Jaeger, J. C., *Elasticity, Fracture, and Flow.* London, Methuen and Co., Ltd., and New York, Wiley, 1962.

Telford, W. M., L. P. Geldart, R. E. Sheriff, and D. A. Keys, *Applied Geophysics.* London and New York, Cambridge University Press, 1976.

Timoshenko, S., and J. N. Goodier, *Theory of Elasticity.* New York, McGraw-Hill, 1951.

White, J. E., *Seismic Waves—Radiation, Transmission, and Attenuation.* New York, McGraw-Hill, 1965.

E. S. Robinson & C. Coruh, "Basic Exploration Geophysics", John Wiley & Sons, 1988

THREE

Refracted Seismic Waves and Earth Structure

Seismic refraction surveying is one of our most powerful methods for detecting subsurface structure. This method had its beginnings in the second half of the nineteenth century when the first measurements were made of the speed, or velocity, of seismic waves through various earth materials. Early in the twentieth century, seismologists were able to discern the principal interior zones of the earth from analysis of refracted earthquake waves. During World War I German scientists used some of their methods to detect enemy artillery positions by means of refracted seismic waves produced by cannon recoil. In the decade following the war, these ideas were modified and expanded for application to geologic problems. During that time seismic refraction surveying emerged as one of our most useful tools in the exploration for natural resources.

Our aim in this chapter is to describe how refracted seismic waves can be used to detect the thicknesses and physical properties of buried rock layers. We have already introduced basic procedures for detecting seismic waves with geophones placed in a line extending away from an energy source (Figure 1–2), and for displaying the results on a seismogram (Figure 2–26). How do we identify refracted seismic waves on such a seismogram, and how do we analyze them to measure seismic wave velocities and layer thicknesses?

We learn to identify refracted waves from characteristic alignments of pulses on a seismogram. From the positions of these pulses we find the times required for refracted waves to travel from a common source to geophones at various distances. These travel times are then plotted at the appropriate geophone distances on a graph that we call a *travel time curve*, or a *time–distance curve*, often abbreviated *t–x curve*.

Alignments of points on a *t–x* curve indicate the velocities of seismic waves through different rock layers and provide the information needed to calculate layer thicknesses. We will consider some examples that illustrate how the information on a *t–x* curve is related to earth structure by application of Snell's law, the principles of Huygens and Fermat, and the methods of trigonometry and geometry.

THE SINGLE-LAYER REFRACTION PROBLEM

Critical Refraction

We begin by describing a simple structure consisting of a single horizontal layer of thickness h_1 that lies on another deeper material (Figure 3–1). To keep things as uncomplicated as possible, let us consider only the direct and refracted *P*-waves that are produced at the source S. The *P*-wave velocities will be V_1 in the top layer and V_2 in the underlying material. In addition, let the velocity be faster in the deeper material, that is, $V_2 > V_1$.

Now, suppose that *P*-waves produced at S travel outward in all directions such as those indicated by the rays in Figure 3–1. These rays refract at the boundary according to Snell's law (Equation 2–15), which tells us that larger angles of refraction correspond to larger angles of incidence. For example, because the incident angles are $i'_1 > i_1$, it follows that the angles of refraction will be $i'_2 > i_2$, and so on. Recall from Chapter 2 that we have a limit where the angle of refraction $i_2 = 90$ degrees corresponds to the critical angle of incidence i_c. For that particular case Snell's law takes the form

$$\sin i_c = V_1/V_2 \qquad (3\text{–}1)$$

which is the same as Equation 2–16. Remember also the principles of Huygens and Fermat which tell us that the critically refracted wave travels at velocity V_2 along the boundary of the deeper material, at the same time producing waves that refract back into the upper layer at the critical angle i_c (Figure 2–13). In this way it is possible for the critically refracted wave in Figure 3–1 to travel from the source S to a receiver at R along a path that reaches the boundary of the deeper material. This boundary is also called the *refractor* because it is here that the paths of seismic waves are refracted.

Now let us draw attention to an important fact. In order to detect a refracted wave, we must place a receiver far enough from the source for critical refraction to be possible. The minimum distance, indicated by X_{crit} in Figure 3–2, is called the *critical distance*. Observe that a refracted wave reaching a receiver at R′ would have traveled only an infinitesimally short distance along the refractor at point A before refracting upward to R′. A similarly refracted wave reaching a receiver at R would have traveled the distance AB along the refractor. The triangle SAO tells us that the critical distance is related to the critical angle of incidence and the layer thickness. We see that

$$\tan i_c = \frac{X_{crit}/2}{h_1} \qquad (3\text{–}2)$$

which can be rearranged to give

$$X_{crit} = 2h_1 \tan i_c \qquad (3\text{–}3)$$

Because we are concerned with critical refraction, we can use Equation 3–1 and some well-known trigonometric identities to show that

$$\cos i_c = (1 - \sin^2 i_c)^{1/2} = \left[1 - \left(\frac{V_1}{V_2}\right)^2\right]^{1/2} = \left(\frac{V_2^2 - V_1^2}{V_2^2}\right)^{1/2} \qquad (3\text{–}4)$$

and therefore,

$$\tan i_c = \frac{\sin i_c}{\cos i_c} = \frac{V_1}{(V_2^2 - V_1^2)^{1/2}} \qquad (3\text{–}5)$$

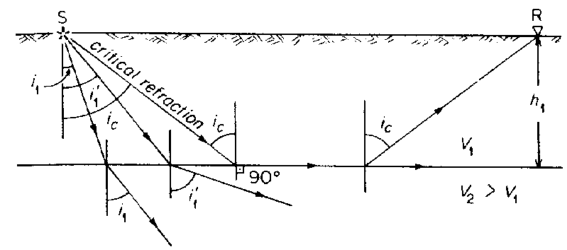


Figure 3–1
Refraction of seismic waves in a structure consisting of an upper layer in which wave velocity is V_1 separated by a plane horizontal boundary from underlying material in which wave velocity is V_2. The first layer has thickness h_1, and the velocity V_2 is greater than the velocity V_1. Three rays departing from the energy source (S) illustrate refraction at the boundary. The ray corresponding to a refraction angle of 90 degrees is called the critically refracted wave and can be observed at a receiver (R) on the surface.

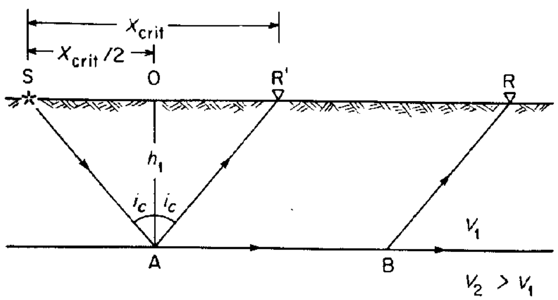


Figure 3–2
Minimum observation distance for waves refracted from a horizontal plane boundary. Because velocity V_2 is greater than V_1, the critical ray from the source (S) is refracted along the interface and back to the surface to be recorded at receivers R' and R. The wave refracted from the point A to the surface receiver R' is the first observable refracted wave, and the minimum distance for detecting the refracted wave is called the critical distance. Any point on the refractor beyond the point A refracts rays to the surface. The critical distance (X_{crit}) is controlled by the thickness of the layer and the velocity contrast between V_1 and V_2.

This equation can be rearranged to obtain

$$X_{crit} = \frac{2h_1}{\left[\left(\frac{V_2}{V_1}\right)^2 - 1\right]^{1/2}} \qquad (3–6)$$

This equation tells us that the critical distance is directly proportional to the layer thickness and indirectly proportional to the velocity ratio V_2/V_1. For a given layer thickness, the larger the velocity contrast, the shorter the critical distance along which refracted waves cannot be detected. Equation 3–6 can be useful in planning the spacing of geophones for a seismic refraction survey if we can make a preliminary estimate of the velocity contrast we expect to encounter. For example, suppose that V_1 = 1000 m/s and V_2 = 1414 m/s, so that the ratio $V_2/V_1 = 1.414 = \sqrt{2}$. For this case, Equation 3–6 tells us that

$$X_{crit} = 2h_1 \qquad (3–7)$$

Therefore, we can expect to detect a refracted wave if a boundary exists at a depth that is less than one-half the distance of a geophone from the source.

Preparing a Travel Time Curve

So far, we have discussed how the path of a refracted seismic wave is related to the wave velocities above and below a refractor, and the layer thickness. However, before we do a seismic refraction survey, we do not know the values of V_1, V_2, and h_1. The survey will measure

these values. The first step, then, is to place several geophones in a line, and produce the seismic waves, perhaps by detonating an explosive and recording a seismogram. The next step is to prepare a travel time curve using information obtained from the seismogram.

Look again at the seismogram in Figure 2–26. Notice that several pulses appear on each of the traces. For the purposes of many seismic refraction surveys, only the first pulse on each trace is used in the analysis. This pulse is called the *first arrival*, because it indicates the first of several waves that reach the geophone.

Now let us examine Figure 3–3, which helps explain how a travel time curve is prepared. Here we see a source S and a line of geophones $R_1, R_2, \ldots, R_{12}$ on the surface of a simple one-layer structure. Above this structure is an idealized seismogram that shows only the first arrivals. After such a seismogram is recorded, the seismologist can write the geophone distances $x_1, x_2, \ldots, x_{12}$ on the seismogram traces. Then the time (t) of onset of the first arrival is measured for each trace. These times, $t_1, t_2, \ldots, t_{12}$, are marked on the seismogram. Points corresponding to these distance and time values are plotted on the graph at the top of Figure 3–3. This graph is the travel time curve.

Measuring Seismic Wave Velocities

What does this travel time curve tell us about the structure? First, look at the travel time points corresponding to the nearest geophones, R_1, R_2, and R_3. Observe that a straight line can be drawn through these points that also passes through the origin of the graph. This fact indicates that the first arrivals to reach these geophones must be the direct waves that have traveled straight along the surface from the source. The straight alignment shows that the additional time required for the wave to travel to a farther geophone is directly proportional to the additional distance to that geophone. Now let us find the slope of the straight line. This can be done by using the time and distance values for any two geophones, for example, R_1 and R_3, in the following way:

$$\text{slope} = \frac{x_3 - x_1}{t_3 - t_1} = \frac{\Delta_x}{\Delta_t} = \frac{1}{V_1} \qquad (3–8)$$

This relationship is very important! It tells us that we can calculate the velocity V_1 of the seismic waves in the top layer from the slope of a straight line drawn through the travel time points.

Next, examine the travel time points corresponding to the more distant geophones R_5, $R_6, \ldots, R_{12}$. These points plot along a different straight line. Rather than intersect the origin of the graph, the straight line intersects the vertical axis at the time value of T_1, which we call the *intercept time*. The importance of T_1 will be discussed later in this chapter.

Because the travel time points corresponding to the more distant geophones plot on a different straight line, they cannot be representing direct waves. Therefore, they must be representing refracted waves. But how can a refracted wave be a first arrival, reaching a geophone ahead of the direct wave? The ray paths in Figure 3–3 make it obvious that the refracted wave must always travel a longer distance than the direct wave. However, the refracted wave has the advantage of traveling at higher velocity through the deeper material. Since $V_2 > V_1$, there will be a point beyond which this advantage of speed overcomes the disadvantage of distance so that the refracted wave arrives before the direct wave.

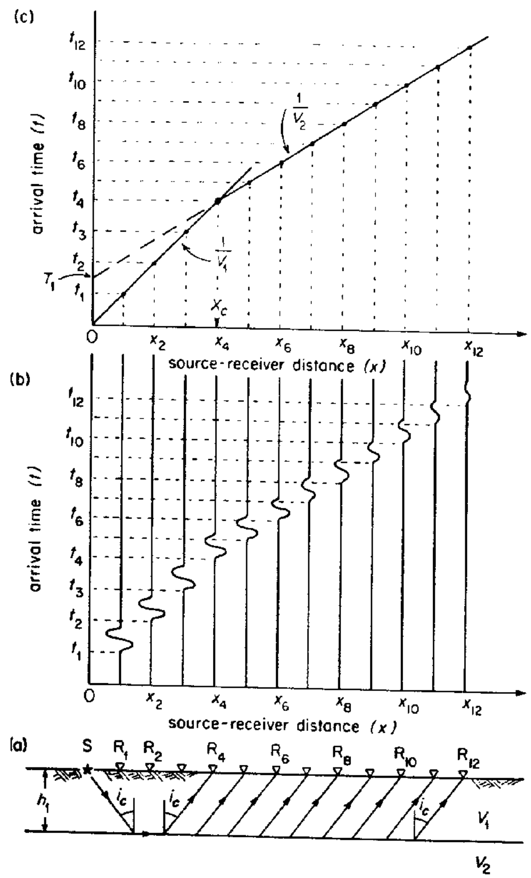


Figure 3–3
Wave paths, seismogram, and travel time curve for direct waves and waves critically refracted from a horizontal plane boundary. The figure shows recording of direct and refracted waves by a 12-channel system. (a) The critical ray from the source (S) refracts along the interface and back to the surface at the receivers from R_3 to R_{12}. (b) The seismogram consists of traces observed at receivers. Each trace represents ground vibration as a function of time. Onset times of the first arriving waves are marked on the vertical time axis, and the distances from the source to receivers are marked on the horizontal axis. (c) Time–distance curves are constructed by drawing lines through alignments of points that show arrival times at different distances. Slopes of the lines indicate the velocities.

The fact that travel time points for the refracted waves plot on a straight line indicates that the additional time required for the wave to reach a farther geophone is directly proportional to the additional distance to that geophone. We see in Figure 3–3 that all the refracted waves follow the same path from the source down to the refractor, and they all follow similar paths from the refractor upward to the geophones. The only difference between their paths is the distance traveled along the refractor. For any two geophones, this difference equals the distance between those geophones, because the upward paths through the layer are parallel. Therefore, the slope of the straight line, found from time and distance values for any pair of geophones, for example, R_5 and R_{10}, can be used to calculate the seismic wave velocity V_2 in the deeper material:

$$\text{slope} = \frac{x_{10} - x_5}{t_{10} - t_5} = \frac{\Delta x}{\Delta t} = \frac{1}{V_2} \qquad (3\text{–}9)$$

Let us summarize the basic procedure for measuring V_1 and V_2 by means of a seismic refraction survey.

1. Place an energy source, such as an explosive, and a line of geophones along the surface.
2. Record a seismogram.
3. Measure the times of first arrivals from the seismogram traces.
4. Plot these times at the corresponding geophone distances on a graph.
5. Draw lines through straight alignments of points to get the t–x curve.
6. Measure slopes of the straight lines and calculate velocities from the reciprocals of these slopes.

Another feature of the travel time curve in Figure 3–3 should be pointed out. The distance at which the two straight lines intersect is called the *crossing distance* X_c. A geophone placed at this distance would receive both the direct wave and the refracted wave at exactly the same time. For all distances beyond X_c, the refracted wave will be the first arrival, and the direct wave will become a later arrival.

What is the relationship between the crossing distance X_c and the critical distance X_{crit} that we introduced earlier? Look again at Figure 3–2. We see that a direct wave reaching R′ travels the distance SR′ = X_{crit} at velocity V_1. But the refracted wave must follow the longer path SAR′, traveling at the same speed. Because it is immediately refracted back to R′ at the point of incidence A, it does not travel along the refractor at velocity V_2. Therefore, the refracted wave must arrive later than the direct wave at R′, implying that $X_{crit} < X_c$. At distances between X_{crit} and X_c, the direct wave will be the first arrival, and the refracted wave will be a later arrival on the seismogram.

Calculating Layer Thickness

Next, let us find out how to calculate the layer thickness h_1 defined by a horizontal interface from information on the travel time curve. Two approaches can be used to analyze this problem. One makes use of the crossing distance X_c, and the other utilizes the intercept time T_1. We will begin with the analysis based on X_c.

Suppose that a source S and a receiver R are separated by the distance x, as in Figure 3–4. Then the travel time t_D for a direct wave will be simply

$$t_D = x/V_1 \qquad (3\text{–}10)$$

The travel time for the refracted wave will be

$$t_R = \frac{SA}{V_1} + \frac{AB}{V_2} + \frac{BR}{V_1} \qquad (3\text{–}11)$$

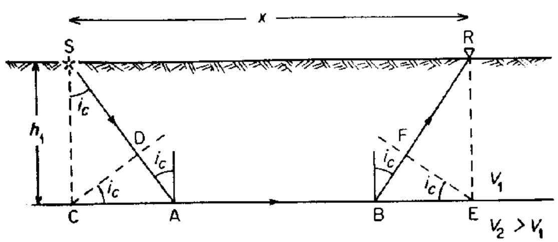


Figure 3–4
Geometrical features of the travel path and the wave front of a seismic wave critically refracted along a horizontal plane boundary. The critically refracted wave between the source (S) and receiver (R) can be studied by the geometry of wave fronts. If SA and BR are the paths of the critically refracted ray in the first layer, then CD and EF are the critically refracted wave fronts at the times when the critical wave reaches the interface and departs from the interface, respectively. The travel time for the ray path defined by SABR is equivalent to the travel time for the wave front moving between S and D, C and E, and F and R.

Now examine the right triangles SCA and BER that show us that

$$SA = BR = h_1/\cos i_c \qquad (3\text{–}12)$$
$$CA = BE = h_1 \tan i_c \qquad (3\text{–}13)$$

and finally,

$$AB = x - CA - BE \qquad (3\text{–}14)$$

Substituting from Equations 3–12, 3–13, and 3–14 into Equation 3–11 gives

$$t_R = \frac{2h_1}{V_1 \cos i_c} + \frac{x - 2h_1 \tan i_c}{V_2} \qquad (3\text{–}15)$$

Then, using the identity $\tan i_c = \sin i_c/\cos i_c$, and rearranging the terms, we obtain

$$t_R = \frac{x}{V_2} + \frac{2h_1}{V_1 \cos i_c}\left(1 - \frac{V_1}{V_2}\sin i_c\right) \qquad (3\text{–}16)$$

According to Equation 3–1, however, we can write

$$1 - \frac{V_1}{V_2}\sin i_c = 1 - \sin^2 i_c = \cos^2 i_c$$

and therefore,

$$t_R = \frac{x}{V_2} + \frac{2h_1}{V_1}\cos i_c \qquad (3\text{–}17)$$

We know that at the crossing distance X_c the direct and refracted waves arrive at the same time, that is, $t_D = t_R$. Therefore, if $x = X_c$, we can set Equation 3–10 to equal Equation 3–17 to obtain

$$\frac{X_c}{V_1} = \frac{X_c}{V_2} + \frac{2h_1}{V_1}\cos i_c$$

Substituting from Equation 3–4 for $\cos i_c$, we get

$$X_c\left(\frac{1}{V_1} - \frac{1}{V_2}\right) = X_c\left(\frac{V_2 - V_1}{V_1V_2}\right) = \frac{2h_1}{V_1}\left(\frac{V_2^2 - V_1^2}{V_2^2}\right)^{1/2} \qquad (3\text{–}18)$$

Rearranging to solve for h_1, we have

$$h_1 = \frac{X_c}{2}\left(\frac{V_2 - V_1}{V_1V_2}\right)\left[\frac{V_1V_2}{(V_2^2 - V_1^2)^{1/2}}\right] \qquad (3\text{–}19)$$

and finally,

$$h_1 = \frac{X_c}{2}\left(\frac{V_2 - V_1}{V_2 + V_1}\right)^{1/2} \qquad (3\text{–}20)$$

This very important result can be used for interpreting a seismogram such as the one shown in Figure 3–3. After the travel time curve has been prepared, the velocities V_1 and V_2 can be found from the slopes of the two straight lines. Then the crossing distance X_c can be observed from the intersection of these lines. These three values are used in Equation 3–20 to calculate the layer thickness.

Now let us look at the other method for finding layer thickness that is based on the intercept time T_1. Recall from Figure 3–3 that T_1 is the point at which the straight line representing the refracted waves intersects the vertical axis of the travel time graph. This method involves analysis of wave fronts that were introduced in Chapter 2. It would be helpful to review the discussion of Figure 2–9 which introduces some important relationships between wave fronts and refracted rays.

In Figure 3–4 the line CD lies on the wave front of a wave traveling downward from the source S to the refractor. Similarly, the line EF lies on the wave front of a wave moving upward from the refractor. Note that these wave fronts are perpendicular to the lines SA and BR, which indicate the directions of the advancing waves. Now, observe carefully that whereas one point on the wave front is at D, another part of that wave front has already encountered the refractor at C. Then, as one part advances from D to A at velocity V_1, another part of the wave front is sweeping along the refractor from C to A at velocity V_2. Similarly, part of the upward traveling wave front, moves from B to F at velocity V_1 while another part sweeps along the refractor from B to E at velocity V_2. Therefore, we have the relationship

$$\frac{DA}{V_1} = \frac{CA}{V_2} = \frac{BF}{V_1} = \frac{BE}{V_2} \qquad (3\text{–}21)$$

The travel time of the refracted wave reaching a geophone at R in Figure 3–4 can be expressed as

$$t_R = \frac{SD}{V_1} + \frac{DA}{V_1} + \frac{AB}{V_2} + \frac{BF}{V_1} + \frac{FR}{V_1} \qquad (3\text{–}22)$$

or by substituting from Equation 3–21 we can obtain

$$t_R = \frac{SD + FR}{V_1} + \frac{CE}{V_2} \qquad (3\text{–}23)$$

But the triangles SDC and RFE tell us that

$$SD = FR = h_1 \cos i_c \qquad (3\text{–}24)$$

and we know that $CE = x$. Therefore, the travel time can be expressed as

$$t_R = \frac{x}{V_2} + \frac{2h_1}{V_1}\cos i_c \qquad (3\text{–}25)$$

which is the same as Equation 3–17. For the structure we have been studying, the velocities V_1 and V_2, the layer thickness h_1, and the critical angle are fixed. Therefore, the second term must be equal to a constant k,

$$\frac{2h_1}{V_1}\cos i_c = k \qquad (3\text{–}26)$$

so that

$$t_R = \left(\frac{1}{V_2}\right)x + k \qquad (3\text{–}27)$$

We recognize that this formula is used for a straight line on a graph of t and x, which has a slope = $1/V_2$ and an intercept k. Equation 3–27, then, is the expression for the straight line in Figure 3–3 that represents the refracted waves. The constant k turns out to be the intercept time, so that

$$k = T_1 = \frac{2h_1}{V_1}\cos i_c \qquad (3\text{–}28)$$

Solving for the layer thickness, we get

$$h_1 = \frac{T_1}{2}\frac{V_1}{\cos i_c} \qquad (3\text{–}29)$$

and substituting from Equation 3–4 for $\cos i_c$ gives

$$h_1 = \frac{T_1 V_1 V_2}{2(V_2^2 - V_1^2)^{1/2}} \qquad (3\text{–}30)$$

This result is just as important as the one in Equation 3–20. From the travel time curve (Figure 3–3) velocities V_1 and V_2 are found, as before, from the slopes of the lines. Then the intercept time T_1 can be observed from the intersection of the line representing the refracted waves and the vertical axis of the graph. These three values can be used in Equation 3–30 to calculate the layer thickness.

Relationships Between Intercept Time and Crossing Distance

We know that layer thickness can be calculated by means of the velocities V_1 and V_2, and either the crossing distance X_c or the intercept time T_1. If we set Equations 3–20 and 3–30 equal to each other, we obtain

$$h_1 = \frac{X_c}{2}\left(\frac{V_2 - V_1}{V_2 + V_1}\right)^{1/2} = \frac{T_1 V_1 V_2}{2(V_2^2 - V_1^2)^{1/2}} \qquad (3\text{–}31)$$

The equation can be rearranged to express the crossing distance in terms of the intercept time and the velocities:

$$X_c = T_1\left(\frac{V_1 V_2}{V_2 - V_1}\right) \qquad (3\text{–}32)$$

Next, by rearranging Equation 3–6 to solve for h_1 and setting the result equal to Equation 3–20, we get

$$h_1 = \frac{X_{crit}}{2}\left(\frac{V_2^2 - V_1^2}{V_1^2}\right)^{1/2} = \frac{X_c}{2}\left(\frac{V_2 - V_1}{V_2 + V_1}\right)^{1/2} \qquad (3\text{–}33)$$

which can be rearranged to relate the critical distance and the crossing distance

$$\frac{X_{crit}}{X_c} = \frac{V_1}{V_1 + V_2} \qquad (3\text{–}34)$$

This proves our earlier conclusions that $X_{crit} < X_c$, so that in the interval between X_{crit} and X_c the first arrivals on a seismogram must be the direct waves and the refracted waves will be later arrivals.

Let us look once again at Figure 3–3. We see that for geophones $R_5, R_6, \ldots, R_{12}$, the first arrivals are refracted waves. For each receiver let us calculate

$$\Delta t_x = t_x - \frac{x}{V_2} \qquad (3\text{–}35)$$

where t_x is the travel time of the refracted wave and x is the distance of that receiver.

The term Δt_x is called the *delay time*. We might think of it in this way: If the wave could travel directly to the receiver at velocity V_2, it would arrive sooner than the actual travel time t_x. Because the wave must travel at the slower velocity V_1 through the upper layer, it is "delayed." We know from Equations 3–25 and 3–28 that the intercept time

$$T_1 = t_R - \frac{x}{V_2} \qquad (3\text{–}36)$$

where the travel time t_R of a refracted wave is the same as t_x in Equation 3–35. Thus, we know that the delay times calculated for all the receivers should have the same value, which is identical to the intercept time. This would be true for an ideal structure for which the values of h_1, V_1, and V_2 are everywhere constant. But in nature there are no perfectly uniform rock layers. Small changes from place to place can produce small differences in the delay times calculated for different receivers. Where such irregularities exist, we can calculate an average layer thickness by using an intercept time found from the average of the delay times for the different receivers.

Application

A practical application of these methods is a short refraction survey of the layer of soil and alluvium that overlies bedrock at a construction site. The procedure and results of such a survey are presented in Figure 3–5. For this example the field procedure began by arranging 12 geophones in a line at 2-meter intervals. The seismic cable connects them to the portable amplifier–recorder unit. *P*-waves were then produced at the source by pounding a hammer on a steel plate. A switch mounted on the hammer turns on the amplifier–recorder unit at the moment of impact, and a seismogram is recorded. First arrival times, read from this seismogram, were entered in the table of values at the corresponding geophone distances. These data were plotted on the travel time curve, and straight lines were drawn through the alignments of points.

From the slopes of the lines we obtain

$$V_1 = 415 \text{ m/s} \quad \text{and} \quad V_2 = 2055 \text{ m/s}$$

and we observe that the intercept time is 0.025 second and the crossing distance is close to 12.8 meters. Using Equation 3–20, we calculate

$$h = 5.2 \text{ m}$$

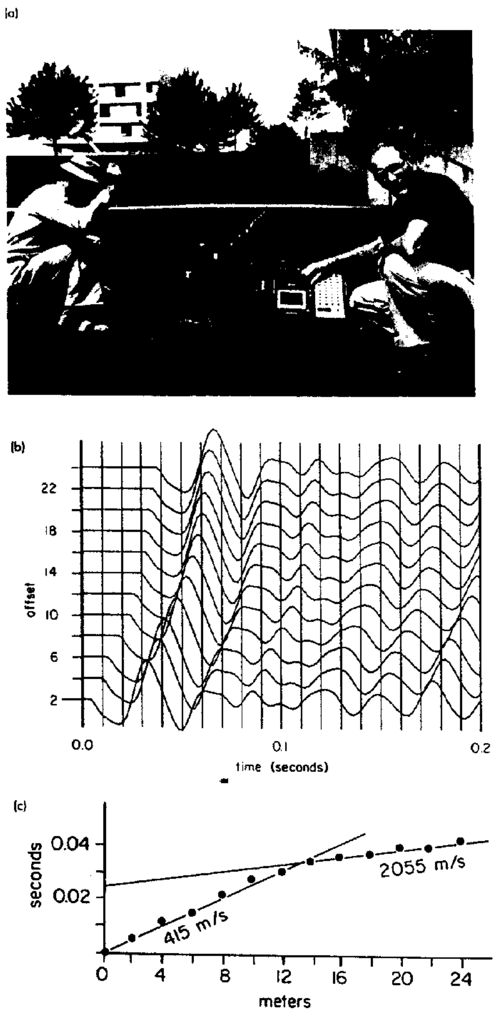
and from Equation 3–30 we get

$$h = 5.3 \text{ m}$$

There is only a small difference in these values because we cannot read the crossing distance and the intercept time more accurately from the travel time curve.

This survey indicates that the thickness of soil and alluvium overlying bedrock is about 5¼ meters. Field operations took approximately 15 minutes, and the analysis required about the same amount of time. Altogether, then, this sounding was done in about half an hour by means of refraction surveying.

This small-scale survey demonstrates a procedure that is followed in larger surveys that are used to probe deeper into the earth. Here a geophone line 24 meters long was used to detect a shallow refractor. The same basic procedure can be used with much longer geophone lines. On a very large scale, geophones have been placed in lines reaching several hundred kilometers to measure the thickness of the earth's crust and to detect structure in the upper mantle. For such a long line, short-wave radios rather than a seismic cable are



(c)

x (meters)	t (seconds)
2	0.005
4	0.011
6	0.014
8	0.020
10	0.025
12	0.028
14	0.031
16	0.033
18	0.034
20	0.036
22	0.037
24	0.039

Figure 3–5
(a) Portable seismic recording equipment, (b) refraction seismogram, and (c) travel time curve of first arrival times for a short refraction survey in southwestern Virginia. (a) Twelve receivers with 2-meter intervals are used to record the direct and refracted arrivals. Waves were generated by hammering a metal plate. (b) The seismogram horizontal axis is the recording time that is marked by the vertical lines at 0.01-second intervals. (c) The time–distance curve prepared from the seismogram. Straight lines through the travel time points show that the crossing distance is at 12.8 meters and intercept time is 0.025 second.

used to connect geophone positions. Otherwise, the procedure is quite similar to that illustrated in Figure 3–5.

REFRACTED WAVES IN MULTILAYERED STRUCTURES

We can use the same basic method to analyze refracted seismic waves in a multilayered structure that we have introduced for a single layer. Snell's law, critical refraction, and the geometry of rays and wave fronts are used in the same way.

The Ray Parameter

The multilayered structure in Figure 3–6 consists of four horizontal layers with thicknesses of h_1, h_2, h_3, and h_4, in which the seismic wave velocities are V_1, V_2, V_3, and V_4. Below these layers is the deeper rock in which the wave velocity is V_5. This structure has four refractors, which are the boundaries of the layers. Suppose that we have the following relationships for the velocities,

$$V_1 < V_2 > V_3 < V_4 < V_5$$

Let us consider a ray path originating at the source S that is critically refracted at the deepest refractor. At the other refractors noncritical refraction occurs. By applying Snell's law at each refractor, we obtain

$$\frac{\sin i_{12}}{\sin i_{23}} = \frac{V_1}{V_2}, \quad \frac{\sin i_{23}}{\sin i_{34}} = \frac{V_2}{V_3},$$
$$\frac{\sin i_{34}}{\sin i_{45}} = \frac{V_3}{V_4}, \quad \frac{\sin i_{45}}{\sin i_5} = \frac{V_4}{V_5}$$

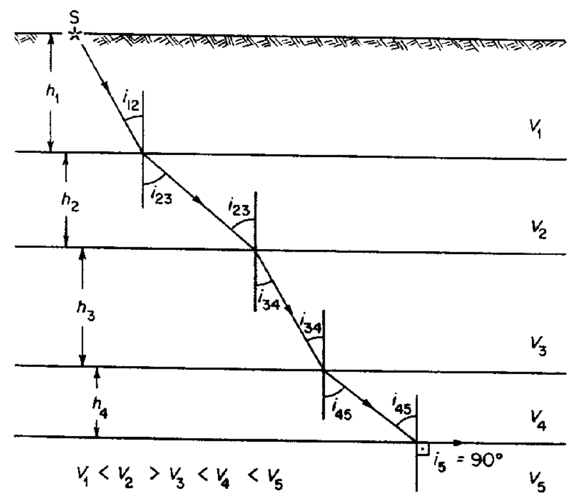


Figure 3–6
Path of a seismic wave refracted through a multi-layered structure with horizontal plane refractors. The ray path is defined by the departure angle (i_{12}), velocities, and layer thicknesses. The ray path represents the critically refracted wave at the interface between layers where the velocities are V_4 and V_5.

Because we can rearrange the terms in the following way,

$$\frac{\sin i_{12}}{V_1} = \frac{\sin i_{23}}{V_2} = \frac{\sin i_{34}}{V_3}, \text{ etc.}$$

and because i_5 = 90 degrees, we can write

$$\frac{\sin i_{12}}{V_1} = \frac{\sin i_{23}}{V_2} = \frac{\sin i_{34}}{V_3} = \frac{\sin i_{45}}{V_4} = \frac{\sin i_5}{V_5} = \frac{1}{V_5} = p \quad (3\text{–}37)$$

where p is a constant called the *ray parameter* or the *slowness* of the ray path. We see that p is the same for all parts of the path that a refracted wave follows through a multilayered structure.

Wave Fronts and Rays

Now let us consider how a wave front advances through a multilayered structure. This discussion will allow us to recognize some very important geometrical relationships that can be used later to find layer thicknesses. To keep the discussion as uncomplicated as possible, let us consider the structure in Figure 3–7 which consists of two horizontal layers resting on a deeper material in which the

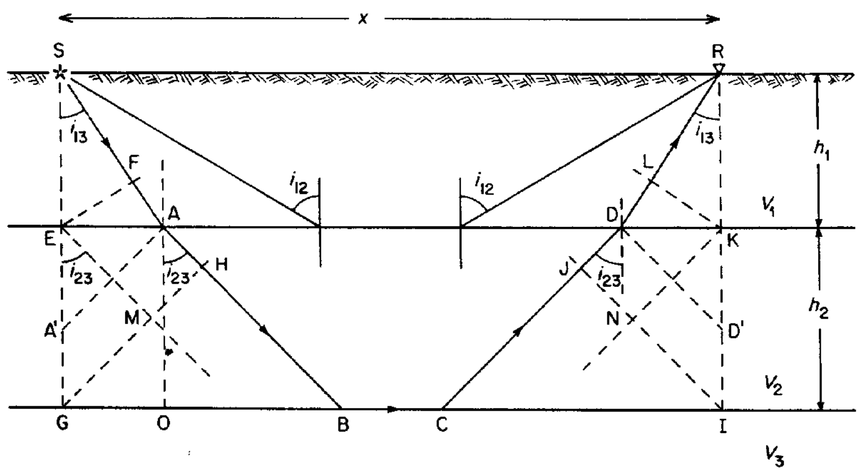


Figure 3–7
Geometrical features of the travel paths and the wave fronts of seismic waves critically refracted at two horizontal plane boundaries. The critically refracted wave from the second interface is defined by the ray path SABCDR. EF, AA', GH, JI, DD', and LK are the wave fronts. The wave represented by the path SABCDR can be viewed as a critically refracted wave front traveling from S to E with the velocity V_1, E to G with the velocity V_2, G to I with the velocity V_3, I to K with the velocity V_2, and K to R with the velocity V_1.

wave velocities are $V_1 < V_2 < V_3$. The receiver at distance x from the wave source S is far enough away to receive the direct wave, a wave critically refracted along the upper refractor, and a wave critically refracted along the deeper refractor. Critical refraction along the upper refractor is the same as we have already discussed for the single-layer structure. Therefore, let us concentrate on the wave that is critically refracted along the deeper refractor. Its ray parameter is

$$p = \frac{\sin i_{13}}{V_1} = \frac{\sin i_{23}}{V_2} = \frac{1}{V_3} \quad (3\text{–}38)$$

Several lines perpendicular to its ray path in Figure 3–7 illustrate an advancing wave front. When we follow the ray path, we see that the travel time from S to R is

$$t = \frac{\text{SA} + \text{DR}}{V_1} + \frac{\text{AB} + \text{CD}}{V_2} + \frac{\text{BC}}{V_3} \quad (3\text{–}39)$$

To understand more clearly how the wave is traveling, we must look closely at different parts of the wave front. First, let us examine how the wave front EF advances downward to A'A. As one part moves from F to A at velocity V_1, other parts are simultaneously sweeping from E to A and from E to A' at velocity V_2. In addition, look at the geometrically similar upward advance of the wave front DD' to LK. In the same time that one part moves from D to L at velocity V_1, other parts sweep from D to K and from D' to K at velocity V_2. Therefore, we have the following relationships:

$$\frac{\text{FA}}{V_1} = \frac{\text{EA}}{V_2} = \frac{\text{EA}'}{V_2} = \frac{\text{DL}}{V_1} = \frac{\text{DK}}{V_2} = \frac{\text{D}'\text{K}}{V_2} \quad (3\text{–}40)$$

Next, examine the advance from A'A to GMH at velocity V_2. We see that one part moves from A to H during the same time that another part sweeps from A' to G. Similarly, the wave front JNI moves to DD' at velocity V_2. Therefore, it moves from J to D at the same time that it sweeps from I to D'. This shows that

$$\frac{\text{AH}}{V_2} = \frac{\text{A}'\text{G}}{V_2} = \frac{\text{JD}}{V_2} = \frac{\text{ID}'}{V_2} \quad (3\text{–}41)$$

From Equations 3–40 and 3–41 and the geometry in Figure 3–7, we can see that

$$\frac{\text{EA} + \text{AH}}{V_2} = \frac{\text{EM}}{V_2} = \frac{\text{JD} + \text{DK}}{V_2} = \frac{\text{NK}}{V_2} \quad (3\text{–}42)$$

What we have shown so far is that while one part of the wave front moves from S to H, another part sweeps from S to G. Moreover, while one part of a wave front moves from J to R, another part of that same wave front sweeps from I to R.

Finally, we observe that as the wave front advances from H to B at velocity V_2, it is also sweeping from G to B at velocity V_3. It also sweeps from C to I at velocity V_3. Therefore,

$$\frac{\text{HB}}{V_2} = \frac{\text{GB}}{V_3} = \frac{\text{CJ}}{V_2} = \frac{\text{CI}}{V_3} \quad (3\text{–}43)$$

The positions of wave fronts in Figure 3–7 show us that the travel time of the refracted wave from S to R is

$$t = \frac{\text{SF} + \text{LR}}{V_1} + \frac{\text{EM} + \text{NK}}{V_2} + \frac{\text{GI}}{V_3} \quad (3\text{–}44)$$

Travel Time and Layer Thicknesses

Continuing with our study of Figure 3–7, we find from the triangles SEF and RKL that

$$SF = LR = h_1 \cos i_{13} \qquad (3\text{–}45)$$

and from the triangles EMG and NIK, it is evident that

$$EM = NK = h_2 \cos i_{23}$$

In addition, we observe that

$$GI = x$$

We can insert these terms into Equation 3–44 to obtain

$$t = \frac{x}{V_3} + \frac{2h_1}{V_1}\cos i_{13} + \frac{2h_2}{V_2}\cos i_{23} \qquad (3\text{–}46)$$

Because of the similarity in the form of the second and third terms, we can express them as a summation:

$$t = \frac{x}{V_3} + 2\sum_{k=1}^{2}\frac{h_k}{V_k}\cos i_{k3} \qquad (3\text{–}47)$$

We can rearrange Equation 3–38 to get

$$\sin i_{13} = \frac{V_1}{V_3} \text{ and } \sin i_{23} = \frac{V_2}{V_3}$$

or, in general,

$$\sin i_{k3} = \frac{V_k}{V_3} \qquad (3\text{–}48)$$

and since

$$\cos i_{k3} = (1 - \sin^2 i_{k3})^{1/2} = \left[1 - \left(\frac{V_k}{V_3}\right)^2\right]^{1/2} \qquad (3\text{–}49)$$

we can obtain

$$t = \frac{x}{V_3} + 2\sum_{k=1}^{2}\frac{h_k}{V_k V_3}(V_3^2 - V_k^2)^{1/2} \qquad (3\text{–}50)$$

Now we have an equation that relates layer thicknesses and velocities with the travel time of a refracted wave traveling between a source and a receiver separated by the distance x.

Next, we will examine the travel time curve that corresponds to a structure with two horizontal refractors. The example in Figure 3–8 illustrates the paths of refracted waves reaching 12 receivers spaced at intervals of Δx. The travel time curve above this structure shows the arrival times of these waves. We already know that the critical distance for the upper refractor can be calculated from Equation 3–6. Receiver R_4 is situated at that distance. How can we determine the critical distance for the deeper refractor? Ray paths in Figure 3–8 show that receiver R_5 is at this distance, which turns out to be twice the distance CA + DB. The triangles SCA and ADB show that

$$X_{crit} = 2(CA + DB) = 2h_1 \tan i_{13} + 2h_2 \tan i_{23}$$

which can be rewritten as the summation

$$X_{crit} = 2\sum_{k=1}^{2} h_k \tan i_{k3} \qquad (3\text{–}51)$$

Because $\tan i_{k3} = \sin i_{k3}/\cos i_{k3}$, we can use Equations 3–48 and 3–49 to get

$$\tan i_{k3} = \left[\left(\frac{V_3}{V_k}\right)^2 - 1\right]^{-1/2}$$

Inserting this into Equation 3–51 gives us

$$X_{crit} = 2\sum_{k=1}^{2} h_k \left[\left(\frac{V_3}{V_k}\right)^2 - 1\right]^{-1/2} \qquad (3\text{–}52)$$

Now let us consider the arrival time difference between the receivers R_8 and R_{10} where the interval is $2\Delta x$. If we follow the ray paths from S to R_8 and R_{10}, we see that SABE is a common ray path and the ray paths ER_8 and FR_{10} are equal. Therefore, the arrival time difference between the receivers R_8 and R_{10} is due to the ray path EF. This time difference may be given twice Δt where

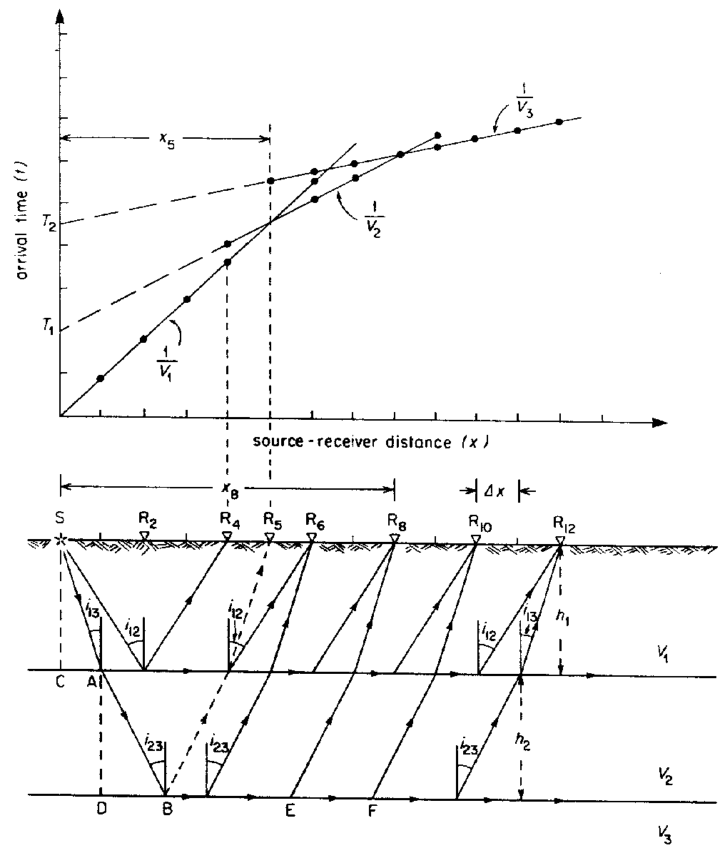


Figure 3–8
Travel paths and corresponding travel time curve for seismic waves critically refracted in a structure consisting of horizontal layers in which the wave velocities are V_1 and V_2 overlying deeper material in which the wave velocity is V_3. The first critically refracted arrivals from the top and bottom refractors can be observed at receivers R_4 and R_5, respectively. The first arrivals at the receiver groups R_1 to R_5, R_5 to R_8, and R_8 to R_{12} represent the direct wave from the top interface and refracted waves from the middle interface and the bottom interface, respectively. T_1 and T_2 are the intercept times for the first and second interfaces.

$$\Delta t = \Delta x/V_3$$

We can now see that the arrival times of waves critically refracted along the deeper refractor will plot on the travel time graph in a straight line with the slope of $1/V_3$. Equation 3–47 tells us that if $x = 0$, we have

$$t = T_2 = 2 \sum_{k=1}^{2} \frac{h_k}{V_k} \cos i_{k3} \qquad (3\text{–}53)$$

where T_2 is the intercept time found by extending that straight line to the vertical axis of the travel time graph.

Let us review the important features of the travel time graph in Figure 3–8.

1. There are three straight-line segments with slopes of $1/V_1$, $1/V_2$, and $1/V_3$.
2. There are intercept times T_1 and T_2 that correspond to waves critically refracted along the upper refractor and the deeper refractor, respectively.
3. There are crossing distances corresponding to each refractor.

At the nearest receivers, the direct waves traveling at velocity V_1 are the first arrivals. Between the first critical distance and the first crossing distance, the direct waves are first arrivals, and the waves traveling along the upper refractor at velocity V_2 are later arrivals. Beyond the first crossing distance, these refracted waves become the first arrivals, and the direct waves become later arrivals. Then at the second critical distance, waves traveling at velocity V_3 along the deeper refractor also begin to reach the receivers as later arrivals. Beyond the second crossing distances, these deeper refracted waves become first arrivals.

In a multilayered structure, either crossing distances or intercept times can be used for calculating layer thicknesses. For simplicity, we will rearrange Equation 3–53 and use the intercept times. To illustrate the procedure, let us suppose that the slopes of the straight lines on the travel time graph in Figure 3–8 indicate that V_1 = 1500 m/s, V_2 = 2500 m/s, and V_3 = 3200 m/s. Furthermore, suppose that we extend the upper two lines to the vertical axis and find intercept times of T_1 = 0.2 second and T_2 = 0.45 second. Now we can use Equation 3–29 to obtain

$$h_1 = \frac{T_1 V_1 V_2}{2(V_2^2 - V_1^2)^{1/2}} = 187.5 \text{ m}$$

Then we rearrange Equation 3–53 to get

$$h_2 = \left(\frac{T_2}{2} - \frac{h_1}{V_1} \cos i_{13}\right) \frac{V_2}{\cos i_{23}} \qquad (3\text{–}54)$$

We use Equation 3–49 to solve for $\cos i_{13}$ and $\cos i_{23}$, which are used with our previously calculated value of h_1 and the measured values of V_1, V_2, and T_2 to obtain

$$h_2 = 635.8 \text{ m}$$

This procedure illustrates how a travel time curve prepared from seismic refraction survey data can be interpreted to detect three different materials by means of seismic wave velocities and to determine the depths at which these materials will be encountered.

The same reasoning that we have used to analyze one-layer and two-layer structures can be used to find thicknesses and wave velocities in structures possessing any number of horizontal layers. Whatever the number of layers, the intercept time is expressed by an equation having the same general form as Equation 3–53. For example, in a structure with three refractors, the intercept time for waves refracted along the deepest one is

$$T_3 = 2 \sum_{k=1}^{3} \frac{h_k}{V_k} \cos i_{k4} \qquad (3\text{–}55)$$

This equation can be rearranged to obtain the thickness of the third layer:

$$h_3 = \left(\frac{T_3}{2} - \frac{h_1}{V_1} \cos i_{14} - \frac{h_2}{V_2} \cos i_{24}\right) \frac{V_3}{\cos i_{34}}$$

$$= \left(\frac{T_3}{2} - \sum_{k=1}^{2} \frac{h_k}{V_k} \cos i_{k4}\right) \frac{V_3}{\cos i_{34}} \qquad (3\text{–}56)$$

Finally, if a structure has n horizontal layers with thicknesses $h_1, h_2, \ldots, h_n$, and wave velocities $V_1, V_2, \ldots, V_n$ resting on a deeper material in which the wave velocity is V_{n+1}, we would expect a travel time curve with $n + 1$ straight-line segments. The intercept time determined from waves reaching the deepest refractor will be

$$T_n = 2 \sum_{k=1}^{n} \frac{h_k}{V_k} \cos i_{k(n+1)} \qquad (3\text{–}57)$$

and the thickness of the deepest layer will be

$$h_n = \left[\frac{T_n}{2} - \sum_{k=1}^{n-1} \frac{h_k}{V_k} \cos i_{k(n+1)}\right] \frac{V_n}{\cos i_{n(n+1)}} \qquad (3\text{–}58)$$

REFRACTION IN STRUCTURES WITH DIPPING LAYERS

To be able to account for more of the variety found in nature, we need to analyze how refracted seismic waves travel in a structure with inclined refractors. Recall that in horizontal layers the downgoing and upgoing parts of a refracted raypath are similar. This is not true for waves that are critically refracted in a structure possessing dipping layers. Therefore, the calculation of layer thickness becomes more complicated.

Reciprocity

One feature of refracted wave travel time the same for both dipping layer structures an horizontal layer structures. If the positions c a source and a receiver are interchanged, th travel time remains the same. This fact called the *condition of reciprocity* and is illu trated in Figure 3–9a for a horizontal refra tor. Here we have two opposite travel tir curves. Observe that the travel time t_D for refracted wave traveling from the source S_1 t the farthest detector R_{12} is exactly the same a the travel time t_R for the refracted wave tra eling from the source S_2 to the receiver mo distant from it. That $t_D = t_R$ should be ol vious, because both waves follow exactly th same path. This is an example of reciproci and is a necessary condition for geometric manipulations of seismic waves.

For the other receivers such as R_6 in Figu 3–9a, the waves from S_1 travel through diffe ent parts of the layer than those from S_2, bu the paths are geometrically similar. Therefor the opposite travel time curves indicate ide tical velocities and intercept times.

Now look at Figure 3–9b, which shows a inclined refractor. Here we again see th waves from S_1 and S_2 reach the most dista receivers R_{12} in identical travel times $t_D = t$ That is obvious because these waves follow e actly the same path, but in opposite direction Here we have the condition of reciprocity in structure with a dipping refractor.

Figure 3–9b also shows that critically r fracted waves from S_1 follow paths to inte mediate receivers that are not geometrical similar to those of waves from S_2. This is e dent from the paths reaching R_6 which is mi way between S_1 and S_2. We see that the o positely traveling waves arrive at differe times. The effect of the inclination of the r

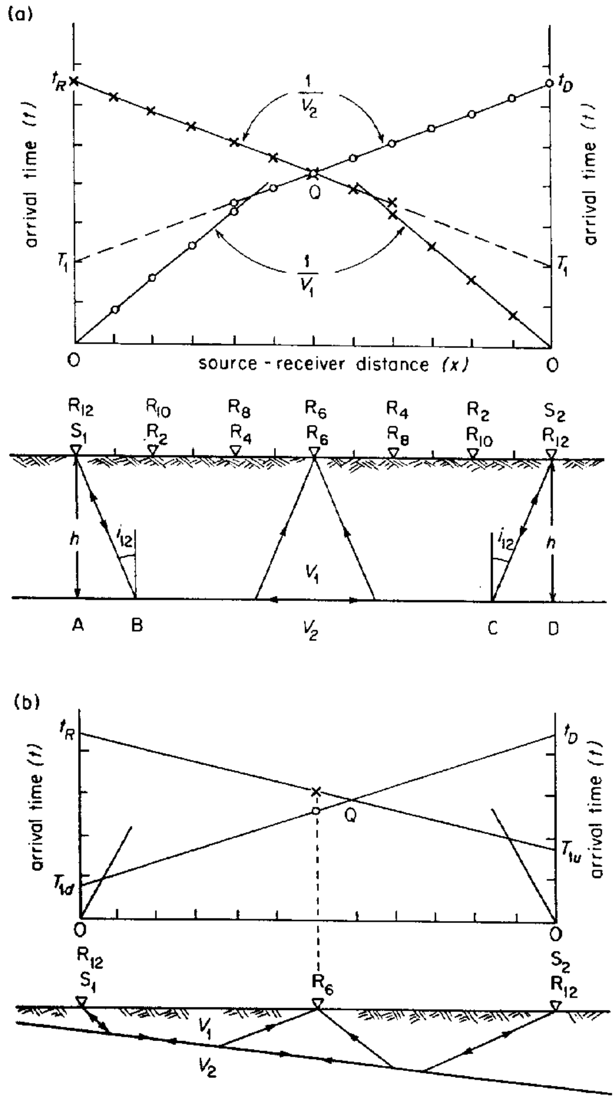


Figure 3–9
Travel paths and corresponding travel time curves for reversed refraction profiles (a) in a structure with a horizontal plane refractor and (b) in a structure with an inclined plane refractor. Intercept time T_1 is the same from the direct and reversed recordings, and the crossing point (Q) of the time–distance curves is at the center of the profile over (a) the horizontal interface. The intercept times (T_{1d}, T_{1u}) are different in the case of (b) a dipping interface. The crossing point (Q) is shifted away from the center of the profile in the down-dip direction.

fractor is to produce these differences, and similarly to produce different intercept times T_{1d} and T_{1u} on the opposite travel time curves. Notice that waves from S_1 travel along the refractor in the down-dip direction. Consequently, d is the subscript of the intercept time T_{1d}, corresponding to the waves that are refracted down dip. For the waves from S_2 that are refracted up dip, we use u in the subscript of the corresponding intercept T_{1u}.

We must have information from opposite travel time curves to analyze the depth and inclination of a dipping refractor. The seismic refraction survey field procedure is to set out the geophones in a line. A seismogram is recorded with the source at one end of this line. Then, another seismogram is recorded using a source at the opposite end of the line. This procedure is called "reversing the line." A survey done in this manner is called a *reversed refraction survey*.

Travel Time and Layer Thickness

For structures with dipping refractors, the geometry of refracted waves follows the same rules that we have already introduced in our analysis of horizontal layers. But where layers are inclined we must use more terms in the time–distance equations. In this book we will limit the discussion to the structure in Figure 3–10, which has a single refractor inclined at the angle α.

Let us begin with the source at S′ and a receiver at R′. With this arrangement we see that the critically refracted waves travel in the down-dip direction. As before, for critical refraction we have $\sin i_{12} = V_1/V_2$. The ray path in Figure 3–10 is S′BCR′, and the advance of wave fronts shows that

$$\frac{EB}{V_1} = \frac{AB}{V_2} \quad \text{and} \quad \frac{CF}{V_1} = \frac{CD}{V_2}$$

Therefore, the travel time will be

$$t = \frac{S'B}{V_1} + \frac{BC}{V_2} + \frac{CR'}{V_1} = \frac{S'E + FR'}{V_1} + \frac{AD}{V_2} \qquad (3\text{–}59)$$

But the triangles S′AE and R′FD show that

$$S'E = h_{1d}\cos i_{12} \quad \text{and} \quad FR' = h_{1u}\cos i_{12}$$

Furthermore, because S′G is parallel to the refractor, we have

$$S'G = AD = x\cos\alpha$$

where x is the distance between S′ and R′. Therefore, the travel time can be expressed as

$$t = \frac{x\cos\alpha}{V_2} + \frac{h_{1d} + h_{2d}}{V_1}\cos i_{12} \qquad (3\text{–}60)$$

Observe next that

$$GR' = h_{1u} - h_{1d} = x\sin\alpha$$

so that

$$h_{1u} = h_{1d} + x\sin\alpha \text{ and } h_{1d} = h_{1u} - x\sin\alpha$$

Now let us call the travel time for waves refracted in the down-dip direction the down-dip travel time t_d. Substituting the identities into Equation 3–60 gives

$$t_d = \frac{x\cos\alpha}{V_2} + \frac{2h_{1d} + x\sin\alpha}{V_1}\cos i_{12} \qquad (3\text{–}61)$$

Now, if we set $x = 0$, we obtain an expression for the intercept time,

$$T_{1d} = \frac{2h_{1d}}{V_1}\cos i_{12} \qquad (3\text{–}62)$$

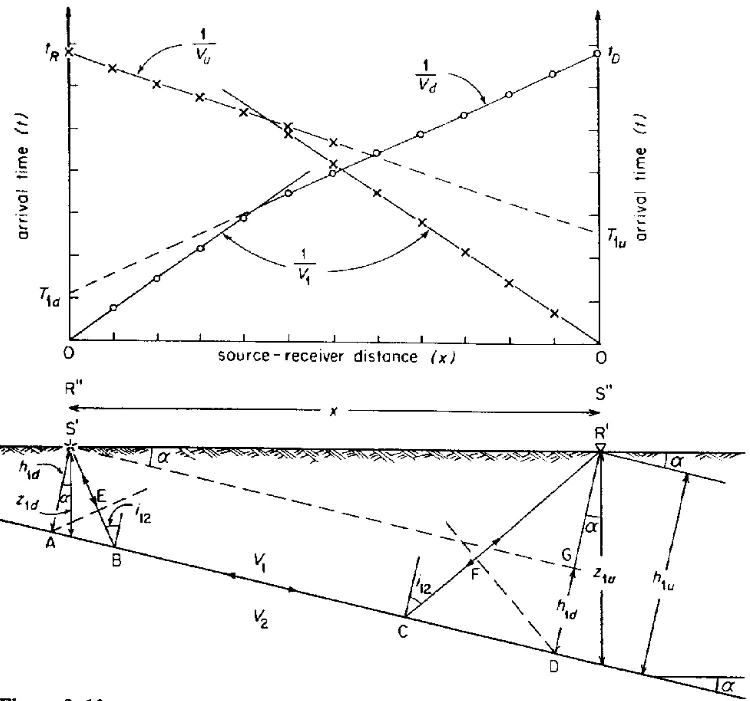


Figure 3–10
Geometrical features of the travel paths, the wave fronts, and the corresponding travel time curves for seismic waves critically refracted along an inclined plane refractor. Reversed time–distance curves over a dipping interface indicate different intercept times (T_{1d}, T_{1u}) and apparent velocities (V_d, V_u). Because of the reciprocity, $t_D = t_R$. AE and FD represent the wave fronts of critically refracted waves reaching and departing from the sloping interface. The depths to interface at S' and S", z_{1d} and z_{1u}, are respectively.

Then by rearranging Equation 3–60 into the form

$$t_d = \frac{x}{V_1}\left(\frac{V_1}{V_2}\cos\alpha + \sin\alpha\cos i_{12}\right) + \frac{2h_{1d}}{V_1}\cos i_{12}$$

and substituting from Equation 3–62, the relationship $\sin i_{12} = V_1/V_2$, and the identity

$$\sin i_{12}\cos\alpha + \sin\alpha\cos i_{12} = \sin(i_{12} + \alpha)$$

we obtain

$$t_d = \frac{x}{V_1}\sin(i_{12} + \alpha) + T_{1d} \qquad (3\text{–}63)$$

Now suppose that we reverse the positions of the source and receiver. For this arrangement we see in Figure 3–10 that critically refracted waves produced at S" travel to the receiver at R" following a path along the refractor in the up-dip direction. To obtain an expression for the up-dip travel time t_u, we now substitute using $h_{1d} = h_{1u} - x\sin\alpha$ in Equation 3–60, which gives

$$t_u = \frac{x\cos\alpha}{V_2} + \frac{2h_{1u} - x\sin\alpha}{V_1}\cos i_{12} \qquad (3\text{–}64)$$

By setting $x = 0$, we find the up-dip intercept time

$$T_{1u} = \frac{2h_{1u}}{V_1}\cos i_{12} \qquad (3\text{–}65)$$

Rearranging and making substitutions similar to those used to get Equation 3–63, we obtain the expression

$$t_u = \frac{x}{V_1}\sin(i_{12} - \alpha) + T_{1u} \qquad (3\text{–}66)$$

Now let us compare Equations 3–63 and 3–66 with the result presented earlier in Equation 3–27 for a horizontal refractor. By setting the dip angle $\alpha = 0$, we see that because $\cos 0° = 1$, both Equations 3–63 and 3–66 reduce to

$$t_u = t_d = \frac{x}{V_2} + T_1$$

which is Equation 3–27 where $t_R = t_u = t_d$ and $k = T_1$.

Features of Reversed Travel Time Curves

Look again at the opposite travel time curves in Figure 3–10. We know from the condition of reciprocity that $t_R = t_D$. But we can also see from Equations 3–62 and 3–65 that the intercept times $T_{1u} > T_{1d}$ because $h_{1u} > h_{1d}$. We know that the line on the travel time curve extending from T_{1d} to t_D shows the alignment of travel times for waves refracted in the down-dip direction. The slope of this line is $1/V_d$ where V_d is the apparent speed of these waves. Similarly, the apparent speed V_u indicated by waves refracted in the up-dip direction is found from the reciprocal of the slope of the line extending from T_{1u} to t_R. It is clear from the facts that $t_D = t_R$ but $T_{1u} > T_{1d}$, that $V_u > V_d$. We can verify this result by means of Equations 3–63 and 3–66. Arranged in the following way,

$$t_d = x\left(\frac{1}{V_d}\right) + T_{1d} \quad \text{and} \quad t_u = x\left(\frac{1}{V_u}\right) + T_{1u}$$

we can recognize that both expressions have the familiar form of equations for straight lines that have slopes:

$$\frac{1}{V_d} = \frac{\sin(i_{12} + \alpha)}{V_1} \quad \text{and} \quad \frac{1}{V_u} = \frac{\sin(i_{12} - \alpha)}{V_1}$$

Inverting these expressions gives

$$V_d = \frac{V_1}{\sin(i_{12} + \alpha)} \qquad (3\text{–}67)$$

and

$$V_u = \frac{V_1}{\sin(i_{12} - \alpha)} \qquad (3\text{–}68)$$

Again, it is clear that $V_u > V_d$.

These relationships are also evident from inspection of Figure 3–9b. Compare the paths of down-dip and up-dip refracted waves reaching receiver R_6 which is midway between S_1 and S_2. The down-dip path is altogether shorter but has a longer portion along the refractor where velocity is higher compared with the up-dip path. Therefore, it is obvious that at R_6 we should find $t_u > t_d$. A line extending through t_R and t_u clearly has a larger intercept and a smaller slope than a line extending through t_D and t_d. This implies that $T_{1u} > T_{1d}$ and $V_u > V_d$.

We should also point out that the direct waves are not affected by the dipping refractor. Therefore, the straight lines on opposite travel time curves that represent direct waves should have identical slopes of $1/V_1$.

Suppose that we have just completed a reversed refraction survey and that we have plotted travel times read from the seismograms and drawn straight lines through the alignments of points. Simply by inspecting the newly prepared travel time curves, we can reach one or another of the following conclusions.

1. If corresponding straight lines on opposite travel time curves have identical slopes, we have a horizontal refractor.
2. If corresponding straight lines on opposite travel time curves have different slopes, we have a dipping refractor.
3. The refractor dips downward toward the source at which the intercept time is largest.
4. The refractor dips downward toward the source for which the crossing distance is greatest.

Calculating Velocity, Thickness, and Dip

By means of a reversed refraction survey, we can find V_1 from the slope of the direct wave line. But how do we find the velocity V_2 in the material below the refractor? We begin with a trigonometric inversion of Equations 3–67 and 3–68, which gives us

$$i_{12} + \alpha = \arc \sin \frac{V_1}{V_d} \quad \text{and}$$

$$i_{12} - \alpha = \arc \sin \frac{V_1}{V_u} \qquad (3\text{–}69)$$

We can add these expressions to get the critical angle

$$i_{12} = \frac{1}{2}\left(\arc \sin \frac{V_1}{V_d} + \arc \sin \frac{V_1}{V_u}\right) \qquad (3\text{–}70)$$

and then from Equation 3–1 we get

$$V_2 = \frac{V_1}{\sin i_{12}}$$

By trigonometric inversion of this result, we have

$$i_{12} = \arc \sin \frac{V_1}{V_2}$$

which can be used in Equation 3–70 to get

$$\arc \sin \frac{V_1}{V_2} = \frac{1}{2}\left(\arc \sin \frac{V_1}{V_d} + \arc \sin \frac{V_1}{V_u}\right)$$

For small angles we know that $\sin \alpha \cong \alpha$. For this case we can obtain the following approximation,

$$\frac{V_1}{V_2} \cong \frac{1}{2}\left(\frac{V_1}{V_d} + \frac{V_1}{V_u}\right)$$

or

$$V_2 \cong 2\left(\frac{V_d V_u}{V_d + V_u}\right) \qquad (3\text{–}71)$$

Now, by rearranging Equations 3–62 and 3–65, we find

$$h_{1d} = \frac{V_1 T_{1d}}{2 \cos i_{12}} \qquad (3\text{–}72)$$

and

$$h_{1u} = \frac{V_1 T_{1u}}{2 \cos i_{12}} \qquad (3\text{–}73)$$

where the $\cos i_{12}$ can be found from Equation 3–4.

By subtracting the expressions (3–69), we obtain the dip of the refractor

$$\alpha = \frac{1}{2}\left(\arc \sin \frac{V_1}{V_d} - \arc \sin \frac{V_1}{V_u}\right) \qquad (3\text{–}74)$$

Finally, we observe in Figure 3–10 that h_{1d} and h_{1u} are distances to the nearest points on the refractor beneath S′ and R′, respectively. The vertical depths to the refractor are

$$z_{1d} = h_{1d}/\cos \alpha \qquad (3\text{–}75)$$

and

$$z_{1u} = h_{1u}/\cos \alpha \qquad (3\text{–}76)$$

This completes the analysis of how to measure the depth and inclination of a single sloping refractor. The reasoning is similar to that introduced in our study of a horizontal refractor, but the procedure is necessarily more complicated. It should be obvious that this kind of analysis can be extended to a structure with multiple dipping refractors, but we will not discuss that topic in this book. For an analysis of multiple dipping layers, see *Exploration Seismology*, Volume 1, by Sheriff and Geldart (1982).

Application

The method for analyzing a single dipping refractor was applied in a reversed refraction survey in central Virginia. The purpose of the survey was to find the depth and inclination of the bedrock surface beneath the weathered zone. The geophones were placed in a line 140 meters long, and two 2-kg explosive charges were detonated at both ends of this line. First arrival times read from the direct and reverse seismograms were plotted, and straight lines were drawn through the aligned points. The results are shown in Figure 3–11.

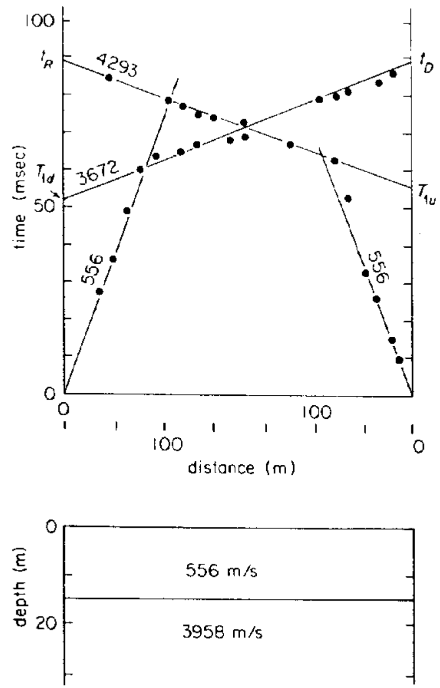


Figure 3–11
Results of a reversed seismic refraction survey in central Virginia. (From M. S. Bahorich, C. Coruh, E. S. Robinson, and J. K. Costain, Geophysics, v. 47, p. 1543, 1982.) The intercept times are T_{1d} = 0.052 and T_{1u} = 0.056 and $t_D = t_R$. Using the apparent up-dip and down-dip velocities, we can determine a true velocity of 3958 m/s for the second layer, which requires a critical angle of 8.075 degrees and a dip angle of 0.6 degree. The refractor depths z_{1d} = 14.6 m and z_{1u} = 15.7 m.