ratio was computed from a weighted residual bathymetry (Fig. 8c) so that the separation could be optimized near Howland. The data domain for DiM-based filtering was extended by about 2° in all directions from Fig. 8(a) to avoid possible edge effects of the filtering. Because the background seemed very gentle, we chose four sectors for the DiM filtering and smoothed the DiM-filtered results using a spatial median filter with 40 km filter width. The ratio was estimated for a range of filter widths and maximized for a 100-km filter width [see Kim & Wessel (2008) for details]. The Howland surface load (i.e. the residual), therefore, was obtained by subtracting the DiM-filtered bathymetry from the observed data (Fig. 9a).

Although there is no available data (e.g. seismic refraction profiles) indicating the existence of a core inside Howland, we can infer from previous studies (e.g. Watts et al. 1997; Araña et al. 2000; Minshull & Charvis 2001; Kopp et al. 2004; Camacho et al. 2009; Contreras-Reyes et al. 2010) that the core is similar in shape to the seamount edifice. O’Higgins (Kopp et al. 2004) and La Reunion (Gallart et al. 1999), for example, both have edifices with low P-wave velocity (∼3–4 km s⁻¹) and core structures outlined by ∼5 km s⁻¹. However, Great Meteor (Weigel & Grevenmeyer 1999) and Louisville Guyot (Contreras-Reyes et al. 2010) show higher P-velocities for the edifice (∼5 km s⁻¹) and the core (∼6.5 km s⁻¹). Thus, we consider the dense core to be correlated with a range of P-velocities (i.e. 5–6.5 km s⁻¹), which separates the core from the volcanic edifice. In addition, the observed cores are generally about 1–2 km below the outer edifice. With extrapolation from these observed cases, we constructed a possible core for Howland by subtracting 2 km from the residual and only keeping positive values (Fig. 9c; dashed line). Then, the equivalent load (Fig. 9b) of the peripheral density was obtained by eq. (18) and used to compute flexure for various elastic thicknesses.

For the gravity calculation, the regional depth, \( d = 5.4 \) km, was taken from the mean depth of the regional bathymetry. The thickness of the oceanic crust, \( t \), was 6.5 km and the densities of the water, the crust and the mantle were 1030, 2900 and 3330 kg m⁻³, respectively.
Flexure modelling with dense core loads

Figure 8. Observed geophysical data near Howland Island. (a) SRTM30+ bathymetry (Becker et al. 2009). (b) Satellite-derived free-air gravity version 18.1 (Sandwell & Smith 2009). (c) Weighting function used for the regional-residual separation and rms estimates for the flexure modelling to emphasize Howland. The depth contours of 1 km interval are overlaid.

(Lambeck 1981b; Adam & Bonneville 2008). For the dense core model, the densities for the core, the edifice and the moat were the same as those we used in the previous flexure calculation (listed in Table 1), assuming that the core and the edifice of Howland are similar to those of Great Meteor (Weigel & Grevemeyer 1999). The layers of the core, the edifice and the flexure predicted with the equivalent load were used to calculate the spatial variation of the subsurface materials needed for the third term in eq. (15). Then, the gravity anomalies due to the core, the edifice and the subsurface were summed. For the uniform density model, we computed flexure for a range of load densities and elastic plate thicknesses using the residual bathymetry. For both models, however, the regional bathymetry (i.e. the DiM-filtered data) was added back to both the residual and predicted flexure before the gravity calculation so that the predicted gravity can exhibit regional gravity components similar to the observed.

The flexure and gravity computations were carried out with all features inside the domain; however, the rms misfits were computed from the weighted difference (Watts et al. 2006) between the predicted and the satellite-derived gravity data of version 18.1 (Sandwell & Smith 2009). The lower resolution of the satellite-derived gravity (compared to shipboard measurements) typically manifests itself as lower amplitudes over short-wavelength features like seamounts. For example, Watts et al. (2006) showed that the satellite-derived data of version 14.2 only recovered $n = 1$ over Wahoo Guyot. Because of the improved resolution for short-wavelength features in the gravity data of version 18.1, however, setting $n = 2$ in eq. (15) was necessary to recover a comparable amplitude of the predicted signals to that of the observed data (Marks & Smith 2007). In particular, the peak amplitude at Wahoo Guyot is amplified from $\sim 55$ mGal (version 14.2) to $\sim 82$ mGal (version 18.1).

The uniform density model minimizes the rms misfits at $T_e = 12$ km (Fig. 10a), which is substantially less than the predicted thickness based on the age data. The upper and lower bounds of the uniform density model prediction were estimated at a 5 per cent tolerance level from the rms minimum and determined along the best load density (see Fig. 12). In contrast, the dense core model is minimized at $T_e = 26$ km (Fig. 10e). However, the changes in the rms misfits from this global minimum are subtle ($< 0.01$ mGal) so that the upper bound of the best dense core model is unresolved (Fig. 12; question mark).

The difference in $T_e$ estimates between the uniform density and dense core models is most apparent in the predicted deflections of Figs 10(b) and (f). As the uniform density model favours a thin plate and a typical crustal density (Fig. 10a), it predicts more flexural deflections than the dense core model. This overestimated flexure increases southwards towards Baker Island. However, the gravity anomaly predicted by the uniform density model (Fig. 10c) is generally more comparable with the observed data (Fig. 8b) than that of the dense core model, especially if we consider the location of the zero contour between the Howard and Baker Islands. This zero contour is not apparent in the predicted gravity by the dense core model (Fig. 10g). In Fig. 10(d), however, the uniform density model shows large negative residual anomalies over Baker. This indicates that the best uniform density model for Howland is not adequate for Baker. Although our computation excluded Baker by using a weight function centred on Howland (Fig. 8c), such negative residuals are large enough to suspect that the uniform density model might decrease $T_e$ even more if both islands were included in flexure modelling. The dense core model instead exhibits a broad negative anomaly between two islands, where the zero contour is observed (Fig. 10f).

In Fig. 11(a), the comparison of the predicted gravity anomalies for the dense core (dotted line) and uniform density (dashed line) models along profile AB of Fig. 9(a) shows that both predictions generally agree with the observed data. Although the best load density of the uniform density model is dense enough to overshoot the peak of the observed anomaly, the dense core model predicts less than the peak because the densities of the core and the edifice are fixed during calculation. As demonstrated analytically in the preceding examples, the uniform density model predicts larger deflections beneath Howland than the dense core model does (Fig. 11b).
Watts (1978) first showed the correlation between the modelled elastic thicknesses beneath seamounts and the depth to the 450 ± 150 °C isotherm of the plate cooling model (Parsons & Sclater 1977). Such relationships have been consistently supported by many subsequent studies (e.g. Calmant 1987; Kruse et al. 1997; Watts et al. 2006) and used as a proxy for age prediction of either plate or seamount (Calmant et al. 1990). In addition, any outliers from this trend (i.e. elastic plates anomalously thinner than expected) have been the focus of many controversies and have invoked proposals of other additional geodynamic processes, including lithospheric thinning (Crough 1983), lithospheric reheating (McNutt 1984) and small-scale convection (Dalloubeix & Fleitout 1989) to explain such anomalously thin plates.

We compare the elastic thickness estimates for Howland with the plate cooling model of Parsons & Sclater (1977) in Fig. 12, which includes also the age prediction curve (dotted line) of Calmant et al. (1990). The estimate of the uniform density model is above the 300 °C isotherm, while that of the dense core model is close to the 500 °C isotherm. The underestimation of the uniform density model, thus, is manifested by a thinner $T_e$ prediction that is consistently observed from the preceding synthetic examples. Because the previously proposed correlation between $T_e$ and age at seamounts are based on the uniform density model estimates, the dense core model prediction does not follow the age prediction curve described by the 400 °C isotherm (Calmant 1987). In addition, the significant disparity between the $T_e$ estimates beneath seamounts and those obtained at trenches and fracture zones have suggested that the former predictions are systematically...
Flexure modelling with dense core loads

5 CONCLUDING REMARKS

We have demonstrated how the uniform density model underestimates $T_e$ and overestimates deflections using various synthetic and real examples. When flexure modelling is constrained by gravity data alone, such biases are often concealed because the predicted gravity with distorted $T_e$ and deflections can nevertheless be the best fit for the observed data. This non-uniqueness of solutions to the gravity field can be circumvented by using additional observations (e.g. seismic profiles) or mitigated by a more rigorous analysis. In this study, we have shown underestimated (e.g. McNutt 1984; Wessel 1992). Therefore, it is noteworthy that the dense core model with a thicker plate can account for the gravity anomaly at Howland (Figs 10 and 11) and follows a hotter isotherm (Fig. 12). This result is in agreement with findings from flexural studies at fracture zones and trenches, without needing additional reheating mechanisms to reset the thermal age of the lithosphere.
Figure 12. Elastic thickness estimates for Howland Island as a function of age of the lithosphere at the time of loading. The 5 per cent uncertainty bounds for the $T_e$ values (vertical bars) are sampled along the best load density for the uniform density model and from the rms misfits for the dense core model. However, the upper bound of the dense core model is not resolved (question mark) because the rms misfits show subtle changes for thicker plates (Fig. 10e). The age uncertainty bounds (horizontal bars) are the sum of the uncertainties in the $^{40}$Ar/$^{39}$Ar age data (Koppers et al. 2007) and the digital age grid of the seafloor (Müller et al. 2008). Theoretical isotherms (Parsons & Sclater 1977) and empirical relationship (Calmant et al. 1990) are shown as dashed and dotted lines, respectively.

that the inclusion of a dense core is important when approximating the first-order inhomogeneous internal seamount structure. Thus, the dense core model appears to be more suitable for investigating the thermal and mechanical properties of the oceanic lithosphere.

Our study does not imply that all published $T_e$ estimates are underestimated. However, some of them could be biased to some degree simply because of the load density used in the flexure modelling. The flexural studies on Tenerife in the Canary Islands are such a case. Watts et al. (1997) predicted the elastic thickness beneath Tenerife to be 20 km, which was 10 km less than the expected thickness based on the dated rock samples. Later, Collier & Watts (2001) revised the flexure model by reducing the load density and obtained the expected elastic thickness. Another important factor is the quality of the bathymetry and gravity data around seamounts of interest. For example, Lambeck (1981b) computed analytically the flexural deformation of the lithosphere beneath the Cook Islands by approximating the topographic features as the stacks of uniform density discs. Although he used a load density less than the crustal density and included a core in his calculation, his $T_e$ estimates were the same as the study of Calmant & Cazenave (1986) who used the crustal density as the load density. Because both studies relied on a few SEASAT altimetry tracks and had poor bathymetric constraints [e.g. SYNAPAS (Van Wyckhouse 1973)], the careful details of the modelling space incorporated by Lambeck (1981b) were not effectively propagated into the solution space.

Finally, we consider underplating beneath large seamounts or ocean islands that is quite commonly observed seismically (e.g. Weigel & Grevemeyer 1999; Ali et al. 2003). In flexure modelling, the role of underplating is to reduce the total flexural depressions because lighter underplated material adds to the buoyancy and hence works against the surface load. This process, in particular, is necessary for the uniform density model to compensate for the overestimated deflections so that a modeler can achieve a better fit between the predicted and observed flexure. However, for such sizeable volcanoes, we can presume the existence of a dense core as a constructional feature. If the dense core model alone can predict the flexed crust, then the subsurface load due to underplating becomes less important in calculating deflections; however, its contribution to the gravity anomaly will remain. Therefore, we suggest the role of underplating in flexure modelling may need to be reassessed with a dense core model.

ACKNOWLEDGMENTS

The constructive reviews from Tony Watts, Garrett Ito and Scott Rowland greatly helped us to improve this manuscript. We thank the Editor Ingo Grevemeyer and the Editorial Assistant Valerie Dennis for expediting the review process. This research was funded by NSF grant OCE-0526496 and the IAMG Student Research Grant (SSK). This is SOEST contribution 7949.

REFERENCES


© 2010 The Authors, *GJI*, 182, 583–598

Journal compilation © 2010 RAS

Flexure modelling with dense core loads 597


