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LAVA FLOWS ARE FRACTALS

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Abstract. We have conducted a preliminary investigation of the fractal nature of the plan-view shapes of lava flows in Hawaii (based on field measurements and aerial photographs) as well as in Idaho and the Galapagos Islands (using aerial photographs only). Our results indicate that the shapes of lava flow margins are fractals. In other words, lava flow shape is scale-invariant (at least within the range of scale measured, 0.5m to 2.4km). This observation has important implications for understanding the fluid dynamics of lava flows. It suggests that nonlinear forces are operating in them because nonlinear systems frequently produce fractals. Furthermore, a'a and pahoehoe flows can be distinguished by their fractal dimensions (D). The majority of the a'a flows we measured have D between 1.05 and 1.09, whereas the pahoehoe flows generally have higher D (1.14 - 1.23). We have extended our analysis to other planetary bodies by measuring flows from orbital images of Venus, Mars and the Moon . All are fractal, and have D consistent with the range of terrestrial a'a and pahoehoe values. Combining the terrestrial and extraterrestrial data, the fractal nature of lava flow outlines holds for over five orders of magnitude in scale (0.5m to 60km).

Introduction

Important quantitative parameters have been developed during the past two decades that relate to the rheology, eruption and emplacement mechanisms of lavas. These include flow lengths, volumes and areas, channel widths, flow-lobe dimensions, and ridge heights and spacings (e.g., Walker, 1973; Hulme, 1974; Hulme and Fielder, 1977; Moore et al., 1978; Malin, 1980; Theilig and Greeley, 1986; Rowland and Walker, 1990; Lopes and Kilburn, 1990; Crisp and Baloga, 1990). Based on this study, we propose to add additional, unique parameters to this list of properties: the fractal properties of lava flows.

Fractals are objects that look similar at all scales; they are "self-similar" or "scale-invariant" (Mandelbrot, 1967, 1983). Many geologic features are fractals (e.g., rocky coastlines, topographic contours and river networks), and they can be described by a property called their fractal dimension. The fractal dimension (D) of a curve (such as a lava flow margin) is a measure of the curve's deviation from linearity, with a straight line defined as D=1. The fractal properties of lava flows have not previously been studied, but we believe this new way of looking at lava flows will provide fresh insight into our understanding of volcanism. We have found that lava flow outlines appear to be fractals. This has important implications because fractals frequently reflect nonlinear, or chaotic, processes (e.g., Campbell, 1987). Our data also suggest that the a'a and pahoehoe flows have different D. This could be a useful remote sensing tool, and lead to a better understanding of physical volcanologic processes, since the formation of a'a versus pahoehoe depends on a critical relation between volumetric flow rate and lava viscosity.

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Methodology

Fractal dimensions of objects such as lava flow margins can be calculated in several ways (e.g., Mandelbrot, 1983). In this study, we employed the "structured-walk" method (Richardson, 1961). In this method, the apparent length (L) of a flow margin is measured by walking rods of different lengths (r) along the margin. For each rod length, L is estimated according to the equation L=Nr, where N is the number of rod lengths, each of length r, needed to approximate the margin. By plotting L as a function of r on a log-log plot (Richardson plot), fractal behavior can be determined. A linear trend on such a plot indicates the data are fractal. The exact cut-off between fractal and non-fractal behavior is arbitrary and subjective; we consider a data set to be fractal if the square of the residuals exceeds 0.95. The fractal dimension (D) can then be calculated as D=1-m, where m is the slope of the linear least squares fit to the data on the Richardson plot. In other words, D is defined as follows:

$$\log L = C + (1-D) \log r,$$

where C is the y intercept of the Richardson plot.

Consider the theoretical case in which a lava flow margin is a straight line of apparent length L when measured with a rod of length r (N rod lengths are used: L=Nr). If we again measure its length, this time using a rod of half the original length (r/2), we count twice as many lengths (2N). Similarly, dividing the rod length by four results in four times as many lengths (4N). In each case, r and N are inversely proportional; L remains constant. Thus, the Richardson plot is a straight line with slope m=0 (D=1). Lava flow margins are nonlinear, however. Instead, they are characterized by embayments and protrusions. Since smaller rod lengths traverse more of these smaller-scaled features, L increases as r decreases. As the nonlinearity of the curve is increased, D is increased until we reach a limit at the dimension of the space, D=2 (m=-1). Therefore, all plan-view shapes have D between 1 and 2. For comparison, plan views of rocky coastlines and topographic contours typically have D=1.25 (Mandelbrot, 1967).

Field measurement technique

In the field, we used a chain to define the rod length. The measurement begins with one person holding one end of the chain (of length 16m) at the arbitrarily chosen starting point (x) along a flow margin as a second person walks along the boundary until the other end of the taut chain exactly intersects the outline. This new point (y) becomes the next starting point. With one end of the chain now fixed at y, the next intersection point (z) is found. This process continues until a given number of lengths (N) are measured, and the ending point is marked. We have used 5 lengths (N=5) of a 16-m chain as a minimum length. To maximize accuracy, we replicate the measurement using the same chain length, but this time we walk in the opposite direction (from the ending point to the starting point). We find that the N values from both directions match well. The results (N) are averaged and L is calculated. We then divide the chain length by two and repeat the procedure, using the same starting and ending points. Our field measurements each consist of five data points, corresponding to chain lengths of 1, 2, 4, 8, and 16m. In one measurement, we also used a chain length of 0.5m.

To assess the precision of this field technique, we conducted 5 replicate measurements of a typical Hawaiian pahoehoe margin. We began each measurement at the same starting point, and measured off 5 lengths of a 16-m chain; therefore, the ending points of each measurement did not necessarily coincide. The results of this error analysis showed negligible variation in D: $\sigma = 0.008$.

Photographic measurement technique

The structured walk method was also utilized to determine D of lava flows from aerial photographs at scales of 1:24,000 and 1:60,000. We tried to use flow margins in the centers of the images to avoid distortion. After digitizing the flow margins, we calculated D using the EXACT algorithm (Hayward et al., 1989). Computerization facilitates changing the rod lengths in small increments, improving the precision of the calculated D. Consistent with our field methodology, the minimum flow length included in the aerial photographic data set corresponds to N=5 for the longest rod length. Care was taken to choose rod lengths sufficiently large as to exceed both the noise inherent in the digitization process as well as the spatial resolution of photographic images.

Results and Discussion

This analysis is based on two types of data: (1) field studies of lava flows on Kilauea, Mauna Loa and Hualalai volcanoes on Hawaii and (2) aerial photographs of lava flows on Hawaii, Idaho and the Galapagos Islands. In this study, we selected flows that are unaffected by external controls, such as a steep ground slope or preexisting local undulations. To date, we have measured D for 28 terrestrial flows: 15 pahoehoe and 9 a'a margins in Hawaii, 3 a'a flows in the Galapagos Islands, and a pahoehoe flow in Idaho. We also studied 6 extraterrestrial flows. On Venus, we calculated D for two radar-bright flows in Mylitta Fluctus and a radar-dark flow south of Ishtar Terra. We also measured two flows in Elysium Planitia, Mars and a lunar flow in Mare Imbrium.

Lava flows are fractals

Our data indicate that lava flow shapes are fractals. Plots of log L versus log r are linear (Figure 1), which demonstrates self-similarity. Figure 1 illustrates typical plots for a'a and pahoehoe flows. Both are fractal and, as discussed below, can be distinguished by their fractal dimensions. In two cases, we made both field and aerial photograph measurements, and obtained similar values of D. Combining these two data sets for terrestrial flows, the self-similarity holds for rod lengths ranging from 0.5m (field) to 2.4km (aerial photographs).

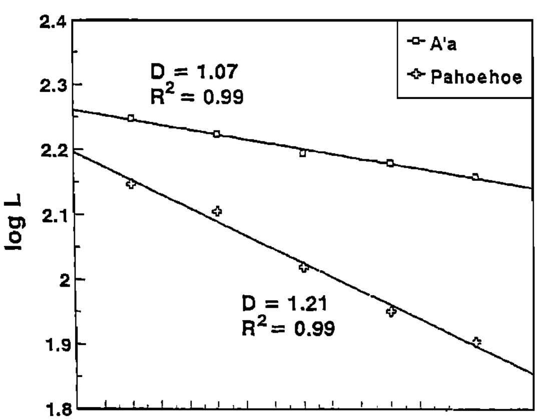

Fig. 1. Plots of L vs. r, in meters, for typical pahoehoe and a'a flows. Note the pahoehoe has a steeper slope (higher D).

Including the extraterrestrial data (equivalent rod lengths up to 60km), the fractal nature of lava flow outlines holds for over five orders of magnitude in scale.

Implications for lava flow dynamics

The fractal properties of lava flows can provide insight into the dynamical processes operating during flow emplacement. The mere fact that flows have fractal outlines suggests that the forces responsible are nonlinear (e.g., Campbell, 1987). We have begun to investigate the nature of these nonlinear processes (Taylor et al., 1991) in collaboration with S. Baloga (JPL/Caltech). Deterministic fluid dynamic models (e.g., Baloga and Pieri, 1986; Baloga, 1987) depict variations in the free surface of a lava flow as due to a balance between the gravitational driving force, important when the slope is steep, and the fluid dynamic pressure gradients that push the flow on a relatively flat surface.

Volume-conservation considerations indicate that the thickness of the flow satisfies a first-order kinematic wave equation, consistent with our observation that flow outlines are not fractals when slopes are steep (as discussed below). On flat surfaces, the thickness and width of a flow depend on a nonlinear diffusion equation of the same form as studied by Nagatani (1991) for random particle deposition on flat surfaces, accompanied by subsequent lateral diffusion of the particles. Nagatani's approach is similar to Family's (1986), but the diffusion is explicitly nonlinear. Although derived for understanding surface deposition processes, many aspects of the governing physics are similar. The processes of momentum and volume transport in lavas are similar to random deposition except that the instabilities in lava flows form inside the bounding surface, rather than onto it. Nagatani's (1991) studies indicate formation of fractal surfaces, suggesting to us that the lobate shapes of many lava flows result from instabilities in a nonlinear diffusive process associated with the conservation of lava volume.

Lava flow outlines may also be modeled by the same processes operating in systems showing viscous fingering. In such systems, a low-viscosity fluid is injected into one of higher viscosity and instabilities develop (Saffman and Taylor, 1958); the instabilities lead to the development of fingers of the low-viscosity fluid in the more viscous one. Lava flows are like this, having a cool, viscous outer layer surrounding a hot fluid interior. Instabilities result in formation of lobes and toes, analogous to viscous fingers. The process of viscous fingering has been modeled by diffusion-limited aggregation (e.g, Feder, 1988) in which random walking particles finally attach to a growing cluster. In lava flows, the random walkers can be viewed as carrying little chunks of melt with them, thus moving the boundary. Viscous fingering is transitional to viscoelastic fracturing (Lemaire et al., 1991), which might also be important in lava flows.

A New Remote Sensing Tool

The outlines of a'a and pahoehoe flows are qualitatively different (Figure 2): pahoehoe flows tend to have many more protrusions and embayments. Since the flow outlines of both types are fractals as well, it should be possible to distinguish them by their fractal dimensions. Our preliminary study suggests that this is the case (Figures 1, 3). Most (six of nine) of the Hawaiian a'a flows have D between 1.05 and 1.09. The Hawaiian pahoehoe flow margins tend to be more nonlinear, and thus have higher fractal dimensions: all have D between 1.13 and 1.23. The single Idaho flow measured (Hell's Half Acre, pahoehoe) has D=1.21, consistent with the range of Hawaiian pahoehoe flows. The three Galapagos flows (all a'a) measured yield D values of 1.05, 1.07 and 1.09, in agreement with the range of Hawaiian a'a flows. Therefore, despite differences in geographic location, the pahoehoe flows consistently have higher D than a'a flows.

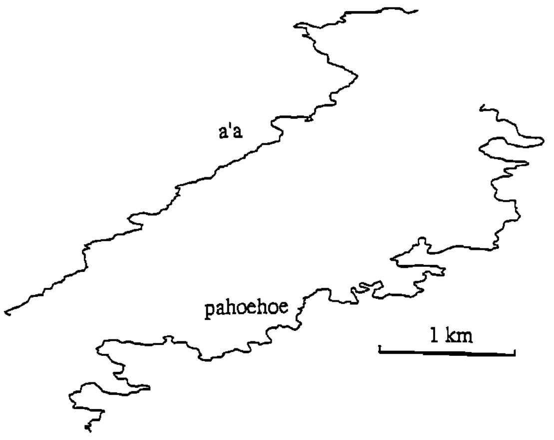

Fig. 2. Digitized outlines of a'a and pahoehoe flows. Typically, pahoehoe margins have more embayments and protrusions, corresponding to a higher D.

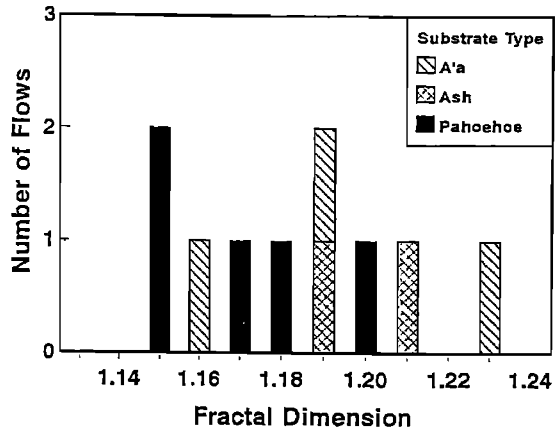

Fig. 4. Histogram of D values of terrestrial pahoehoe flows on three different substrates based on field data. Note the lack of correlation between D and substrate type.

These D values are also unaffected by differences in substrate. We made 10 field measurements of Hawaiian pahoehoe flows. Some of these flowed upon preexisting a'a lava flows, others upon preexisting pahoehoe flows, still others atop ash deposits. Figure 4 shows the lack of correlation between D and substrate for these 10 flows. In one case (a pahoehoe), we performed a controlled experiment on the effect of substrate on D. We measured D in one location where this pahoehoe flowed atop a preexisting pahoehoe, and again nearby, where the same flow had an a'a substrate. The D values obtained for this flow overlying pahoehoe and a'a substrates (1.17 and 1.19, respectively) are well within the observed variation of D along a flow margin. (We performed a rigorous analysis to study variation along a flow margin by measuring D of 7 adjacent segments of a flow margin in the field and found $\sigma = 0.05$). However, preexisting topography may have a significant effect on D. Positive topography (e.g., hills) may deflect or bifurcate flows, thus increasing D; negative topography (e.g., channels) serves to confine or channelize flows, thereby decreasing D. We have therefore restricted this analysis to those flows whose shapes appear unaffected by topographic controls. Similarly, we have excluded those flow margins emplaced on a steep ground slope from this analysis. (We have begun to assess the effect of slope on D by measuring an a'a flow on three different slopes: 12°, 15° and 28°. We found that the fractal behavior held for 12° and 15° slopes, but broke down at 28°.)

We call attention to the region of overlap between a'a and pahoehoe at D of 1.13 to 1.15 (Figure 3). Two of the twelve a'a flows have anomalously high D of 1.13 and 1.15. One of these is a field measurement of a portion of the Hualalai 1800 a'a flow. This flow locally has some pahoehoe characteristics, and may be somewhat transitional. On the other hand, the aerial photograph measurement of a different, larger portion of this flow yielded D=1.06, suggesting that the high D measured in the field may be a local anomaly. The second a'a anomaly corresponds to a portion of the Mauna Loa 1899 a'a, measured from an aerial photograph, which we have yet to ground-truth.

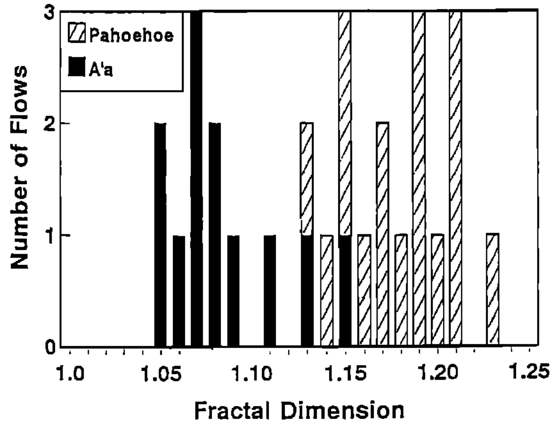

Fig. 3. Histogram of D values of terrestrial lava flows based on aerial photographic and field data. Note that a'a and pahoehoe generally have different D.

Despite these few exceptions, a pattern emerges among the terrestrial basaltic lava flows, with pahoehoe flows having higher D than a'a flows. Distinguishing a'a and pahoehoe by their fractal dimensions may prove to be a useful remote sensing tool, for inaccessible areas of the Earth as well as for other planets. This can lead to a better understanding of physical volcanologic processes. Whether an erupting lava is a'a or pahoehoe depends on a critical relationship between volumetric flow rate and viscosity (Peterson & Tilling, 1980; Kilburn, 1981; Rowland & Walker, 1990). Pahoehoe flows are generally associated with low volumetric flow rates and/or relatively fluid lavas, whereas a'a formation is favored by high volumetric flow rates and/or viscous lavas. These lava flow types are also usually associated with particular eruptive styles (e.g., fountaining, no fountaining, channel formation, lava tube formation), and may even be indicative of the plumbing system supplying that particular eruption.

We have applied our analysis to other planetary bodies by measuring six extraterrestrial flows: three on Venus, two on Mars and a single lunar flow. We measured the Venusian flows from Magellan images using equivalent rod lengths of 1.5 to 60km. Two of the flows appear bright and one is dark on these radar backscatter images. Surface roughness is a major contributor to the backscatter signal: rougher surfaces tend to produce brighter images. Thus, one would expect a'a flows, with their rough, clinkery surfaces, to be relatively radar-bright and smooth-surfaced pahoehoe flows to be radar-dark. Interestingly, the Venusian radar-bright flows yielded D of 1.04 and 1.09, consistent with the range of terrestrial a'a values, whereas the radar-dark flow has a D=1.20, typical of terrestrial pahoehoe flows (Figure 3). These data suggest that a'a and pahoehoe flows on other planets can be distinguished remotely by D.

On Mars, we measured two flows in Elysium Planitia using rod lengths on Viking Orbiter images equivalent to 300m to 10km. One of these yields D=1.06, consistent with the range of terrestrial a'a values; the second has D=1.19, well within the terrestrial pahoehoe range. We also measured a lunar flow in Mare Imbrium, using rod lengths on Apollo 15 metric photographs equivalent to 3 to 60km. We calculated D as 1.20, which (based on terrestrial analogy) suggests pahoehoe. However, direct analogy might not apply. The relatively young flows in Mare Imbrium appear fairly thick (average 30-35m) and cover an area similar to that covered by the Columbia River flood basalts in Washington and Oregon (Schaber, 1973). Schaber (1973) suggests that the lunar flows were emplaced at high effusion rates, which, in spite of the low viscosities of lunar lavas, would favor formation of a'a. An important consideration is that perhaps flood basalts behave differently from either typical a'a or pahoehoe (Self et al. 1991). Alternatively, the Mare Imbrium flow field may have been emplaced as numerous thin pahoehoe flows. Clearly, before we can use D to remotely discriminate between a'a and pahoehoe, we must understand the dynamics that give rise to the apparent difference between the two types of lava.

Complexities and Future Work

Thus far, we have been concentrating on the simplest cases to ascertain whether there is a fractal nature to lava flows, and to study the differences in fractal properties between lava flow types. We have not yet fully considered the effects of preexisting topography and ground slope on fractal properties. Furthermore, we have only considered pahoehoe and a'a, the end member types of basalt lava flows. There are transitional types, such as slab pahoehoe and spiny pahoehoe (e.g., Peterson and Tilling, 1980), which are loosely defined on morphological grounds. To date, we have measured a single transitional flow (1855 Mauna Loa) in the field, and found D=1.09. Clearly, more measurements of transitional flows are needed to better understand the transition from a'a to pahoehoe with respect to fractal behavior. Perhaps we can apply this fractal approach to quantify differences among transitional flow types. Finally, with a single exception (the 1800 Hualalai a'a flow, an alkali basalt), our measurements have been restricted to tholeiite basalts. Interestingly, the field measurement of the 1800 Hualalai lava flow has an unusually high D for an a'a flow, suggesting a possible dependence of D on viscosity. To study a greater range in viscosities and explore the effect of yield strength on lava morphology, we plan to study andesitic and rhyolitic lavas.

Conclusions

This preliminary investigation allows us to make the following tentative conclusions:

(1) Lava flow margins are fractals, exhibiting self-similarity at scales ranging from 0.5m (possibly smaller) to 60km (possibly larger). This has important implications for understanding the fluid dynamics of lava flows.

(2) Pahoehoe flow margins tend to have higher fractal dimensions (1.13 - 1.23) than most a'a flow margins (1.05 - 1.09). This may prove to be a useful remote sensing tool.

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