Homework 11: Heat Flow and Continental Geotherms

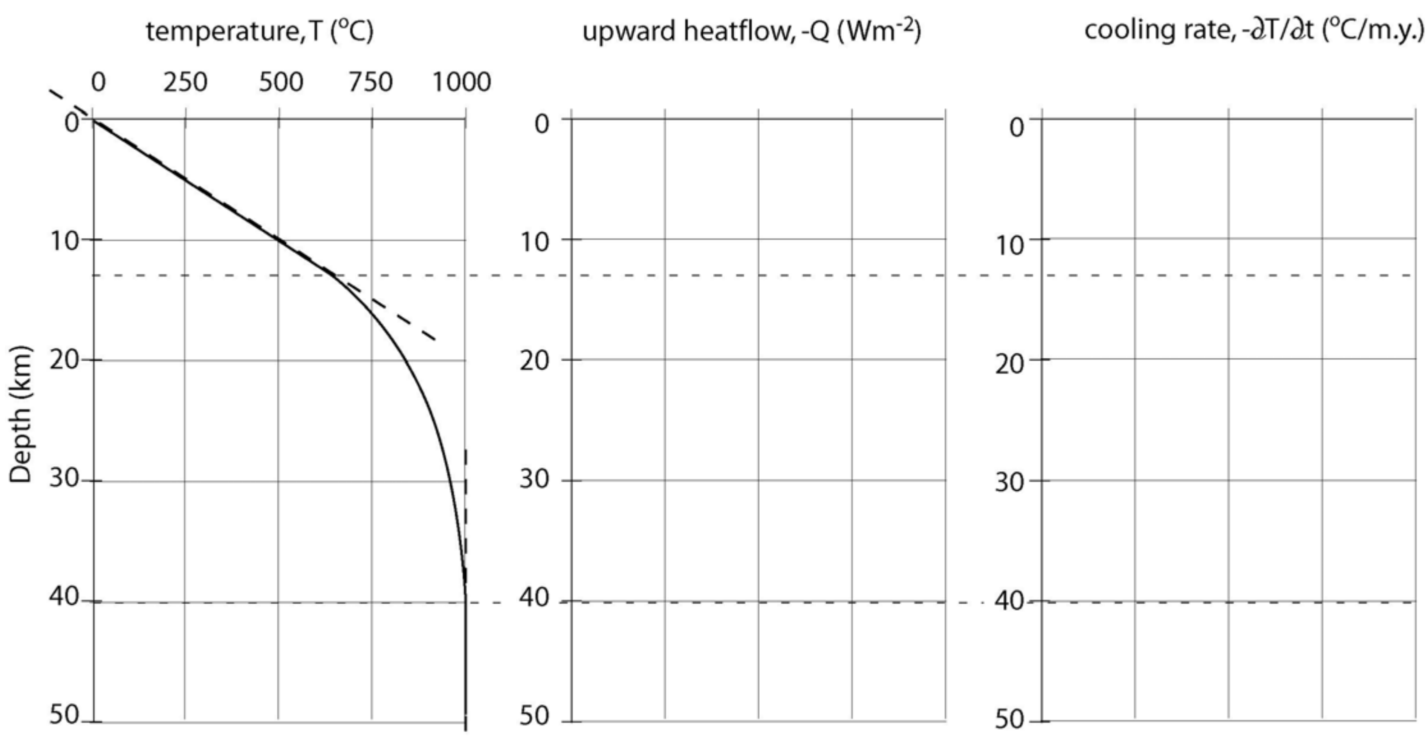
Part A due Wed. 4/22; Part B due Fri. 4/24

<u>Part A: Heat Flow</u> Based on Fowler Handout, pp. 219-226 (due for discussion Wed. 4/22)

1) Fourier's Law of conduction states that heat flows down the temperature gradient, $\mathbf{Q} = -k\vec{\nabla} T$. The plot on the left shows a temperature profile at an instant in time in the earth. In this case, temperature only varies with depth such that $\vec{\nabla} T = \partial T/\partial z\,\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector pointing <u>down into the earth</u>. Thus, the heat flow is 1-D, i.e., $\mathbf{Q} = -\partial T/\partial z\,\hat{\mathbf{z}}$, which means that <u>+|**Q**| is positive</u> when heat is flowing <u>down</u> (+$\hat{\mathbf{z}}$ direction) into the earth and <u>-|**Q**| is positive</u> when heat is flowing <u>up</u> (-$\hat{\mathbf{z}}$ direction) out of the earth. In the middle diagram, sketch <u>the upward</u> heat flow (i.e., -|**Q**|) as a function of depth. Use a thermal conductivity of k = 2 W m^{-1}°C^{-1} to estimate magnitudes and to thus label your horizontal axis accordingly.

2) Equations 7.5-7.6 show that the amount of heat leaving an infinitesimal volume of rock by conduction is proportional to the divergence of **Q**. In 3-D, the divergence is $\vec{\nabla}\bullet\mathbf{Q} = \left(\partial Q_x/\partial x + \partial Q_y/\partial y + \partial Q_z/\partial z\right)$ but, as described for Eq. 7.5-7.6, if **Q** is only in the $\hat{\mathbf{z}}$ direction (i.e. $\mathbf{Q} = Q_z\hat{\mathbf{z}}$) and only varies with z, then $\vec{\nabla}\bullet\mathbf{Q} = \partial Q_z/\partial z$. Without radioactive heating (A = 0), the rate at which the rock cools (i.e., the negative rate the rock heats up) is controlled only by the amount of <u>heat lost</u> by conduction, i.e., $-c_p\rho\,\partial T/\partial t = \vec{\nabla}\bullet\mathbf{Q} = \partial Q_z/\partial z$ (see Eq. 7.10). This equation describes how the temperature profile is changing with time at the instant the temperature in the leftmost plot is measured. In the right diagram, sketch <u>cooling rate</u> $-\partial T/\partial t$ as a function of depth. Assume $c_p\rho = (10^3\text{J kg}^{-1}\,°\text{C}^{-1})(3000\text{ kg m}^{-3})$ to estimate magnitudes and to label the horizontal axes.

3) Based on your results for problem 2, sketch a new temperature profile after a time of ~1 m.y. has passed in the left diagram.



<u>Part B: Continental geotherms</u> Fowler pp. 226-230, Due Friday 4/24

Surface heat flow at a given location on the Colorado Plateau is measured to be Q_0 = 90 mW/m^2. Various lines of evidence indicate that at a depth of z = 35 km, the temperature of the crust was recently $T(z{=}35)$ ~ 800°C. The thermal conductivity is 3 W/(m°C), and the surface temperature averages to be 0°C. Calculate and plot the equilibrium geotherm (i.e., T versus depth, z) by solving the one-dimensional, steady-state heat flow equation subject to the above boundary conditions.

4) Plot the equilibrium geotherm.

5) Estimate the average rate of heat production A in the crust beneath the above location.

6) At what rate is heat flowing from the underlying mantle into the crust?, i.e., what is $Q(z{=}35)$ km? Briefly explain why the difference between Q_0 and $Q(z{=}35)$ does (or does not) make sense given your estimate of A.